DELHI COLLEGE OF ENGINEERING

LIBRARY
Kashmiri Gate, Delhi-110006

Accession No. 36170

Class No. 621.3

Book No. TEA

DELHI COLLEGE OF ENGINEERING
Kashmiri Gate, Delhi-110006
LIBRARY

DATE DUE

For each day's delay after the due date a fine of 10 Paise per Vol. shall be charged for the first week, and 50 Paise per Vol. per day for subsequent days.

Borrower's No.	Date Due	Borrower's No.	Date Due

THE TECHNICAL COLLEGE SERIES

General Editor: W. E. FISHER, O.B.E., D.Sc.

(Formerly Principal, Wolverhampton and Staffordshire College of Technology.
First Director of the National Foundry College)

ELECTRO-TECHNOLOGY

VOLUME I

THE TECHNICAL COLLEGE SERIES

General Editor: W. E. FISHER, O.B.E., D.Sc.
(Formerly Principal, Wolverhampton and Staffordshire College of Technology)

A Selection of the Books in this Series:

NATIONAL CERTIFICATE MATHEMATICS
(In three volumes)
P. Abbott, B.A.; C. E. Kerridge, B.Sc.;
H. Marshall, B.Sc.; G. E. Mahon, B.Sc.

EXAMPLES IN PRACTICAL MATHEMATICS
FOR NATIONAL CERTIFICATE
C. C. T. Baker, B.Sc.

MECHANICAL ENGINEERING
SCIENCE FOR NATIONAL CERTIFICATE
(In two volumes)
J. D. Walker, B.Sc.(Eng.), A.M.I.Mech E.

ELECTRO-TECHNOLOGY FOR NATIONAL CERTIFICATE
(Volume I)
H. Teasdale, B.Sc., M.Ed., A.M.I.P.E.
E. C. Walton, B.Eng., Ph.D., M.I.E.E.
(Volumes II and III)
H. Buckingham, M.Sc., Ph.D.
E. M. Price, M.Sc.

WORKSHOP ENGINEERING
CALCULATIONS AND TECHNICAL SCIENCE
(In two volumes)
J. T. Stoney, B.Sc.

ADVANCED NATIONAL CERTIFICATE MATHEMATICS
(Volumes I and II)
J. Pedoe, M.A.(Cantab.), B.Sc.(Lond.)

HIGHER NATIONAL CERTIFICATE WORKSHOP TECHNOLOGY
T. Nuttall, A.M.I.Mech.E., A.M.I.P.E.

PRACTICAL MATHEMATICS
(In two volumes)
C. C. T. Baker, B.Sc.

MAGNETISM AND ELECTRICITY
(In two volumes)
J. M. R. Sutton, B.Sc.

APPLIED PHYSICS
(Volumes I and II)
W. G. Darvell, M.Sc., F.Inst.P.

APPLIED MECHANICS
J. D. Walker, B.Sc.(Eng.), A.M.I.Mech.E.

THE TECHNICAL COLLEGE SERIES

LECTRO-TECHNOLOGY

FOR NATIONAL CERTIFICATE COURSES

VOL. I

BY

H. TEASDALE
B.Sc., M.Ed., A.M.I.P.E.
*(Sometime Head of Department of Technology and Science of
The Wakefield Technical College.)*

AND

E. C. WALTON
B.Eng., Ph.D., M.I.E.E.
*(Head of Department of Electrical Engineering and Physics
at The College of Technology, Leeds)*

THE ENGLISH UNIVERSITIES PRESS LTD
102 NEWGATE STREET
LONDON, E.C.1

First printed 1947
New Edition 1957
Reprinted 1958

ALL RIGHTS RESERVED

Printed in Great Britain for the English Universities Press, Limited,
by Richard Clay and Company, Ltd., Bungay, Suffolk

GENERAL EDITOR'S FOREWORD

The Technical College Series today includes many books which are outstanding in their particular fields, and it is the aim of the publishers to maintain and develop the worthy tradition of the Series while meeting in full the increasing needs of technical and scientific education.

An outstanding contribution of the technical colleges to education has been the system of National Certificates under which the Ministry of Education and the colleges work in association with leading professional institutions. The system has progressed from its early pre-occupation with engineering until the schemes now cover practically the whole field of higher technology and applied science. The major engineering institutions, the Royal Institute of Chemistry, the Institute of Physics, the Institution of Metallurgists are all associated with National Certificate schemes. There are National Certificates in Building and in Commerce, with each of which a group of professional institutions is associated. Though the pattern of National Certificate Courses was originally dictated by the needs nd limitations of the evening student, the system of endorsements tainable by further study has now brought about the result that se courses have been extended to meet the full requirements of ctice in the subjects with which they deal. During recent years system of part-time-day release of apprentices and learners has come common in all branches of industry as well as in the public rvices. This has effected something like a revolution in technical ducation; and in particular the treatment of National Certificate dies up to the standard already indicated has become much ader.

The books included in the Series will be planned to suit the uirements of three main groups: (i) the part-time and full-time dents working in technical colleges for professional qualifications university degrees; (ii) technologists, managers, and research kers in industry; (iii) teachers in technical colleges and elsere who require text-books of high standard, but broad enough treatment of their subjects to be readily adaptable to local roved schemes of study.

W. E. Fisher

PREFACE TO SECOND EDITION

SINCE the publication of the first edition of this book, the rationalised m.k.s. system of electrical and magnetic units has become more widely used in the teaching of electrical technology. The reasons for the adoption of this system of units by the teaching staffs of a large number of colleges are as follows. In 1950 the International Electrotechnical Commission recommended the rationalised m.k.s. system of units for general use, this recommendation being later endorsed by the Institution of Electrical Engineers in this country. As a result of these recommendations, the rationalised m.k.s. system received widespread publicity, and most members of electrical-engineering teaching staffs are now agreed that, in due course, this system will replace entirely the more cumbersome c.g.s. electromagnetic and electrostatic systems of units.

The authors consider, however, that students of electrical technology should be introduced, from the beginning, to both the c.g.s. and the m.k.s. systems of units. Such students will then be able to appreciate more readily the advantages of the rationalised m.k.s. system, and at a suitable stage in their studies may discard the c.g.s. systems in their general work.

In the second edition of this book, therefore, the c.g.s. and rationalised m.k.s. systems of units have been introduced, side by side. Since, in Volume II, the treatment of the subject is to be exclusively in terms of the rationalised m.k.s. system, a brief outline of the c.g.s. electrostatic system of units has been added to Chapter IX of Volume I. The numerical exercises at the end of certain chapters have also been augmented by the addition of problems involving the use of rationalised m.k.s. units.

July 1956.

PREFACE TO FIRST EDITION

THIS work is intended to meet the need for a comprehensive and up-to-date text-book in Electrical Technology suitable for students taking electrical or mechanical engineering courses in Technical

Colleges. It is divided into three volumes: these being arranged to correspond very roughly with the work normally done in the three years (S1, S2 and S3) of the Ordinary National Certificate Course in Electrical Engineering. The work also covers the S1, S2 and S3 examination syllabuses in Electro-technology (sometimes described as Electrical Engineering Science, Electro-technics or Electrical Engineering) drawn up by the Union of Lancashire and Cheshire Institutes, the Union of Educational Institutions and the Northern Counties Technical Examinations Council. The syllabus for the Joint Part I examination in Principles of Electricity conducted by the Institutions of Civil, Mechanical and Electrical Engineers is also adequately covered.

In preparing this text-book the authors have two main purposes. The primary aim is to introduce the fundamental *facts* of Electro-technology to students in such a manner that, even with no previous knowledge, they can obtain, without undue difficulty, a firm grasp of the framework of their subject. The secondary aim is to provide a more comprehensive treatment of Electrical Technology, chiefly by means of appendices to the various chapters. These appendices contain additional explanatory work of a fundamental nature, and are included for the benefit of the enquiring type of student and for those whose previous training in mathematics and science has been more liberal than usual. Whereas it is desirable that the student should thoroughly understand the reasons for the existence of the various phenomena, it is *imperative* that he should know what the phenomena are—he must know the facts of his subject. It is comparatively easy to learn the facts; it is not always so easy to appreciate the ideas involved in their explanation. For these reasons, where explanations are not considered absolutely essential, they are relegated to the appendices. An Addendum containing the proofs of important formulæ is provided at the end of this volume, these formulæ having been stated in the text in a rather arbitrary manner, since their proof involves a knowledge of the calculus. The authors are well aware that the majority of students reading this volume for the first time will not possess a knowledge of the calculus, but the proofs are included for future reference.

The method of presenting the subject-matter in Vol. 1 is that which has been evolved by the authors after many years of trial and error. The student is first introduced to the elementary facts of current electricity, and is led on to a consideration of magnets and magnetic fields only when this work is needed to deal with the principles of the generator and motor. The more academic sections of magnetism and electrostatics, which have little importance in electrical engineering, have been omitted. The electron aspect of electrical phenomena has been introduced from the commencement, since the authors consider that owing to the growing importance of electronics in electrical engineering, the student should become accustomed, from the beginning of his studies, to visualising the behaviour of electric circuits in terms of electrons.

A certain number of *essential* laboratory experiments are included; these the authors consider should be carried out by all students, either individually or in small groups. Normally many additional useful experiments would be performed by students taking this subject in electrical engineering courses. In the descriptions of these essential experiments specific instructions as to instrument ranges, etc., have been avoided as far as possible—such details naturally depend on the actual equipment available in any laboratory.

A set of numerical exercises, of a practical nature wherever possible, is included at the end of each chapter, thus avoiding the necessity on the part of the teacher or student for the use of a separate book of problems. Certain exercises are arranged so that the student must search for the data required for their solution by referring to the tables given at the end of the book. The exercises thus have something in common with the practical calculations the engineer has to make. Exercises of a descriptive character are not included, as it is considered that these lose their value by reason of their proximity to the subject-matter to which they relate.

The recommendations of the British Standards Institution regarding terminology, symbols and abbreviations have been adopted as far as possible. A list of the British Standards dealing with materials and equipment to which direct reference has been made in Vol. 1 is included in Table X at the end of the book; it is considered desirable that the young electrical engineer should become acquainted with the existence and general scope of these British Standards at an early stage in his career.

CONTENTS

CHAP. PAGE

I. PRELIMINARY PHYSICAL SCIENCE 11

II. THE ELECTRIC CIRCUIT 22

III. RESISTANCE 42

IV. ELECTRICAL ENERGY AND ITS CONVERSION INTO HEAT 105

V. ELECTRO-CHEMISTRY 147

VI. ACCUMULATORS 178

VII. MAGNETIC FIELDS 211

VIII. THE MOTOR AND DYNAMO EFFECTS . . . 248

IX. ELECTROSTATICS 286

ADDENDUM 310

TABLES 319

INDEX 327

ACKNOWLEDGMENTS

The authors' and publishers' acknowledgments and thanks are due to the following firms, who have supplied information and lent photographs or blocks for illustrations.

The Bowen Instrument Co., Ltd., for Fig. 53; The British Electric Resistance Co., Ltd., for Figs. 22, 24, 26(*a*) and 27; The Chloride Electrical Storage Co., Ltd., for Figs. 102, 105, 110(*a*) and 112; The Chloride Electrical Storage Co., Ltd., and Lewis's, Ltd., for Fig. 109; The Copper Development Association for Fig. 23; Crompton Parkinson Ltd. for Fig. 159(*a*); The D.P. Battery Co., Ltd., for Figs. 101, 104, 106, 107, 108 and 110(*b*); The Dubilier Condenser Co. (1925), Ltd., for Fig. 183; Edison Storage Battery Distributors, Ltd., for Figs. 116(*b*) and (*c*), 117; The Electric Lamp Manufacturer's Association, Ltd., for Figs. 70, 72 and 73; The Electric Construction Co., Ltd., for Fig. 28; The Electric Furnace Co., Ltd., for Figs. 57, 64 and 65; The General Electric Co., Ltd., for Figs. 61, 75, 77, 78, 144 and 146; Griffin and Tatlock, Ltd., for Fig. 178; Isenthal and Co., Ltd., for Fig. 30; G. B.-Kalee, Ltd., and A. Kershaw and Sons, Ltd., for Fig. 69; Laurence, Scott and Electromotors, Ltd., for Fig. 158; Metropolitan Vickers Electrical Co., Ltd., for Figs. 59, 60 and 67; Nife Batteries, Ltd., for Figs. 116(*a*) and 118; Premier Electric Heaters, Ltd., for Fig. 58; W. G. Pye and Co., Ltd., for Figs. 25, 31, 33 and 143; Standard Telephones and Cables, Ltd., for Fig. 148, and the British Standards Institution for permission to include the data given in Table X.

The authors also wish to acknowledge their thanks to a colleague —Mr. A. Taylor—for valuable suggestions in connection with the manuscript of Chapters V and VI of Vol. 1.

CHAPTER I

PRELIMINARY PHYSICAL SCIENCE

1. The Nature of Matter

CHEMISTS have found that there are only 92 elementary materials; these they call **elements.** All other substances, no matter how complicated, are built up from a number of these elements. Thus copper, iron and carbon are elements; so are oxygen and nitrogen. But wood is more complicated, being built up of several elements, chiefly carbon, hydrogen and oxygen.

An element is composed of minute but identical particles known as **atoms.** Each element has its own particular type of atom; thus there are 92 different kinds of atoms. Some idea of the minute size of an atom can be gathered from the fact that there are roughly 1,400,000,000,000,000,000,000,000 atoms in a cubic inch of copper.

Although atoms are so small, they have a somewhat complicated structure. The hydrogen atom is the simplest, consisting of two particles only, a **proton** and an **electron.** The latter can be regarded as spinning round the former in an orbit like the earth around the sun. All other atoms are constructed entirely from protons, electrons and **neutrons.** An electron is the lightest particle in existence. It is actually *a particle of electricity*, said to be a unit electric charge. An electric current flowing along a wire consists of a stream of billions of electrons. A proton behaves in a manner opposite to an electron. When, for example, an electron tends to move one way, a proton would have a tendency to move in the opposite direction.

So, to differentiate between a proton and an electron we say that the proton is a unit positive charge and the electron a unit negative charge. It should be remembered, however, that the words positive and negative merely indicate this tendency to behave in opposite ways. Actually protons do not move as readily as electrons, since a proton is more than 1800 times as heavy as an electron. A neutron is really equivalent to a proton and an electron merged together, the resultant particle having no perceptible electric charge. This is due to the fact that the proton and electron have electric charges which are equal in size but opposite in nature. In the neutron the effects of these charges cancel each other out.

An atom consists almost entirely of empty space. If the hydrogen atom could be magnified until the proton and electron were as large as golf-balls the orbit of the atom would be as large as a golf-course more than 2 miles in diameter.

In all atoms but the hydrogen atom the core or **nucleus** consists not of a single proton but of a number of protons and neutrons. Spinning round the nucleus in orbits, like planets round the sun, are electrons. Helium is the next simplest atom to the hydrogen atom and possesses two orbital electrons, and two protons in the nucleus. The atom of uranium is the most complicated, having 92 electrons in various orbits and therefore a nucleus with a positive charge also of 92.

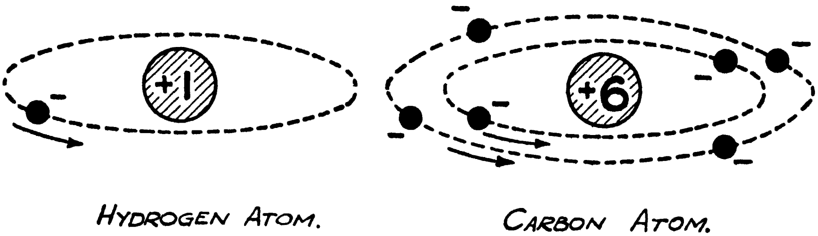


FIG. 1.—SIMPLE EXAMPLES OF ATOMIC STRUCTURE.

Thus we see that one atom differs from another primarily because of the different numbers of particles in the atom, the simplest atom having one orbital electron, the next simplest two and so on until the most complicated atom possesses 92. Further, there are exactly as many protons in the nucleus as there are electrons in orbits, so that as a whole the atom has no electrical property, the negative charges on the electrons cancelling the positive charges on the protons—see Fig. 1. Finally the more electrons and protons the atom possesses the greater also is the number of neutrons in the nucleus. The neutrons are not, however, of much importance in electrical phenomena, since they carry no net electrical charge.

The atoms of some elements such as copper, iron, helium and neon can exist alone, but in the case of oxygen, hydrogen and nitrogen they can exist only in pairs. The smallest group of atoms which can exist alone is known as a **molecule.** Thus a molecule is the smallest possible amount of any substance. Molecules may consist of one, two, three or more atoms. Thus, whereas a copper atom is monatomic, an oxygen atom is diatomic. In the case of elements all the atoms in a molecule are alike. With other substances, known as **compounds**—e.g., water and sulphuric acid—the molecules consist of atoms of different types. Thus the water molecule contains two atoms of hydrogen and one of oxygen, whereas the sulphuric acid molecule contains two hydrogen atoms, one sulphur atom and four oxygen atoms.

2. Systems of Units

In mechanical science all the quantities encountered may be expressed in terms of three fundamental units, those generally adopted being of **length, mass** and **time.** Using the British system, the fundamental units are the **foot, pound** and **second** respectively, and mechanical engineers in Britain usually employ a system based on these units.

In scientific work the **centimetre, gram** and **second** have been used as the three fundamental units for a very long time. As electrical science developed during the last century, two systems of units were devised, both being based on the centimetre, gram and second. For dealing with magnetic fields and electromagnetism the *c.g.s. electromagnetic system* of units was developed, and a separate system, the *c.g.s. electrostatic system*, was devised to suit the quantitative treatment of electrostatics. A third set of units, known as *practical units*, was found to be necessary when dealing with the practical applications of electrical science, these units being multiples or sub-multiples of the c.g.s. electromagnetic units.

More recently there has been a tendency to use, in electrical science, a system of units based on the **metre, kilogram,** and **second** (the m.k.s. system). In this volume the reader will be introduced to both the c.g.s. and the m.k.s. systems of units.

3. Mass, Weight, Force and Torque

The **mass** of an object is a measure of the amount of matter in it; that is, it is a measure of the number of protons, electrons and neutrons forming the object. Mass is measured in **pounds** or **grams** (**1 kilogram** = 1000 grams).

The **weight** of an object is a measure of the earth's **force** of gravitation on the object. The weight of a 1 pound mass is

1 pound weight and the weight of a 1 gram mass is **1 gram weight.**

Weight or gravitational force is only one type of force. Other important types of force met with in engineering are frictional forces, occurring when one surface slides over another, forces of tension, compression and shear existing in certain parts of structures or machinery, and electrical forces encountered in various types of electrical equipment. It is convenient to measure all forces in the same units—namely, in grams wt. or in pounds wt.

Force may be defined as *the product of mass and acceleration.* Thus, in the c.g.s. system, the **dyne** is that force required to give a mass of 1 gram an acceleration of 1 cm. per sec.^{2}. Since the acceleration due to gravity has a value in this country of 981 cm. per sec.^{2}, it follows that the earth's force of gravitation on a mass of 1 gram is 981 dynes. Hence a force of **1 gram wt. is equivalent to 981 dynes.**

In the m.k.s. system the unit of force is termed the **newton** and is that force required to give a mass of 1 kilogram an acceleration of 1 metre per sec.^{2}. Taking the acceleration due to gravity as 9·81 metres per sec.^{2}, it follows that a force of **1 kilogram wt. is equivalent to 9·81 newtons.** The relationship between the newton and the dyne may be derived as follows:

$$\begin{aligned} 1 \text{ newton} &= 1 \text{ kg.} \times 1 \text{ metre/sec.}^2 \\ &= 1000 \text{ grams} \times 100 \text{ cm./sec.}^2 \end{aligned}$$

or,
$$\mathbf{1\ newton = 10^5\ dynes.}$$

The student will find later that electrical forces are often employed to impart a rotary movement to a body. It is, therefore, necessary to examine the principles underlying this effect. When a body is capable of turning freely about an axis, the turning effect of a force about a point on this axis is referred to as the **turning moment,** or **torque,** the moment of a force about a point being defined as the product of the force and the *perpendicular* distance from the point to the line of application of the force.

Fig. 2 illustrates a cylindrical body capable of rotating freely about an axis through O and perpendicular to the page. The turning moment about O due to the force F lb. wt. is given by the product of the force and the distance OP—i.e., it is F lb. wt. × d ft. Using these units of force and distance—i.e., the lb. wt. and foot respectively—the turning moment or torque is in *lb.-ft.*, but it may be expressed alternatively in lb.-in., dyne-cm., newton-metres, etc.

A turning moment may be referred to as *clockwise* or *counter clockwise*, the former if it tends to cause rotation in the same direction as the hands of a clock and the latter if its tendency is the reverse. The force F lb. wt. in Fig. 2 gives rise to a clockwise turning moment about O, whilst the force Q lb. wt. gives rise to a counter clockwise turning moment.

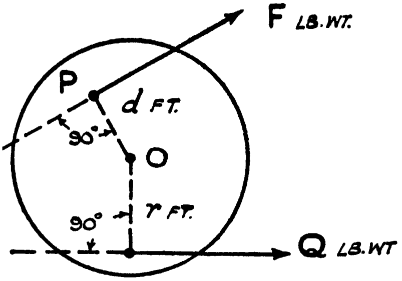


FIG. 2.—THE ROTATIONAL EFFECT OF A FORCE.

The **Principle of Moments** states that, if a body is at rest under the action of a system of forces acting in one plane, then the sum of the clockwise moments about any point in the plane is equal to the sum of the counter clockwise moments about the same point. Thus, in Fig. 2, if the cylindrical body is at rest under the influence of the forces F and Q lb. wt., then the clockwise moment about O due to the force F lb. wt. must be equal to the counter clockwise moment due to the force Q lb. wt., i.e., $F \times d = Q \times r$.

4. Work, Power and Energy

In engineering, **work** is said to be done whenever a force moves an object in any direction. It is necessary to apply a force to move an object because there are usually other forces opposing the motion. The work done when an object is moved a certain distance is calculated by multiplying the force applied by the distance the object moves *in the direction of the force.* Thus:—

Work done = Force applied × Distance moved by object

This formula gives the work done in **foot-pounds** if the force is measured in pounds weight and the distance in feet. If, on the other hand, the force is measured in dynes and the distance in centimetres, the product of dynes and centimetres will give the work done in units called **ergs.** The erg, which is the c.g.s. unit of work, is a very small unit. The m.k.s. unit of work, termed the **joule,** is of more convenient size than the erg, and is of the same order of magnitude as the foot-pound, 1 ft.-lb. being equivalent to 1·356 joules. The fact that **1 joule is equivalent to 10^7 ergs** is deduced as follows:

$$\begin{aligned} 1 \text{ joule} &= 1 \text{ newton} \times 1 \text{ metre} \\ &= 10^5 \text{ dynes} \times 100 \text{ cm.} \\ &= 10^7 \text{ ergs.} \end{aligned}$$

Power is *the rate at which work is done* and so can be measured in **ft.-lb. per second, ft.-lb. per minute, ergs per second** or **joules per second.** A power of 1 joule per sec. is a unit extensively used in electrical engineering, being called **1 watt.**

1 watt = 1 joule per sec.

All the above units of power are small, and so when dealing with the power of machines, larger units are used, namely, the **kilowatt** and the **horse-power** (written H.P.).

1 kilowatt = 1000 watts.
1 H.P. = 550 ft.-lb. per sec. = 33,000 ft.-lb. per min.
1 H.P. = 746 watts.

The horse-power is a unit of power rather in excess of the power at which a strong horse normally works. The value of 550 ft.-lb. per sec. was chosen as a result of experiments carried out by James Watt on strong dray horses, this figure being rather on the high side, since he wished to ensure that an engine stated to be of 5 H.P. could do work at least as rapidly as any five horses.

Energy is the capacity for doing work. Work is never done without energy being used. The quantity of energy used is measured by the amount of work done. Energy very often exists in a chemical form, as for example in the case of food, oil, petrol and coal. But there are also other forms of energy. Heat is a form of energy. Heat is usually produced incidentally when work is done. Certainly whenever friction is involved heat is generated. Moreover, heat may be converted into work, as in the steam engine and turbine. It is from the point of view of energy that electricity is so important in modern life. As will be seen later, energy may be conveyed hundreds of miles in a most convenient manner by means of an electric current.

5. Heat

Heat is usually measured in terms of one of the following three units:—

(a) **The Calorie.** This is the amount of heat required to raise the temperature of 1 gm. of water through 1 degree Centigrade (1 kilo-calorie = 1000 calories).

(b) **The British Thermal Unit (B.Th.U.).** This is the amount of heat required to raise the temperature of 1 lb. of water through 1 degree Fahrenheit. 100,000 British Thermal Units are called a **Therm.**

(c) **The Centigrade Heat Unit (C.H.U.).** This is the amount of heat required to raise the temperature of 1 lb. of water through 1 degree Centigrade.

1 B.Th.U. is equivalent to 778 ft.-lb. of energy.
1 B.Th.U. = 252 Calories = $\frac{5}{9}$ C.H.U.
$\therefore$ **1 C.H.U. = 1400 ft.-lb. of energy.**

If we wish to calculate the amount of heat supplied to a quantity of water we do so by means of the following simple formula:—

Heat supplied = Mass of water × Rise in temperature of the water

i.e., $H = m \times (\theta_2 - \theta_1)$, where θ_1 and θ_2° C. are the initial and final temperatures of the water respectively. θ (pronounced "theta") is the symbol usually employed in connection with temperature (see Table IX).

If, however, we wish to calculate the amount of heat supplied to some substance other than water the above simple formula must be modified thus:—

Heat supplied = Specific heat of substance × Mass of substance × Rise in temperature of substance.

i.e., $H = s \times m \times (\theta_2 - \theta_1)$.

The **specific heat** is a multiplying factor, usually considerably less than 1, which allows for the varying capacities for heat of different substances. The *specific heat of a substance is the number of units of heat necessary to raise the temperature of a unit mass of the substance through* 1 *degree*, i.e., it is the number of calories necessary to raise the temperature of 1 gm. of the substance through 1 degree Centigrade, this number being the same as the number of British Thermal Units which would be necessary to raise the temperature of 1 lb. of the substance through 1 degree Fahrenheit. The specific heats of water, copper and iron are 1·0, 0·09 and 0·10 respectively. Thus it requires 0·09 calorie to raise the temperature of 1 gm. of copper through 1 deg. C. and 0·09 B.Th.U. to raise the temperature of 1 lb. of copper through 1 deg. F. The specific heats of some of the materials used in electrical engineering are included in Table II.

In dealing with the supply of heat to an oven, kettle or other heating appliance a knowledge of the **thermal capacity** of the equipment is often desirable. The thermal capacity of a body is the amount of heat which must be supplied to raise the temperature of the body through 1 degree. If the body is made of a single material the thermal capacity of the body is equal to the product of the mass of the body and its specific heat.

The heat which must be supplied to raise the temperature of a body through a given range may be calculated from the formula:—

Heat supplied = Thermal capacity × Rise in temperature

6. Density and Specific Gravity

The **density** of a substance is the mass of a unit volume of that substance. Thus, the density of water is 1 gm. per c.c. or 62·4 lb. per cu. ft. Similarly, the density of aluminium is 2·63 gm. per c.c. or 164 lb. per cu. ft.

The **specific gravity** of a substance is the ratio of the density of that substance to the density of water. Thus the specific gravity of aluminium is equal to $\frac{2{\cdot}63 \text{ gm. per c.c.}}{1{\cdot}00 \text{ gm. per c.c.}} = 2{\cdot}63$. This figure may be also obtained by dividing the density of aluminium expressed in lb. per cu. ft. by the density of water expressed in lb. per cu. ft. Thus the specific gravity of aluminium is equal to $\frac{164 \text{ lb. per cu. ft.}}{62{\cdot}4 \text{ lb. per cu. ft.}} = 2{\cdot}63$.

Since the density of water is equal to 1 gm. per c.c. the specific gravity of a substance is numerically equal to its density when expressed in gm. per c.c. Whereas the numerical value of the density of a substance depends on the units used, specific gravity possesses no units. Thus there is no ambiguity in using specific gravity, for only one numerical value for the specific gravity of any given substance is possible. For this reason it is usual when dealing with liquids of variable density, e.g., accumulator acid, to discuss the matter in terms of specific gravity rather than density. Table II gives the specific gravities of some of the more common substances.

Exercises on Chapter I

Any additional data required for the solution of these exercises may be obtained from the tables at the end of this book.

1. A straight bar PQR 32 in. long is balanced at the mid-point Q and a load of 15 lb. is attached to the bar at P. (a) What is the moment of the force due to this load about Q? (b) What load must be attached to a point 10 in. from R to maintain the balance? (c) Where must a load of 20 lb. be placed to balance the bar?

[(a) 240 lb.-in.; 20 lb.-ft. (b) 40 lb. (c) 4 in. from R.]

2. A rectangular loop of wire, 6 cm. × 4 cm., is balanced about a central axis parallel to the longer sides. Calculate the turning moment, in dyne-cm., when a force (normal to the wire) of 2·5 gm. wt. is exerted on one of the longer sides in a direction (a) perpendicular to the plane of the loop and (b) making an angle of 45° with the plane of the loop. Express the results also in newton-metres.

[(a) 4905 dyne-cm. or $4{\cdot}905 \times 10^{-4}$ newton-metres; (b) 3468 dyne-cm. or $3{\cdot}468 \times 10^{-4}$ newton-metres.]

3. A motor is developing 45 horse-power. (a) How many ft.-lb. of work does it do every minute? (b) What is the horse-power output of the motor when it does $1{\cdot}2 \times 10^6$ ft.-lb. of work per minute?

[(a) $1{\cdot}485 \times 10^6$ ft.-lb. (b) 36·36 H.P.]

4. A crane lifts a load of 6000 kg. through a height of 7·5 metres in 30 sec. (a) How much work, in joules, is done on the load? (b) What is the output of the hoisting motor (i) in watts and (ii) in horse-power, if the efficiency of the gearing, etc., is 75%?

[(a) 441,450 joules. (b) (i) 19,620 watts, (ii) 26·3 H.P.]

5. The net load on an electric lift is 2 tons and the efficiency of the hoisting mechanism may be taken as 85%. If the driving motor develops 80 H.P., how long will the lift take to ascend a distance of 100 ft.?

[12 sec.]

6. (a) How much work is done by a force of 0·5 gm. wt. when its point of application is moved through a distance of 25 cm.? (b) What force, in newtons, would do 1500 joules of work in moving an object through 7·5 metres? (c) If the operation referred to in (b) takes 0·3 sec., what is the horse-power?

[(a) 12,262 ergs. (b) 200 newtons. (c) 6·70 H.P.]

7. A circular shaft 20 ft. in diameter and 500 ft. deep is half full of water. Determine (a) the work done in emptying the shaft, (b) the H.P. of the motor driving the pump if the efficiency of the latter is 65% and the operation is completed in 12 hours.
[(a) $18{\cdot}41 \times 10^8$ ft.-lb. (b) 119·2 H.P.]

8. A man weighing 12 stones walks a distance of 200 yds. up a hill of gradient 1 in 12. Find the amount of work he does against gravity.
[8400 ft.-lb.]

9. A cyclist weighing, with his machine, 75 kg., rides up a gradient of 1 in 15 at 10 km. per hour. How much work, expressed in joules, does he do per second if the frictional resistance is equal to 5 kg. wt.?
[272 joules.]

10. A loaded tramcar weighing 20 tons travels up an incline of 1 in 10 at a steady speed of 5 m.p.h. If the frictional resistances are equivalent to 15 lb. wt. per ton, find the H.P. output of the driving motors, the efficiency of the gearing being taken as 75%.
[84·94 H.P.]

11. (a) How many gram-calories of heat are required to raise the temperature of 100 gm. of water from 12° C. to 60° C.? (b) How many B.Th.U. and C.H.U. does this result represent? (c) If this quantity of water gives out 6 C.H.U. in cooling from 60° C., find the value to which the temperature has fallen.
[(a) 4800 gm.-cal. (b) 19·04 B.Th.U., 10·58 C.H.U. (c) 32·8° C.]

12. A 20-gallon water heater is to have the temperature of its contents raised from 50° F. to 200° F. (a) How many B.Th.U. are required? (b) If this operation is to be completed in 1½ hours, find the power output in watts of the heating device.
[(a) 30,000 B.Th.U. (b) 5860 watts.]

13. The copper container in a water heater weighs 25 lb. (a) How many B.Th.U. are required to raise its temperature from 40° F. to 210° F.? (b) How many gallons of water would have the same heat capacity as the container?
[(a) 399·4 B.Th.U. (b) 0·235 gall.]

14. A 100-gallon fuel oil tank weighs 120 lb. when empty and is made from steel having a specific heat of 0·110. Find the number of B.Th.U. required to raise the temperature of the tank and its full complement of oil from 45° F. to 90° F., the specific heat of the oil being 0·52 and its specific gravity 0·90. What is the thermal capacity of the tank (a) when empty, (b) when full of oil?

[21,650 B.Th.U. (a) 13·2 B.Th.U. per lb. (b) 481·2 B.Th.U. per lb.]

15. The bearing of an electric motor absorbs 0·35 H.P. Assuming that all this power is converted into heat, find the number of B.Th.U. generated in the bearing per minute.

[14·85 B.Th.U.]

16. The cooling water for an oil engine flows at the rate of 2·8 gallons per minute. The temperature rise of the water is 35° C. Calculate the H.P. given to the water as heat.

[41·6 H.P.]

17. A pound of coal has a calorific value of 5400 C.H.U. How many ft.-lb. of work is this pound of coal capable of doing when used in a steam plant of overall efficiency 20%?

[$1{\cdot}512 \times 10^6$ ft.-lb.]

18. An oil engine consumes 30 lb. of oil per hour, this fuel having a calorific value of 18,000 B.Th.U. per lb. The overall efficiency of the engine is 28%. Determine (a) the useful work done (in ft.-lb.) per lb. of oil used; (b) the H.P. output of the engine.

[(a) $3{\cdot}922 \times 10^6$ ft.-lb.; (b) 59·43 H.P.]

CHAPTER II

THE ELECTRIC CIRCUIT

7. The Nature of an Electric Current

CERTAIN atoms very easily lose an electron, which then drifts away from the rest of the atom. Such electrons are termed **free electrons.** The electron, as we have seen, is a negative charge. An atom which has thus lost an electron has lost a negative charge. This is equivalent to the atom gaining a positive charge. So an atom which has lost an electron is termed a **positive ion,** and this process whereby an atom spontaneously splits up into a negative electron and a positive ion is known as **ionisation.**

If the atoms of a substance behave in this manner, then, at any one instant, there will exist a number of positive ions and, wandering aimlessly from atom to atom, an equal number of free electrons. In addition, there will exist many whole atoms. There is always a tendency for two particles, possessing respectively positive and negative charges, to attract each other. So, when an atom ionises, the free electron drifts away only to become attached to another ion. This process occurs on a very large scale. In a very small piece of wire, for example, billions of electrons will escape from their atoms each second only to become temporarily attached to other atoms. The number of free electrons remains fairly constant, the number of atoms re-forming each second being equal to the number of atoms ionising each second.

Unless an electric current is flowing, the free electrons move at random, as many drifting in one direction as another. This aimless wandering of free electrons is occurring continuously in all pieces of metal. Metals, particularly silver, copper and aluminium, possess large numbers of free electrons, and are called **conductors.** Carbon and certain liquids are also fairly good conductors. Substances like ebonite, dry paper, rubber, glass, porcelain, mica and oil are deficient in free electrons, and are called **non-conductors** or **insulators.** When a current flows through a piece of metal, although there are still electrons moving in every possible direction, there is a general drift in one direction.

If one visits a busy railway terminus, people appear to be swarming from north, south, east and west. When a train arrives at the station, although there are travellers still hurrying from all quarters, there is a definite drift out of the station; more people are moving in one particular direction than in any other. The electrons in a piece of metal behave in a very similar manner when a current is flowing.

8. Electromotive Force

Two conditions are essential before a current can flow. Firstly, there must be a complete circuit of conducting materials, i.e., we must have a closed ring of substances which conduct electricity.

The second requirement is a source of electric pressure to drive the electrons round the circuit. Just as a water-pump will drive a current of water through a system of water-pipes by creating a pressure difference, so a battery of cells (such as the dry cells used in an electric torch, or the lead-acid cells in a motor-car battery) is capable of creating an electrical pressure difference which will cause a drift of electrons if there is a closed circuit of conductors. Just as the pump does not manufacture the water, so the cells do not create electricity. The electricity is in the conducting materials forming the closed circuit. In fact, electricity is the stuff of which everything is made. The problem is simply to get it moving and, as we have seen, this can be done if we use materials possessing free electrons and some equipment which can create an electric pressure.

The electric pressure responsible for the movement of electrons round a circuit is known as **electromotive force,** often referred to as **e.m.f.** for brevity. When water is flowing along a pipe from, say, A to B, there is said to be a pressure difference between these two points, the pressure being greater at A than at B. Similarly, when an electric current is flowing along a wire from, say, C to D, there is an electrical pressure difference between C and D. This electrical pressure difference is known as **potential difference** and is usually referred to as the **p.d.** between the two points. It should be carefully noted that *p.d.* and *e.m.f.* have not the same meaning, the exact difference being explained in Section 10.

A cell possesses an e.m.f. in virtue of the changes taking place within it, and hence the term e.m.f. refers to the electric pressure generated *inside* a cell. A cell possesses two terminals and, if a conducting path is connected across these terminals, thus forming a closed circuit, the e.m.f. generated in the interior of the cell causes electrons to drift from one terminal, called the negative, through the circuit, to the other terminal, known as the positive (see Fig. 3). The reason for this electron flow is that whenever an e.m.f. exists a greater crowding of electrons occurs on the negative terminal than on the positive. So the electrons drift round from the negative terminal, where there is an excess of electrons, to the positive, where there is a deficiency of them.

The negative terminal of a cell is represented diagrammatically by a short, thick line, and the positive terminal by a comparatively long, thin line (see Fig. 3). The actual positive terminal is usually coloured red, and the negative blue or black.

Before it was known that an electric current is due to the drift of negative electrons, an electric current came to be regarded as a movement of positive electricity from the positive to the negative terminal. *Although this is incorrect, it has become conventional to regard the current as flowing from the positive terminal to the negative through that part of the circuit external to the cell.* Actually positive charges are unable to drift round the circuit because they are too heavy; but in order to conform to the practice adopted by all engineers, we shall regard an electric current as consisting of positive charges flowing from the positive to the negative terminal. Although it is unfortunate that this pretence must be retained, there is no real harm in it, since if a positive charge did travel from the positive to the negative terminal it would produce exactly the same effects as an equal negative charge travelling in the reverse direction.

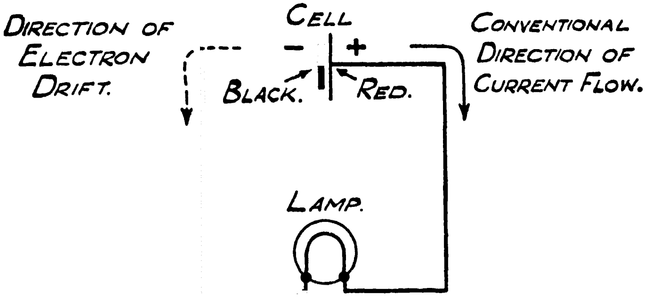

Direction of Electron Drift.
Cell
−
+
Black.
Red.
Conventional Direction of Current Flow.
Lamp.

Fig. 3.—The Simple Electric Circuit.

9. Methods of Connecting Up Cells, Ammeters and Voltmeters

If an e.m.f. is required greater than that produced by a single cell, a number of cells may be connected **in series,** i.e., one after the other, as in Fig. 4a. With this arrangement the positive terminal of the first cell becomes the positive terminal of the battery of cells. Its negative terminal is connected to the positive terminal of the second cell, the negative of the second cell being connected to the positive of the third and so on. This leaves the negative terminal of the last cell free to act as the negative terminal of the battery. The e.m.f. of the battery is the sum of the e.m.f's. of the individual cells. For convenience in drawing circuit diagrams the connecting wires between the individual cells are usually omitted and the battery is represented as in Fig. 4b.

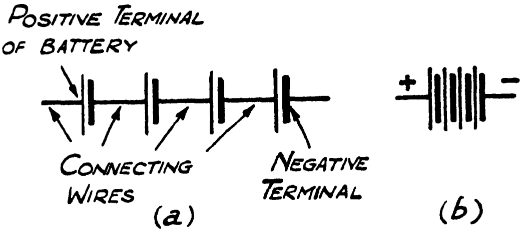


FIG. 4.—CELLS CONNECTED IN SERIES.

The **strength** of an electric current is the **rate of electron drift** round a circuit. But the number of electrons drifting each second past a point in a circuit is usually so huge that a larger unit than the electron must be used for convenience. So a **quantity of electricity** is usually measured in **coulombs** instead of electrons. A charge of 1 coulomb is actually equivalent to that of $6{\cdot}289 \times 10^{18}$ electrons (there is no necessity to remember this colossal number!), but in accordance with the conventional direction of current flow, we regard a coulomb as a unit *positive* charge. **Current strength** is therefore normally measured in **coulombs per second,** or **amperes.** Thus, if 5 coulombs per second flow past a point on a wire, we say that there is an electric current of 5 amperes flowing through the wire.

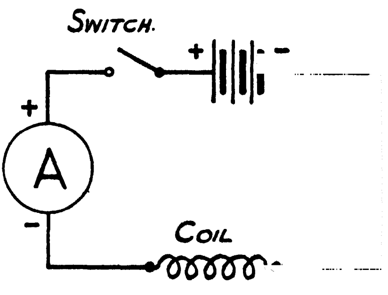


FIG. 5.—THE SERIES CONNECTION OF AN AMMETER.

The strength of the current, in amperes, can be measured very conveniently by means of an **ammeter.** It is unnecessary at this stage to know just how an ammeter works, but it is very important to know how to use such an instrument. An ammeter must always be connected **in series** with the other components of the circuit, as illustrated in Fig. 5, care being taken to connect the ammeter terminal marked POSITIVE to the positive side of the battery. Incidentally, no circuit should be connected up without including a switch, also connected in series with the other components. A switch is merely a device for stopping the flow of current by removing a conducting link from what was originally a complete circuit of conducting materials. It is also essential to include in the circuit some other equipment, such as a coil or lamp; otherwise too large a current would flow and the ammeter be damaged.

The p.d. between any two points is normally measured in volts by means of a **voltmeter.** When wiring up a circuit the student should always leave the voltmeter connections until the last, first completing and checking the main circuit connections. The voltmeter should then be connected across the two points between which a knowledge of the p.d. is required. The instrument is said to be connected **in parallel** with the part of the circuit between the two points in question. For example, in Fig. 6, voltmeter V_1 is connected in parallel with the lamp and measures the p.d. between points L and M. Such a p.d. is often referred to as the **voltage drop** across that particular part of the circuit. Voltmeter V_2, on the other hand, measures the voltage drop in the whole of the circuit external to the battery. Once again it is essential to connect the positive terminal of the instrument to the positive terminal of the battery; otherwise the needle of the instrument will attempt to move backwards.

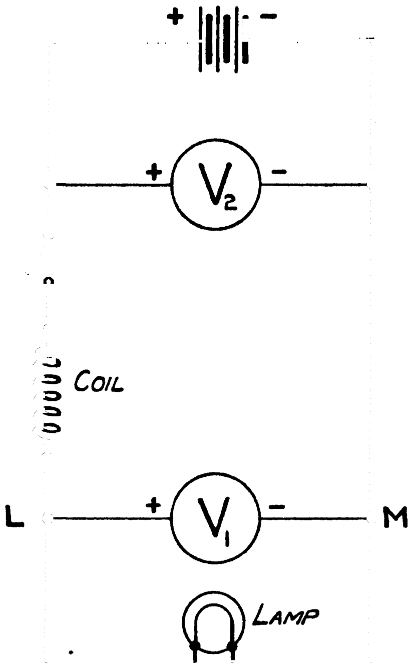


FIG. 6.—THE PARALLEL CONNECTION OF A VOLTMETER.

When a voltmeter is placed across a battery disconnected from all other components, as in Fig. 7, the reading of the instrument gives the p.d. between the battery terminals. This p.d. is a measure of the e.m.f. of the battery. Just why a voltmeter used in this manner indicates the e.m.f. of the battery is explained in Section 25 of Chapter III.

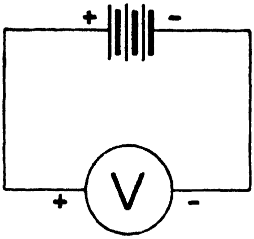


FIG. 7.—THE USE OF A VOLTMETER FOR MEASURING E.M.F.

10. The Electric Current as a Conveyor of Energy

The reason for the importance of electricity in modern life is that an electric charge, such as an electron or a coulomb, is a carrier of energy. We have already discussed one aspect of e.m.f., but the meaning of e.m.f. so far conveyed to the reader is far from complete. Not until the student has a clear knowledge of the relationship between e.m.f. and energy can he be satisfied that he understands the true significance of e.m.f. In order to visualise more perfectly the nature of both e.m.f. and p.d., the following analogy may be helpful.

Imagine a large amount of coal being transferred from a colliery to a power-station in self-propelled trucks, as illustrated in Fig. 8. Coal is energy in a chemical form. At the power-station this chemical energy is transformed into heat energy by burning the coal on the grates of boilers. Thus, energy is being transferred from one place to another at which it is used.

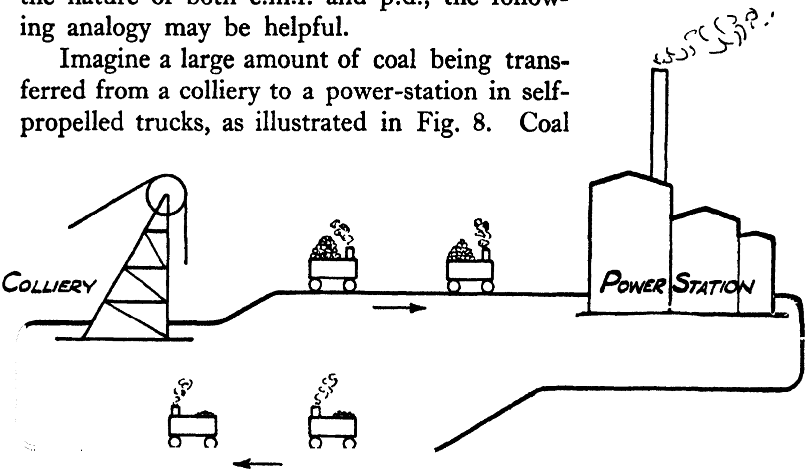


FIG. 8.—THE ELECTRIC CIRCUIT ANALOGY OF SECTION 10.

Imagine a continuous stream of identical trucks of considerable capacity. Each truck, in travelling to the power-station, will have to burn some coal because of the resisting forces. There is a slight expenditure of energy for every yard that each truck travels. Moreover, coal is required for the return journey to the colliery; so all the coal must not be taken from the truck at the power-station. We will suppose that sufficient coal is left in each truck to get it back to the colliery, so that on reaching the colliery there is not a single piece of coal left. At the colliery the truck receives more coal and proceeds again on its journey towards the power-station.

To serve our present purpose, we will suppose that the trucks, in proceeding through the colliery yard, pass under a series of hoppers, each of which can put exactly 1 ton of coal into every truck. If there are two hoppers in operation, each truck will receive 2 tons of coal, and so on. The number of tons of coal put into a truck will depend upon the number of hoppers in use, but each truck will always receive exactly the same amount of coal as every other truck.

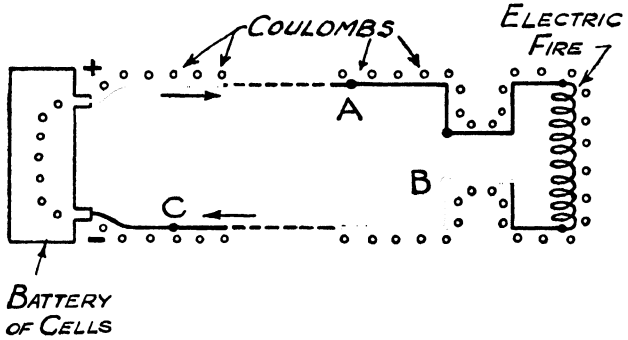


Fig. 9.—The Electrical Counterpart of Fig. 8.

Now consider Fig. 9. It represents a simple electric circuit consisting of a battery with leads to an electric fire. Coulombs, which are carriers of electrical energy, move along the leads in the directions shown, the battery giving energy to these carriers, just as the trucks in our analogy received energy in the form of coal at the colliery. The coulombs give up most of their energy at the fire, but, of course, a little energy is used in taking the coulombs round the connecting wires. When these carriers of energy return to the battery they have lost all their energy, but receive a fresh amount, and once again follow their original path.

Now the electromotive force of the battery is actually a measure of the energy the battery gives to each coulomb. The e.m.f. of a battery is normally measured in volts. Thus if a battery supplies 4 joules of energy per coulomb, we say it has an e.m.f. of 4 volts, a **volt** being an expression meaning a **joule per coulomb.** The number of joules of energy given to each coulomb is determined solely by the number of cells and the type of cell used in the battery, just as in our analogy the number of tons of coal per truck was determined by the number of hoppers used.

As each coulomb travels round the circuit it uses up the energy supplied to it. The potential difference between two points such as A and B (Fig. 9) is a measure of the energy used by a coulomb in moving from A to B. *Thus we see that e.m.f. is a measure of the energy put into a coulomb, and p.d. is a measure of the energy taken out of a coulomb as it moves from one point to another.* Both are normally measured in joules per coulomb or volts.

This energy aspect of e.m.f. and p.d. must not be taken to imply that the description of e.m.f. and p.d. in terms of electrical pressure is untrue; the pressure and energy aspects are simply two different features of the same thing. Thus in Fig. 9, if the p.d. between A and B is 20 times that between B and C, then we can regard this as meaning either that the electrical pressure difference between A and B is 20 times that between B and C, or that 20 times as much energy is taken from a coulomb in travelling from A to B as in travelling from B to C. Again, if two batteries of e.m.f.'s 30 and 10 volts respectively are connected up to identical circuits, the coulombs will travel 3 times as fast in the first case as in the second because the pressure is 3 times as great (see Section 13). At the same time the fundamental difference between the two batteries is that one gives 30 joules of energy to each coulomb and the other only 10 joules per coulomb.

The pressure aspect is certainly an essential feature of e.m.f. and p.d., as is witnessed by such expressions as "*a pressure of* 200 *volts,*" but the most essential fact about a source of e.m.f. is its ability to supply energy to electric charges, which are fundamentally carriers of energy.

11. Quantitative Treatment

Reverting to our analogy (Figs. 8 and 9), suppose we required to know how much coal left the colliery in a certain time. We should require to know how many trucks departed in that time and how many tons of coal were in each truck. Moreover, we should be able to calculate how many trucks left in a given time if we knew how many left per minute. Similarly, a knowledge of the number of coulombs leaving the battery per second is useful in calculations on the amount of energy used in the circuit during a specified period.

We have seen previously that the number of coulombs passing a point in a circuit each second is a measure of the current strength. So the number of coulombs passing a point in a given time can be calculated by the simple relationship:—

No. of coulombs = *No. of amperes* × *No. of seconds*

or **Quantity of electricity** = **Current** × **Time** . . . (I)

This relationship is important, since current and time can be very readily measured by an ammeter and a clock.

The total energy supplied by the battery can be expressed in terms of the current, the e.m.f. and the time. Since each of these quantities can be readily measured, a value for the energy can be obtained thus:—

Number of joules supplied in a given time
= (Number of coulombs leaving the battery in that time)
× (Number of joules in each coulomb)
= (Number of coulombs per second × Number of seconds)
× No. of volts.

i.e., *Total energy supplied by battery*
= *Current strength* × *Time* × *E.M.F.*

The energy absorbed by the fire (Fig. 9) is less than that supplied by the battery, since some of it is used in transporting the coulombs through the leads, but by similar reasoning to the above, it is obvious that:—

The energy absorbed by the fire
= *The current flowing* × *Time for which it flows*
× *P.D. between the terminals of the fire*

Providing the formula is used correctly, we can summarise these results as follows:—

No. of joules = *No. of volts* × *No. of amperes* × *No. of seconds*

or **Energy** = **P.D.** × **Current** × **Time** . (II)

This formula enables one to calculate the work done, or energy used, between any two points in a circuit, or alternatively, the energy supplied by a dynamo or battery if the e.m.f. is substituted for the p.d.

The **power** used by the fire is equal to the energy used in unit time, and so, transposing formula II, we get:—

$$\frac{\textit{Energy}}{\textit{Time}} = \textit{P.D.} \times \textit{Current}$$

or

$$\textbf{Power} = \textbf{P.D.} \times \textbf{Current} \quad . \quad . \quad . \quad \text{(III)}$$

The watt being the unit of power equal to a joule per second, this equation may be expressed in terms of the normal units, thus

$$\textit{No. of watts} = \textit{No. of volts} \times \textit{No. of amperes.}$$

Formula III may also be used to give the total power supplied by the battery if for the p.d. is substituted the e.m.f. of the battery.

The three formulæ I, II and III, giving expressions for quantity of electricity, amount of energy and power, are very important, and the student should become perfectly acquainted with them. Equation III is particularly important. With its aid we can see almost at a glance that a 60-watt lamp designed to work on 200-volt mains will take a current of 60/200, i.e., 0·3 ampere.

12. Experimental Work

In order to obtain a thorough grasp of ideas so perplexing as that of potential difference, it is almost essential for the student to handle electrical apparatus, carrying out appropriate measurements. Furthermore, the scientific laws involved are so important that it is desirable for the student to discover them for himself.

It may be mentioned at this point that a *scientific law* is merely a statement of fact discovered as a result of experiment. An experiment may be carried out solely for the purpose of measuring some quantity, but the usual purpose of experiment is to discover or test the relationship between two or more entities. Such a relationship is known as a scientific law if numerous experiments without exception indicate that the relationship holds good.

Experiment 1. *To show that the sum of the separate p.d's. round a circuit is equal to the e.m.f. of the battery.*

Connect up the circuit shown in Fig. 10. B is a battery of three accumulators, S a switch and V a voltmeter. P, Q, R and X are four dissimilar coils made up from lengths of eureka wire (see Table IV). The switch is closed and the reading of V (i.e., the p.d.

across P) recorded. This is repeated with the voltmeter connected in turn across Q, R and X. The procedure is repeated with three, and then with only two of the coils in the circuit. Finally the e.m.f. of the battery is measured by means of the voltmeter as previously described (see Fig. 7).

No. of coils.	P.D. across				Sum of P.D's.
	P.	Q.	R.	X.	
4					
3				—	
2			—	—	

E.m.f. of battery = . . . volts.

If the results are tabulated as shown, it will be observed that the sum of the p.d's. is, in all three cases, equal to the e.m.f. *within the limits of experimental error.* This expression means that there is always a slight error in taking a reading in the laboratory; neither the instrument nor the observer is capable of precise accuracy. Any slight errors, such as would be obtained with good-quality instruments and a careful observer, are, however, insufficient to hide fundamental scientific facts. Thus the results in the final column of the table in Experiment 1 will be so very nearly equal to the e.m.f. of the battery that we may arrive at the conclusion that, without any experimental error, these results would be precisely equal to the e.m.f. It should be noted that the conclusion drawn

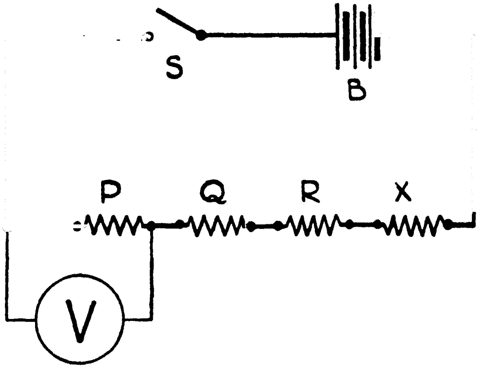


FIG. 10.—THE CIRCUIT EMPLOYED IN EXPT. 1.

from the results of this experiment supports the energy aspect of e.m.f. and p.d. referred to in Section 10.

Experiment 2. *To investigate the relationship connecting the p.d. between any two points in a circuit with the current flowing between those points.*

Connect up four accumulators, a switch, ammeter and two coils P and Q as shown in Fig. 11, and then connect a voltmeter across one of the coils. The aim of the experiment is to test how the p.d. between points X and Y varies with the ammeter reading. First record the ammeter and voltmeter readings with four cells in the battery. Repeat this with three cells, then two cells and finally with a single cell only, tabulating the results as shown. The constancy of the ratio obtained in the last column of the table shows that *the p.d. between X and Y is directly proportional to the current flowing.* Thus the energy used per coulomb in travelling between two points is directly proportional to the number of coulombs travelling per second.

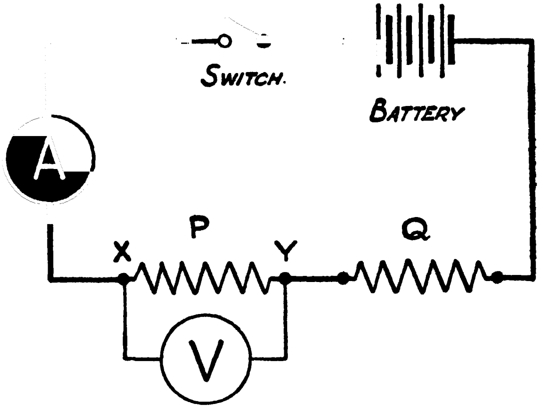


Fig. 11.—The Circuit Employed in Expt. 2.

No. of cells in battery.	Ammeter reading.	Voltmeter reading.	$\frac{\text{Voltmeter reading.}}{\text{Ammeter reading.}}$
4 3 2 1			

The conclusion reached in Experiment 2 is known as **Ohm's Law.** This important relationship, in its complete form, states that **the p.d. between the ends of a conductor is directly proportional to the current flowing, provided that the physical conditions remain unaltered.** (The only important physical condition, as far as solid conductors are concerned, is that of temperature—see Chapter III, Section 22.) Stated mathematically, this law becomes:—

$$\textbf{P.D.} \propto \textbf{Current.}$$

Since, from the results of Experiment 1, the sum of the p.d.'s round a circuit is numerically equal to the e.m.f., it follows that *the current flowing in any given circuit is directly proportional to the e.m.f. of the source.* Doubling the e.m.f. causes the current to be doubled; when the current is doubled the p.d. across any part of the circuit is also doubled.

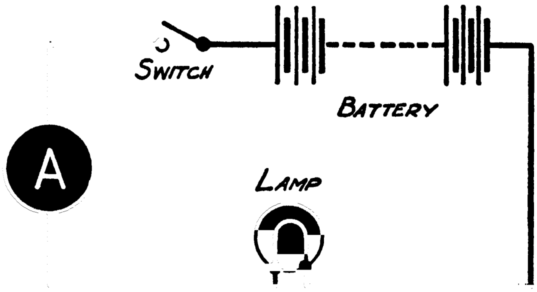


Fig. 12.—The Circuit Described in Section 13.

13. The Ohm's Law Formula

Actually the laws stated above only partly explain what determines the strength of a current. If a series circuit consisting of a battery, a switch, an ammeter and a lamp be connected up as in Fig. 12 and the switch be closed, the ammeter will record a certain current strength; if the lamp is replaced by another of different size, the reading of the ammeter will usually be different. The current strength must, therefore, depend not only on the e.m.f. of the source, but also on the components in the circuit. Evidently the two lamps impede the flow of current to different extents, and the different impeding effects are denoted by the respective **resistances** of the lamps.

It is fairly obvious that if a component *resists* the flow of current, energy will be abstracted from each coulomb as it passes through the component. The greater the resisting force, the greater will be the amount of energy required from each coulomb. An obvious method of measuring this impeding force is, therefore, to determine the voltage drop or p.d. across the component concerned, i.e., the energy used per coulomb. But the p.d. depends not only on the resistance of the component, but also, as we saw from Experiment 2, on the current strength. So it is necessary to stipulate that in such measurements the current must always have the same value. Actually the resistance of any component is normally expressed in terms of the voltage drop across it when 1 ampere is flowing through it. Thus, resistance is usually measured in *voltage drop per ampere* or in **ohms.**

Referring to Fig. 9, we are now able to appreciate that energy is abstracted from the coulombs during their passage through the electric fire *owing to the resistance offered* by the conducting material of the heating element. Further, the value of this resistance controls the flow of coulombs from the battery, and hence the rate at which energy is expended in the fire. In the colliery analogy represented by Fig. 8 the rate at which the trucks are made to move depends on the rate at which energy is required at the power-station. Similarly, if a large amount of power is required at the electric fire, its resistance is made small so that the coulombs will pass along rapidly, each coulomb giving up a definite number of joules at the fire.

The relationship between current, p.d. and resistance can be expressed in the following three ways:—

$$\left.\begin{aligned}\textbf{Resistance} &= \frac{\textbf{P.D.}}{\textbf{Current}}\\ \textbf{P.D.} &= \textbf{Current} \times \textbf{Resistance}\\ \textbf{Current} &= \frac{\textbf{P.D.}}{\textbf{Resistance}}\end{aligned}\right\} \quad . \quad . \quad \text{(IV)}$$

or, in terms of the normal units of resistance, current and p.d.:—

$$\textit{No. of ohms} = \frac{\textit{No. of volts}}{\textit{No. of amperes}}$$

These formulæ are of great importance, as they are fundamental to the whole of electrical engineering. They can be applied to any part of a circuit, or to a complete circuit. In the latter case the value of p.d. is the total p.d. for the complete circuit (numerically equal to the e.m.f. of the source) and the value of the resistance is also that for the complete circuit. In a circuit containing several components it is important, in using the above formulæ, to have clearly in mind the precise part of the circuit being dealt with, and to realise that the formulæ deal with the resistance of *that part* of the circuit, the current taken by that part and the p.d. across that particular part. The following example will illustrate the above-mentioned points.

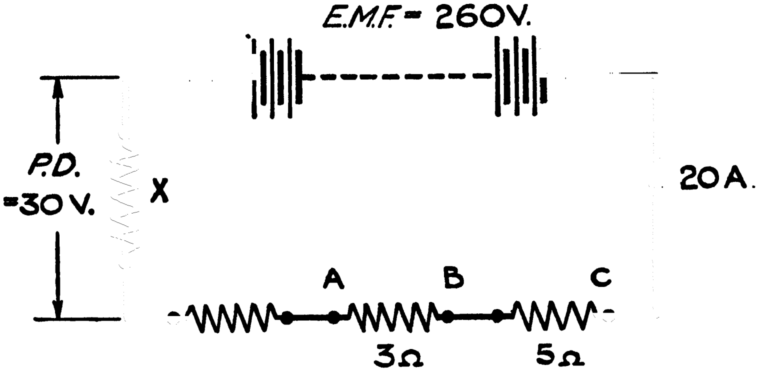


FIG. 13.—CIRCUIT FOR THE EXAMPLE IN SECTION 13.

Example. Determine, for the circuit shown in Fig. 13, (a) the p.d. between the points A and B, also between B and C, (b) the resistance of the component X, and (c) the resistance of the complete circuit.

(a) For the part AB, p.d. = current × resistance
= 20 amp. × 3 ohms = **60 volts.**
For BC, the p.d. = 20 amp. × 5 ohms = **100 volts.**

(b) Resistance of component X $= \frac{\text{p.d.}}{\text{current}} = \frac{30 \text{ volts}}{20 \text{ amp.}}$
= **1·5 ohms.**

(c) Resistance of complete circuit $= \frac{\text{e.m.f.}}{\text{current}} = \frac{260 \text{ volts}}{20 \text{ amp.}}$
= **13 ohms.**

14. Symbols and Formulæ

The British Standards Institution recommend that certain standard symbols be employed in connection with the various entities encountered in electrical technology, those symbols referring to the entities dealt with in Vol. I being listed in Table IX. Using these standard symbols, formulæ I, II, III and IV may be represented as follows:—

Quantity = Current × Time . . . $Q = It$
Energy = P.D. × Current × Time . . $W = VIt$
Power = P.D. × Current . . . $P = VI$
Resistance = P.D. ÷ Current . . $R = \dfrac{V}{I}$

It should be noted that the power, P watts, may be expressed in two other ways, by making use of the Ohm's Law formulæ, as follows:—

$$P = VI;\ \text{but } V = IR, \text{ hence } P = I^2R$$

Alternatively, substituting $I = \dfrac{V}{R}$, $P = \dfrac{V^2}{R}$

In cases where the energy, power or resistance is required for the complete circuit, the p.d., V volts, in the above formulæ is replaced by the e.m.f. of the source, E volts.

15. Multiple and Sub-multiple Units

Often the standard units are of an inconvenient size. For example, in transmitting electric power the p.d. between a pair of conductors may be many thousands of volts. In such a case the **kilovolt,** which is equal to 1000 volts, is a far more suitable unit of p.d. than the volt. Again, an accumulator can usually supply tens of thousands of coulombs with electrical energy. Since an ampere is a coulomb per second, a coulomb may be regarded as an *ampere-second.* Thus when a current of 3 amperes is taken from an accumulator for 20 seconds we say it has supplied 60 amp.-seconds or 60 coulombs. The **ampere-hour** is a much more useful unit than the ampere-second for the purpose of specifying the capacity rating of an accumulator. Since there are 3600 seconds in 1 hour, there will be 3600 coulombs in 1 amp.-hour. Again, many currents in radio circuits are of the order of thousandths or even millionths of an ampere. So the **milliampere,** which is one-thousandth part of an ampere, is frequently used, and sometimes even the **microampere** is found convenient, a microampere being one millionth of an ampere.

The *derived units* of current, charge, p.d. and e.m.f., resistance, power and energy are tabulated below, the letters in brackets being the abbreviations commonly used. These abbreviations will be used in future throughout this volume. The English alphabet contains insufficient letters for the purpose of engineering symbols, and therefore Greek letters are frequently used. Thus we make use in the table of the Greek letters " mu " (μ) and " omega " (Ω). We shall eventually become acquainted with many more electrical entities than the six tabulated below, but in each case the multiple units are either one thousand or one million times the standard unit and the submultiple units are one thousandth or one millionth of the standard unit.

Entity.	Standard unit.	Derived unit.	Value of derived unit in terms of standard.
Electric current	Ampere (A)	Milliampere (mA)	10^{-3} A = 0·001 A
		Microampere (μA)	10^{-6} A = 0·000001 A
Quantity of electricity (i.e., amount of charge)	Coulomb (C)	Amp.-hour (Ah)	3600 C
Potential difference and Electromotive force	Volt (V)	Kilovolt (kV)	1000 V
		Millivolt (mV)	$10\ ^{3}$ V
		Microvolt (μV)	10^{-6} V
Resistance	Ohm (Ω)	Megohm (MΩ)	10^{6} Ω
		Kilohm (kΩ)	1000 Ω
		Microhm ($\mu\Omega$)	10^{-6} Ω
Power	Watt (W)	Kilowatt (kW)	1000 W
		Megawatt (MW)	10^{6} W
Energy	Joule (J)	Watt-hour (Wh)	3600 J
		Kilowatt-hour (kWh)	$3{\cdot}6 \times 10^{6}$ J

16. D.C. and A.C.

It is advisable at this stage to distinguish between d.c. (direct current; sometimes referred to as continuous current) and a.c. (alternating current), since in the succeeding chapters these terms will be encountered quite frequently.

The term d.c. refers to a current having a constant direction, i.e., the conventional direction already referred to in Section 8. Direct current is obtainable from sources of e.m.f. such as primary cells, secondary cells or accumulators and commutator-type dynamos (see Chapter VIII).

Alternating current (a.c.) consists of an oscillation or " surging " of the electrons backwards and forwards in the circuit. The current is continuously varying in strength and reversing at a very rapid rate. One complete forward and backward surge is known as a **cycle,** the number of cycles occurring each second being termed the **frequency.** The standard frequency for mains supplies in Great Britain is 50 cycles per second (50 c/s). In telephony, radio, etc., much higher frequencies are employed, and the following derived units are therefore used:—

The **kilocycle per second** (kc/s), equal to 1000 cycles per sec.

The **megacycle per second** (Mc/s), equal to 10^6 cycles per sec.

Alternating current is usually obtained from a special type of dynamo. known as an alternator (see Chapter VIII) or from a valve oscillator.

Exercises on Chapter II

1. A battery of e.m.f. 4 volts supplies a steady current over a period of 15 seconds, during which time 32 coulombs leave the battery. How much energy is supplied by the battery and what is the strength of the current?

[128 J; 2·13 A.]

2. 800 joules of energy are expended when a current of 3 amperes flows through a coil for 4 minutes. How many coulombs pass in this time? What is the p.d. across the coil?

[720 C; 1·11 V.]

3. The p.d. across the terminals of a lamp is 4·1 volts, the current being 0·3 amp. What quantity of electricity passes in 1 minute? How much energy is supplied to the lamp in half-an-hour? If the e.m.f. of the battery is 4·3 volts, how much energy is wasted per second in the remaining part of the circuit?

[18 C; 2214 J; 0·06 J.]

4. A battery of e.m.f. 6 volts supplies a current of 2·5 amp. for 3 minutes. How much energy is used in this time? What quantity of electricity is supplied by the battery?

[2700 J; 450 C.]

5. A lamp taking 60 watts is run from 200-volt mains for 15 minutes. How many joules are used in this time? How many coulombs pass every minute? What is the current flowing?

[54,000 J; 18 C; 0·3 A.]

6. An electric fire takes 7·5 amp. at 200 volts. What is the power of the fire? How many coulombs pass every hour? How many joules are used in this time?

[1500 W or 1·5 kW; 27,000 C; $5{\cdot}4 \times 10^6$ J.]

7. A battery has an e.m.f. of 12·8 volts and supplies a current of 3·2 amp. What is the resistance of the circuit? How many coulombs leave the battery in 5 minutes? How many joules are used in this period?

[4 Ω; 960 C; 12,288 J.]

8. A lamp takes 0·5 amp. when working on 200-volt mains. How many joules are used every minute? What is the power of the lamp, in watts, and its working resistance, in ohms?

[6000 J; 100 W; 400 Ω.]

9. An automobile lamp is rated at 12 volts, 36 watts. Determine (a) the current taken by the lamp, (b) the resistance of the lamp, and (c) the number of coulombs entering the lamp per minute. The lamp is supplied from a 12-volt source.

[(a) 3 A; (b) 4 Ω; (c) 180 C.]

10. An electric iron is operated from 225-volt mains. The p.d. across the iron is 220 volts and the total power taken is 900 watts. Calculate (a) the current flowing, (b) the resistance of the iron element, (c) the resistance of the connecting leads, (d) the power taken by the iron, and (e) the power wasted in the connecting leads.

[(a) 4 A; (b) 55 Ω; (c) 1·25 Ω; (d) 880 W; (e) 20 W.]

11. The resistance of an electric fire element is 40 ohms. What is the power taken by the fire on 200-volt mains? If the fire is to take 810 watts, what must be the mains pressure?

[1000 W; 180 V.]

12. A valve heater is rated at 4 volts, 2·5 amp. What is the resistance of the heater and the power taken on a 4-volt supply? If the supply pressure is 10% above the rated p.d. of the heater, calculate the power taken.

[1·6 Ω; 10 W; 12·1 W.]

13. A 5000-ohm coil is rated at 1 watt. What is the maximum safe current, in milliamperes? What is the percentage overload (in power) if a p.d. of 80 volts is applied to the coil?

[14·14 mA; 28%.]

14. A component of resistance 2·5 megohms carries a current of 0·6 mA. What is the p.d. across the component and the power expended in it? What are the values of p.d. and current when this component is taking precisely 1 watt?

[1500 V; 0·9 W; 1578 V; 0·632 mA.]

15. The field circuit of a motor has a resistance of 80 ohms and carries a current of 3·5 amp. What is the p.d. across the circuit and the power expended? If the circuit p.d. has a permissible variation from the above value of $\pm 6\%$, calculate the current and power in each of the two extreme cases.

[280 V; 980 W; 3·71 A; 3·29 A; 1101 W; 866 W.]

16. A cadmium-copper trolley-wire has a resistance of 0·24 ohm per 1000 yd. Determine the voltage drop and power loss (in kW) per 1000 yd. when the wire is carrying (a) 100 amp. and (b) 250 amp.

[(a) 24 V; 2·4 kW; (b) 60 V; 15 kW.]

17. A circuit consists of 3 coils A, B and C connected in series across a battery of e.m.f. 50 volts, the current being 1·25 amp. The resistance of coil A is 8 ohms and the p.d. across coil C is 15 volts. Determine (a) the p.d. across coil A, (b) the resistance of coil C and (c) the resistance of the complete circuit.

[(a) 10 V; (b) 12 Ω; (c) 40 Ω.]

18. Using the data given in Exercise 17, calculate (a) the power loss in coil A, (b) the number of joules expended in coil C in 24 hours, and (c) the number of ampere-hours supplied by the battery during this period.

[(a) 12·5 W; (b) $1{\cdot}62 \times 10^6$ J; (c) 30 Ah.]

19.· Two coils A and B are designed to take 100 watts and 50 watts respectively when operating separately on 200-volt mains. Find the power taken by each coil and the p.d. across it when they are joined in series on the 200-volt supply.

[A: 66·7 V; 11·1 W; B: 133·3 V; 22·2 W.]

20. A signalling bell requires a p.d. of 4·5 volts for correct operation and takes a current of 0·75 amp. It is fed by a pair of wires each having a resistance of 2 ohms. Determine (a) the resistance of the bell, (b) the total p.d. across the supply ends of the connecting wires, and (c) the power loss in these wires.

[(a) 6 Ω; (b) 7·5 V; (c) 2·25 W.]

CHAPTER III

RESISTANCE

17. Resistors in Series

SINCE all conductors resist the flow of current to a certain extent, they may also be termed *resistors*. The magnitude of the resisting effect of a resistor is known as its *resistance*, which, as we have seen, is measured in ohms. The reciprocal of resistance is known as **conductance,** for which the standard symbol G is employed. Thus, $G = \frac{1}{R}$. The unit of conductance is the **mho,** a word formed by reversing the letters in *ohm*.

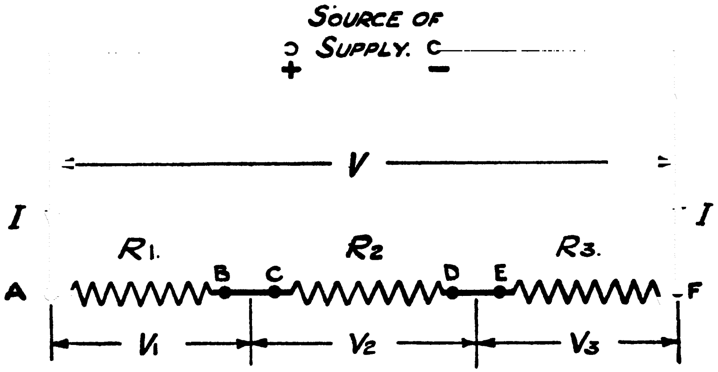


FIG. 14.—RESISTORS JOINED IN SERIES.

Consider three resistors having resistances R_1, R_2 and R_3 ohms respectively joined in series as in Fig. 14. The total p.d. V volts is obviously the sum of the separate p.d.'s, V_1, V_2 and V_3 volts, i.e., $V = V_1 + V_2 + V_3$.

But, $V_1 = IR_1$, $V_2 = IR_2$, and $V_3 = IR_3$, I amp. being the current flowing through each resistor in turn.

$$\text{Thus,}\quad V = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3)$$

$$\text{or}\quad \frac{V}{I} = R_1 + R_2 + R_3.$$

If the total resistance of the series combination between the points A and F be denoted by R_s ohms, then $R_s = \frac{V}{I}$

Thus $$R_s = R_1 + R_2 + R_3$$

No matter what the number of resistors, *the total resistance of a number of resistors in series is equal to the sum of their separate resistances.* Thus, if the voltage applied to the circuit is known, the current which flows may be calculated. From this value of current it is possible to determine the p.d. across each of the separate resistors. It should be noted that, with a series circuit, there is only one possible path for the coulombs, the current has the same value at every point in the circuit and the p.d.'s across the separate resistors are, in general, different.

18. Resistors in Parallel

Consider now the three resistors arranged *in parallel* as in Fig. 15. The terminals A, C and E of the resistors are now joined to one common point, and the other three terminals B, D and F to a second common point, V volts being applied across these points so that the parallel combination takes a current of I_p amperes from the source of supply. This current divides into three parts, I_1 amp. flowing through resistor AB, I_2 amp. through CD and I_3 amp. through EF, the sum of I_1, I_2 and I_3 amp. being equal to the total current I_p amp. taken from the supply.

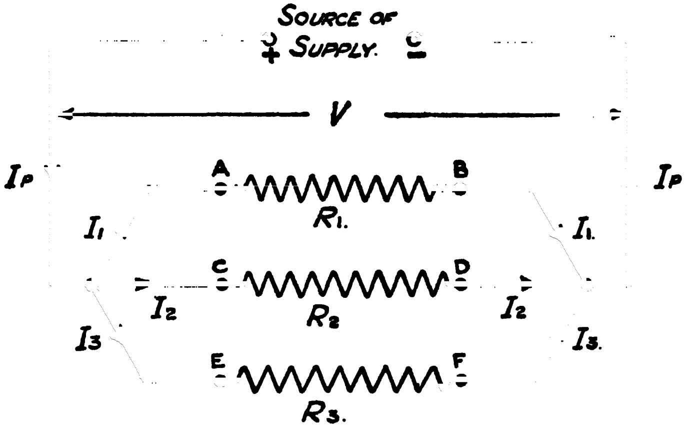


FIG. 15.—RESISTORS JOINED IN PARALLEL.

The p.d. across each resistor is obviously V volts. Therefore, applying the Ohm's law formula to each resistor in turn,

$$I_1 = \frac{V}{R_1},\ I_2 = \frac{V}{R_2} \text{ and } I_3 = \frac{V}{R_3}$$

If we suppose the total resistance of the parallel combination to be R_p ohms, then:—

$$I_p = \frac{V}{R_p}$$

But, $$I_p = I_1 + I_2 + I_3$$

Therefore, $$\frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$
$$= V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

Hence, $$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Suppose, for example, that 3 resistors in parallel have resistances of 2, 4 and 8 ohms respectively.

Then $$\frac{1}{R_p} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{4 + 2 + 1}{8} = \frac{7}{8}.$$

So that $$R_p = 1\tfrac{1}{7}.$$

i.e., the combined resistance is $1\frac{1}{7}$ ohms.

Thus we see that the combination of resistors in parallel has a resistance less than the smallest of the three resistances. This is always the case. If there are, say, five resistors in parallel, *each having the same resistance, R ohms*, the value of the combined resistance, R_p ohms, is one-fifth of that of one of the separate resistors, i.e., $R/5$ ohms. Let the three resistors in Fig. 15 have conductances G_1, G_2 and G_3 mhos respectively. Then $G_1 = \frac{1}{R_1}$, $G_2 = \frac{1}{R_2}$ and $G_3 = \frac{1}{R_3}$. Denote the conductance of the whole arrangement by G_p mhos., i.e., $G_p = \frac{1}{R_p}$. Then, since $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$, the equation may be written

$$G_p = G_1 + G_2 + G_3$$

No matter what the number of resistors, *the total conductance of a number of resistors in parallel is equal to the sum of their separate conductances.* Thus, putting an extra component in parallel with a number already in parallel increases the combined conductance, i.e., it lowers the combined resistance; so more current flows through the arrangement from the supply.

The situation is rather similar to the case of a crowd making its exit from a football ground. Supposing 1000 men pass per minute through a large gate. If a rather smaller gate is opened in addition, allowing 500 men to pass through it each minute, 1500 men now leave the ground every minute. The opening of the second gate does not affect the number passing through the first, but the two together offer less resistance to the crowd than the single gate; three open gates side by side would offer less resistance than two.

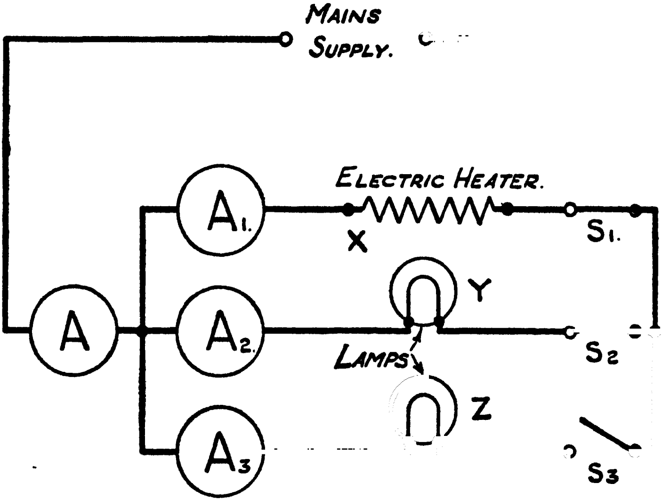


FIG. 16.—VARIOUS COMPONENTS JOINED IN PARALLEL.

All electrical equipment working from a mains supply is in parallel with all other equipment working from that supply. Closing a lamp-switch in a building is similar to opening another gate at the football ground; it does not affect the current taken by each of the other lamps, heaters and motors already operating. The lamp simply takes a certain amount of current itself, thereby increasing the *total* current taken from the mains.

This important principle can be demonstrated with the circuit shown in Fig. 16. When the switches S_1 and S_2 are closed, S_3 being open, the ammeter A records the total current taken by the electric heater X and lamp Y. The reading of ammeter A will be the sum of the readings of A_1 and A_2. On closing S_3 there will be no perceptible change in the readings of A_1 and A_2, but A_3 will now register the current flowing through the lamp Z, and the reading of ammeter A will increase by the amount recorded by A_3.

The parallel system allows each piece of equipment to be fitted with its own switch, and it is because pieces of electrical equipment arranged in parallel are independent of each other that the system is universally used.

Referring again to Fig. 15, since $V = I_1R_1$ and $V = I_2R_2$,

$$I_1R_1 = I_2R_2$$

or,

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}$$

Thus, the component having the smaller resistance takes the larger current. In fact, *the ratio of the currents in any two of the branches of a parallel combination is equal to the inverse ratio of the resistances.* Thus, if a cooker and lamp in parallel take 15 amp. and 0·5 amp. respectively, it is because the lamp has a resistance 30 times that of the cooker.

19. Mixed Grouping of Resistors

Occasionally, circuits may be more complicated than the simple series or parallel arrangements described in the previous two sections. Such circuits can, however, be dealt with by means of the principles already derived; this is illustrated by the following examples.

Example I. The supply to a tramcar is at 500 volts. The interior of the car is illuminated by twenty identical 40-watt lamps arranged in four groups connected in parallel, each group containing five lamps in series. Calculate (a) the p.d. across each lamp; (b) the current flowing through each lamp; (c) the resistance of each lamp and (d) the combined resistance of the twenty lamps forming the complete lighting circuit.

The lamps are arranged as in Fig. 17, the current flowing through group of five lamps being that which flows through a single lamp.

(a) Since five lamps are arranged in series with a total p.d. of 500 volts,

P.D. across each lamp = **100 volts.**

(b) Applying the formula: Power = Current × P.D. to a single lamp,

$$\text{Current taken by a single lamp} = \frac{40}{100} \text{ amp.}$$

$$= \mathbf{0{\cdot}4\ amp.}$$

(c) The resistance of a single lamp $= \dfrac{\text{P.D. across terminals}}{\text{current taken}}$

$= \dfrac{100}{0{\cdot}4}$

$=$ **250 ohms.**

(d) Resistance of five lamps in series $= 5 \times 250$

$= 1250$ ohms.

Resistance, R_p ohms of the four parallel groups of lamps is given by the formula

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

So, $$\frac{1}{R_p} = \frac{1}{1250} + \frac{1}{1250} + \frac{1}{1250} + \frac{1}{1250} = \frac{4}{1250}$$

$\therefore$ Combined resistance $= \dfrac{1250}{4} =$ **312·5 ohms.**

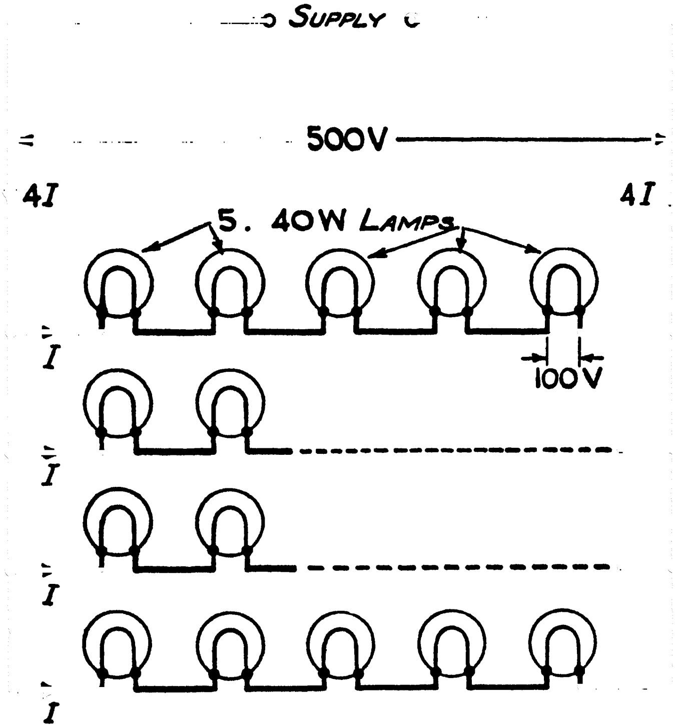


Fig. 17.—Circuit for Example I of Section 19.

Alternatively, current taken by a group of five lamps in series = current taken by a single lamp = 0·4 amp.

∴ Total current taken by the four groups in parallel

$$= 4 \times 0{\cdot}4 = 1{\cdot}6 \text{ amp.}$$

So the resistance of the arrangement

$$= \frac{\text{P.D. across the arrangement}}{\text{current taken}}$$

$$= \frac{500}{1{\cdot}6}$$

$$= 312{\cdot}5 \textbf{ ohms.}$$

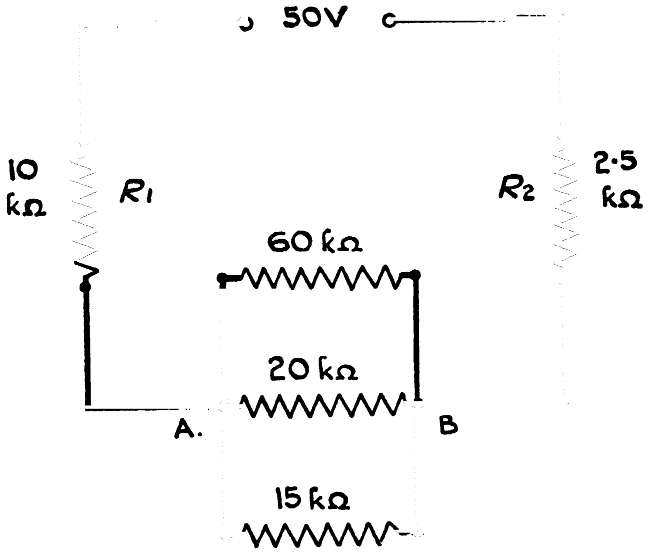


FIG 18—CIRCUIT FOR EXAMPLE II OF SECTION 19

Example II. Resistors of 60 kΩ, 20 kΩ and 15 kΩ are joined in parallel across a pair of terminals A and B. Between A and one supply terminal a resistor R_1 of 10 kΩ is connected, and to complete the circuit a resistor R_2 of 2·5 kΩ is inserted between B and the other supply terminal. The supply voltage is 50. Determine (a) the total current drawn from the supply, (b) the p.d.'s across R_1 and R_2, (c) the p.d. across AB and (d) the current in each of the three resistors connected between A and B.

The circuit is shown in Fig. 18.

(a) Let the effective resistance between A and B be R_p kΩ.

Then, $\frac{1}{R_p} = \frac{1}{60} + \frac{1}{20} + \frac{1}{15} = \frac{1+3+4}{60} = \frac{8}{60}$

$\therefore \quad R_p = \frac{60}{8} = 7{\cdot}5$

$\therefore$ Total resistance of circuit $= 10 + 2{\cdot}5 + 7{\cdot}5$
$= 20$ kΩ.

$\therefore$ Total current flowing $= \frac{50}{20 \times 10^3} = \frac{2{\cdot}5}{10^3}$ amp.

or **2·5 mA.**

(b) P.D. across $R_1 = 10 \times 10^3 \times \frac{2{\cdot}5}{10^3} =$ **25 volts.**

P.D. across $R_2 = 2{\cdot}5 \times 10^3 \times \frac{2{\cdot}5}{10^3} =$ **6·25 volts.**

(c) P.D. across AB = resistance between A and B × total current

$= 7{\cdot}5 \times 10^3 \times \frac{2{\cdot}5}{10^3}$

= **18·75 volts.**

Alternatively:—

Supply voltage = sum of p.d.'s round circuit
$\therefore$ 50 = 25 + 6·25 + (p.d. across AB)
or, p.d. across AB = 50 − 31·25
= **18·75 volts.**

(d) Current in 60-kΩ resistor $= \frac{18{\cdot}75}{60 \times 10^3} = \frac{0{\cdot}3125}{10^3}$ amp.

or **0·3125 mA.**

Current in 20-kΩ resistor $= \frac{18{\cdot}75}{20} =$ **0·9375 mA.**

Current in 15-kΩ resistor $= \frac{18{\cdot}75}{15} =$ **1·25 mA.**

Note that the total current = 0·3125 + 0·9375 + 1·25
= **2·5 mA.**

20. Resistivity

It follows from the theory of resistors in series and parallel (Sections 17 and 18) that *the resistance of a conductor of uniform cross-section will be directly proportional to its length and inversely proportional to its cross-sectional area.* Thus, for example, a wire of length 100 feet may be regarded as consisting of 100 wires, each of length 1 foot, in series; so, if 1 foot of the wire possesses a resistance r ohms, from the formula $R_s = R_1 + R_2 + R_3 + \ldots$, it is clear that the wire of length 100 feet will have a resistance of $100\,r$ ohms, i.e., 100 times that of a similar wire of length 1 foot. Again, a conductor of cross-sectional area 4 sq. mm. may be regarded as consisting of 4 conductors, each of area 1 sq. mm., in parallel. So if one of the conductors of area 1 sq. mm. possesses a resistance of x ohms, i.e., a conductance of $\frac{1}{x}$ mhos, it follows from the formula $G = G_1 + G_2 + \ldots$ that the conductance of the composite conductor will be $\frac{4}{x}$ mhos, and therefore its resistance will be $\frac{x}{4}$ ohms. In other words, a conductor of cross-sectional area 4 sq. mm. will have a resistance which is ¼ that of a conductor of the same length and material, but of cross-section 1 sq. mm. (see Fig. 19).

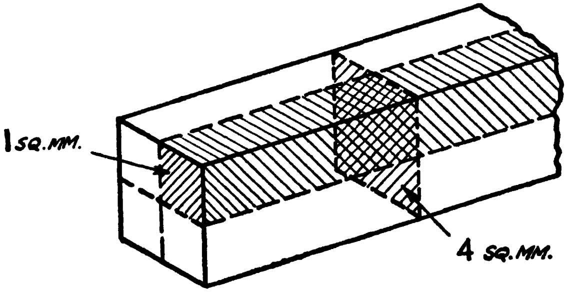


FIG. 19.—THE EFFECT OF CROSS-SECTIONAL AREA ON RESISTANCE.

Thus, for a conductor of length l cm. and cross-sectional area a sq. cm., the resistance R ohms is given by:—

$$R = a\ constant \times \frac{l}{a}$$

The value of this constant will depend upon the material of which the conductor is made, since two conductors of the same dimensions but of different materials are found to possess different resistances. This constant is known as the **resistivity** (or specific resistance) of the material concerned, the standard symbol for resistivity being ρ ("rho"). Thus the formula for the resistance of a conductor becomes

$$R = \frac{\rho l}{a}$$

By putting l and a each equal to unity, we obtain $R = \rho$. Thus the resistivity of a material is equal to the resistance of a conductor of unit length and unit cross-sectional area. A conductor of such dimensions could in fact be made in the form of a unit cube, so *resistivity* is frequently defined as the resistance between opposite faces of a unit cube of the material.

In using the formula $R = \frac{\rho l}{a}$ to calculate the resistance, R ohms, of a conductor care must be taken to ensure that corresponding units are employed throughout, e.g., if the length of the conductor be *l cm.*, and the area *a sq. cm.*, then ρ must represent the magnitude of the resistivity in *ohms per cm. cube*, but if the length be *l inches* and the area *a sq. inches*, then ρ must represent the magnitude of the resistivity in *ohms per inch cube.*

Since a cm. cube regarded as a wire is exceptionally short and of a large sectional area, the resistivity of conducting materials will be exceedingly small. For example, the resistivity of annealed copper is 0·00000172 ohm per cm. cube. To avoid the use of such small numbers, the resistivity of conducting materials is usually expressed in *microhms per cm. cube* or *microhms per inch cube.* The value for the resistivity of annealed copper thus becomes 1·72 microhms per cm. cube.

The reciprocal of resistivity is **conductivity,** the usual symbol being σ (" sigma "). Table V (at the end of Vol. I) gives the resistivity, both in microhms per cm. cube and microhms per inch cube, of conducting materials commonly used in electrical engineering; conductivity is also given both in mhos per cm. cube and mhos per inch cube. Since an inch cube has a length 2·54 and a cross-sectional area $2{\cdot}54^2$ times that of a cm. cube, the resistivity in cm. measure is 2·54 times that in inch measure, the resistivity of annealed copper being approximately 0·68 microhm per inch cube.

A perusal of Table V makes it obvious why copper and aluminium are so widely used as electrical conductors. Pure iron has a resistivity approximately 7 times, and steel with a 0·1% carbon content a resistivity about 12 times that of annealed copper; eureka has a resistivity roughly 30 times, and the nickel-chromium alloy used in certain electric heating appliances a resistivity about 70 times that of annealed copper. Slight traces of impurities in copper and aluminium increase their resistivities very considerably.

Carbon has a resistivity of the order of a few thousand microhms per cm. cube, and most insulators have a resistivity of the order of 10^{15} ohms per cm. cube.

The term **annealed** has been used above in connection with copper conductors, and it is also frequently employed in connection with other kinds of wire and strip used for electrical purposes. In the annealing process the wire is rendered soft and ductile by heating it to a high temperature and allowing it to cool slowly. This process removes the hardness caused by *drawing* the wire through dies in the course of manufacture, and makes the wire more suitable for winding into coils or spirals. Wires which have not been subjected to this annealing process are referred to as **hard-drawn**, and possess tensile strengths higher than the corresponding annealed wires. Hard-drawn conductors are therefore preferred for use in overhead lines and as trolley wires, in which cases mechanical strength is of importance.

In practice, the resistance of a length of conductor of standard gauge may be determined from **wire tables** prepared by the wire manufacturers. Alternatively, for annealed or hard-drawn copper conductors, and hard-drawn aluminium conductors of standard quality, this information may be obtained from the appropriate specifications issued by the British Standards Institution (see Table X). In all these wire tables the resistance of each standard size of conductor is stated in ohms per 1000 yards, or in ohms per mile, at a specified temperature. However, for resistance calculations in conjunction with non-standard sizes of conductor, and materials for which wire tables are not available, the resistivity formula is invaluable.

21. Cables and Bare Conductors

Electric power is transmitted from the power-station and distributed by means of overhead conductors or underground cables. On entering a building the supply is further distributed through smaller-sized cables supported from the fabric of the building.

A conductor, whether bare or forming the core of a cable, usually consists of a number of strands of wire of circular cross-section, in order to provide some flexibility. Groups of 3, 7, 19, 37, 61, 91, 127 and 169 strands are used, since these are the numbers which give a cylindrical form to the conductor (Fig. 20). In the case of 3-strand conductors all the strands are spiralled about the axis of the conductor, whilst for conductors containing 7 or more strands all the layers are spiralled around a central strand, the direction of spiralling being reversed for each successive layer. This causes the resistance of a stranded conductor to exceed that of a solid conductor of equal length and cross-sectional area by about 2% (due to the increased length of all the strands but the central one). The contact between adjacent strands is so poor that the current does not, in general, flow from one strand to another, but follows the spiral paths of the various strands.

For insulated cables the conductor is usually of annealed copper, the individual strands in rubber-insulated cables being "tinned" to eliminate chemical action between the rubber and the copper conductor. Bare stranded conductors used for overhead lines are usually of hard-drawn copper or aluminium, a central core of galvanised steel wire being used in many cases to improve the mechanical strength.

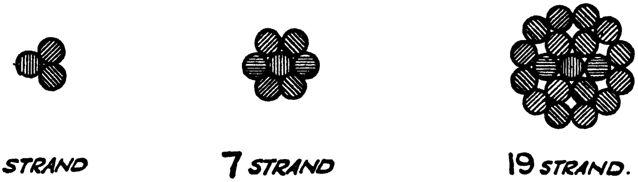


FIG. 20.—CROSS-SECTIONS OF VARIOUS STRANDED CONDUCTORS.

In order to specify a particular conductor it is merely necessary, if all the strands are of the same material, to state the number of strands and the diameter of each, e.g., 3/·029″ or 61/·103″ (the symbol ″ being usually omitted).

22. The Variation of Resistance with Temperature

The resistivity of a material is very much dependent upon the temperature. At temperatures of 0° C. and 100° C. respectively annealed copper has resistivities of 1·59 and 2·26 microhms per cm. cube, the corresponding values for iron being 10·8 and 17·5. But the resistivities of all materials do not *increase* with rise in temperature. Thus, if the resistances of a tungsten and a carbon filament lamp are measured when the lamps are both hot and cold, it will be found that whereas the resistance of the tungsten filament lamp when hot may be more than 10 times the cold resistance, the cold resistance of the carbon filament lamp may be several times greater than the hot resistance. Hence the resistivity of tungsten increases, while that of carbon decreases, with rise in temperature. In general, the resistivities of good conductors increase and those of fair conductors and insulators decrease with rise in temperature.

This behaviour can be explained as follows. When a substance is heated, the molecules vibrate more rapidly, hampering the movement of electrons as they drift from ion to ion. With the rise in temperature of a good conductor there is no increase in the number of free electrons, and therefore the sole effect of temperature rise is increased resistance, due to increased molecular vibration. In the case of mediocre conductors like carbon there are comparatively few free electrons at normal temperature, but with the increased vibration of atoms and molecules at higher temperatures the atoms part with more of their electrons, and this increase in the number of free electrons more than compensates for the increased difficulty each electron experiences in flowing through the material. So as the temperature is raised there is an increase in the electron flow for a

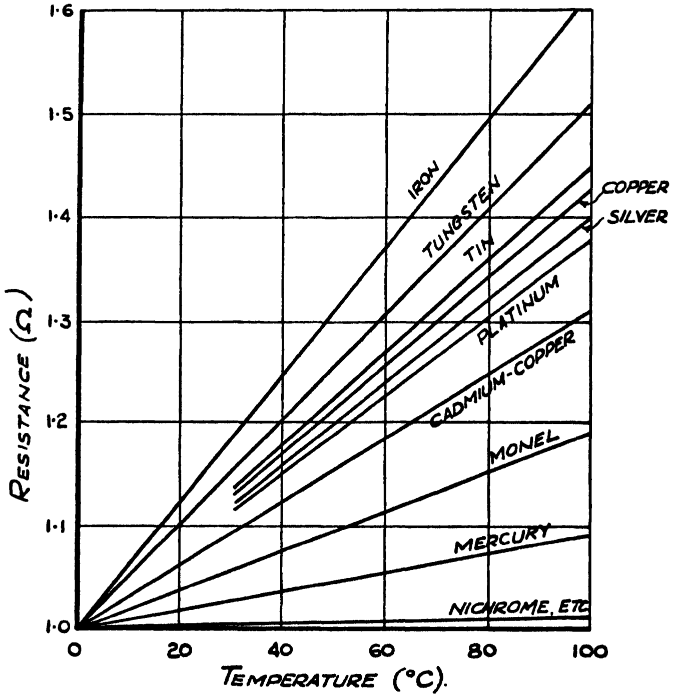


FIG. 21.—RESISTANCE/TEMPERATURE CHARACTERISTICS.

given p.d. across the specimen. An extreme case of decrease in resistivity with temperature rise is that of gutta-percha, a rather poor insulator used in sub-marine cables because of its moisture-resisting properties; its resistivity at 24° C. is only 1/24th of the resistivity at 0° C.

Over a limited range of temperature, change of resistivity is directly proportional to change of temperature. Fig. 21 shows how the resistance of various conducting materials, each having a resistance of 1 ohm at 0° C., increases as the temperature is raised to 100° C. Thus 1 ohm of copper at 0° C. becomes 1·426 ohms at 100° C.

This variation of the resistivity of a material with temperature is expressed quantitatively by means of the **temperature coefficient of resistance,** the usual symbol being α (pronounced " alpha "). The temperature coefficient of resistance may be defined as *the increase in resistance of a specimen of the material having a resistance of* 1 *ohm at* 0° *C. when the temperature is raised to* 1° *C.* A similar definition is of course possible in terms of the Fahrenheit scale of temperature. The temperature coefficient for copper (both annealed and hard-drawn) is 0·00426 ohm per ohm per deg. C. So a copper wire of resistance 1 ohm at 0° C. will have a resistance at 50° C. of $1 + (0{\cdot}00426 \times 50) = 1{\cdot}213$ ohms. Similarly a copper wire of resistance 20 ohms at 0° C. will have a resistance at 50° C. of $20(1 + 0{\cdot}00426 \times 50) = 24{\cdot}26$ ohms. A wire of resistance R_0 ohms at 0° C. will exhibit an *increase* of resistance at θ° C. of $R_0\alpha\theta$; so if the resistance at θ° C. is R_θ ohms:—

$$R_\theta = R_0 + R_0\alpha\theta$$

i.e.,

$$R_\theta = R_0(1 + \alpha\theta) \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{I})$$

It follows that $R_\theta - R_0 = R_0\alpha\theta$

so,

$$\alpha = \frac{R_\theta - R_0}{R_0\theta} \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{II})$$

Let a resistor have resistances R_0, R_1 and R_2 ohms at temperatures 0, θ_1 and θ_2° C. respectively, then

$$R_1 = R_0(1 + \alpha\theta_1)$$

and

$$R_2 = R_0(1 + \alpha\theta_2)$$

so,

$$\frac{R_1}{R_2} = \frac{1 + \alpha\theta_1}{1 + \alpha\theta_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{III})$$

Taking α for copper as $\frac{1}{234{\cdot}5}$, equation III may be written, for *copper*, in the form:—

$$\frac{R_1}{R_2} = \frac{1 + \left(\frac{1}{234{\cdot}5} \times \theta_1\right)}{1 + \left(\frac{1}{234{\cdot}5} \times \theta_2\right)}$$

i.e.,
$$\frac{R_1}{R_2} = \frac{234{\cdot}5 + \theta_1}{234{\cdot}5 + \theta_2} \quad . \; . \; . \; . \; . \; . \; . \quad \text{(IV)}$$

Equations I and II are of use only in cases where the lower temperature is 0° C. This case rarely occurs in practice, and so equation III is more useful in general; where the material concerned is copper, as is so often the case, equation IV may be used.

It should be noted that for materials such as carbon, where there is a decrease in resistance with rise in temperature, the temperature coefficient is negative.

For certain alloys such as manganin, constantan and nichrome (see Tables IV and VI) the temperature coefficients of resistance are very low, particularly in the case of the first-named material. These alloys are therefore employed for fixed and variable resistors whose resistance values are required to remain practically independent of temperature variations.

Table VI gives the values of the temperature coefficient for a number of the metallic conducting materials employed in electrical circuits and apparatus.

23. Construction of Resistors

This section deals briefly with the construction of the various types of resistor. It is important for the student to become familiar at an early stage with the construction and correct usage of the various forms of resistor, particularly those of the variable type, as these components are employed in almost every experiment described in the subsequent pages of this book.

In general, the construction of a resistor depends upon the resistance value to be provided, the power to be dissipated and upon whether a single fixed value or a range of resistance values is required.

Fixed Resistors

(a) *Wire-wound.* In this type the resistor consists of a wire or strip of *constantan* or *nichrome* (chosen on account of their high resistivities and low temperature coefficients of resistance; see Tables IV, V and VI) wound spirally on a tube or bobbin of *ceramic* or other insulating material as illustrated in Fig. 22 (the term

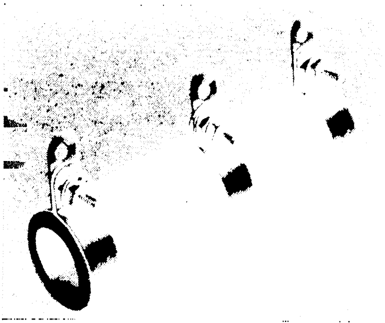


Fig 22 —A Wire-wound Resistor

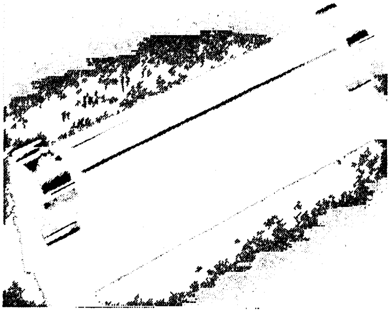


Fig 24 —A Vitreous Enamelled Resisto

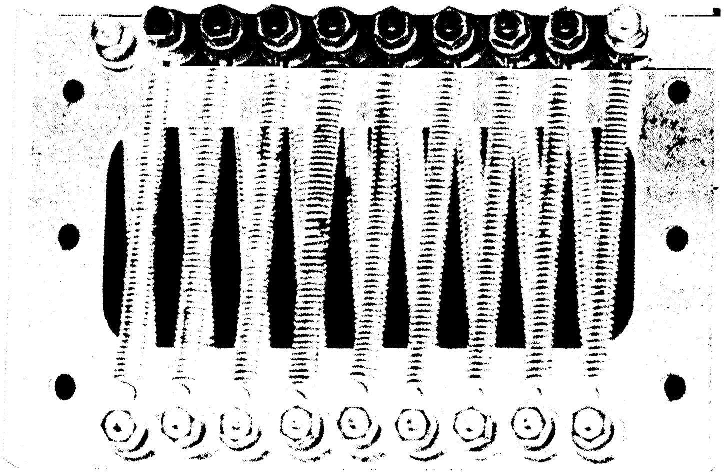


Fig 23 —An Open-wire Spiral Resistor

" ceramic " is applied to materials such as porcelain). Alternatively, the wire may be wound into open spirals, these being stretched between insulated supports as illustrated in Fig. 23. The wire is usually terminated at the ends of each tube or section by connecting it to fixed metal clips or terminals.

(b) *Vitreous enamelled.* This is also a wire-wound type, the wire being wound as a spiral on an insulating tube and the outer surface of the unit subsequently coated with a layer of *vitreous* enamel (i.e., a substance of the same nature as glass). A typical low-wattage resistor of this type is illustrated in Fig. 24, the terminal connections to this unit being made via the metal caps at the ends of the tube. Very often these terminal connections consist merely of flexible copper leads. The advantages of this type over type (a) are: (1) the vitreous coating over the tube and resistance wire protects the latter, which is often very fine, and (2) this coating assists in the dissipation of heat from the resistor, thus enabling it to be reduced in size.

(c) *Carbon composition.* This form of construction is usually employed for low-wattage resistors such as those used in radio circuits. The resistance material consists of carbon mixed with semi-conducting substances to give the desired resistivity, and this is formed into a small cylindrical rod provided with a short wire connection at each end. Alternatively, the carbon composition may be applied as a film to a ceramic rod or glass tube and, if necessary, the resistance increased by spiralling the film with a diamond or carborundum cutter. These resistors are usually coloured in conformity with a recognised code in order to indicate their ohmic values.

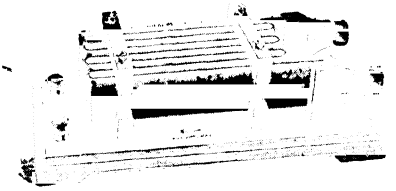
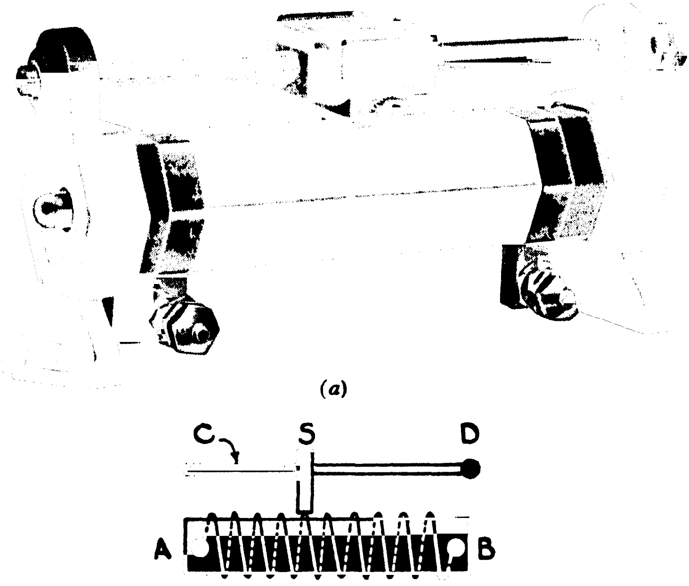


Fig. 25.—A Standard Resistor.

(d) *Standard resistors.* This type of resistor is intended for use in carrying out precise measurements or calibrations. It is manufactured in standard values, e.g., 1 ohm, 0·1 ohm, 0·01 ohm, etc., each standard size being marked with the maximum current it is designed to carry. The resistance element is generally of *manganin* (see Tables IV, V and VI) in the form of wire, rod or strip, on account of the extremely low temperature coefficient of resistance which this material possesses. These standard resistors are usually provided with a pair of substantial terminals for connection to the main current-carrying circuit, and also a pair of smaller terminals, between which the precise value of resistance exists. Fig. 25 illustrates a typical standard resistor of value 0·01 Ω, designed to carry a maximum current of 200 A, the element being constructed of manganin strip. Standard resistors are often immersed in oil to improve the rate of heat dissipation and to ensure that the entire length of the element is at a reasonably uniform temperature.

Variable Resistors

(a) *Rheostats.* This is the term applied to variable resistors in general, apart from those having resistance steps of standard values. A common type of laboratory rheostat is the *tubular pattern* illustrated in Fig. 26a and shown diagrammatically in Fig. 26b. The resistance wire (usually composed of a copper-nickel alloy) is wound in the form of a close spiral on a vitreous enamelled steel tube and connected to the terminals A and B at the ends of the tube. Insulation between adjacent turns of the spiral is provided by an oxide coating on the wire. A contact brush S, capable of movement along the metal rod C, enables the terminal D at one end of this rod to be connected to a series of points along the length of the spiral.



(a)

(b)

FIG. 26.—THE TUBULAR TYPE OF RHEOSTAT.

Thus, when using this rheostat as a variable series resistor, connection is made to terminal D and either A or B. The current then enters the spiral at, say, terminal A, traverses the portion of it between A and the sliding contact S, finally leaving via this contact, the metal rod C and the terminal D. In the particular type of rheostat illustrated in Fig. 26a the contact brush S is of moulded copper graphite, i.e., graphite with fine particles of copper included to reduce the contact resistance with the wire spiral. In other types of tubular rheostat the contact brush is of laminated metal.

An alternative form of rheostat is the *radial pattern* illustrated in Fig. 27; this is a more convenient type for panel-mounting and operates on the same principles as the tubular pattern described above. For high values of resistance the wire-wound element in the radial type of rheostat is usually replaced by a *track* of the carbon composition type.

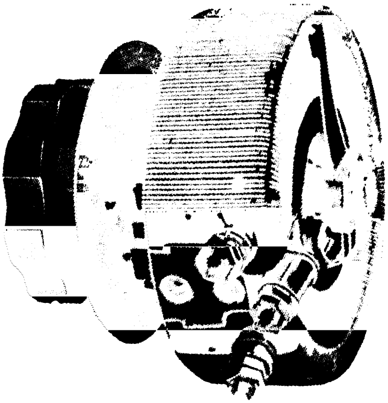


Fig. 27.—The Radial Type of Wire-wound Rheostat.

A slightly different principle is employed in the *stud type* of rheostat frequently used where greater values of power dissipation are involved. The complete rheostat consists of a number of wire-wound elements joined in series and contained in a ventilated metal casing (see Fig. 28). Tappings from these elements are taken to a number of contact studs mounted on the insulating panel. This series group of resistance elements corresponds to the spiral of wire in the tubular pattern of rheostat, but in the stud type only a comparatively small number of definite resistance values are possible, these being made available by moving the radial contact arm over the studs.

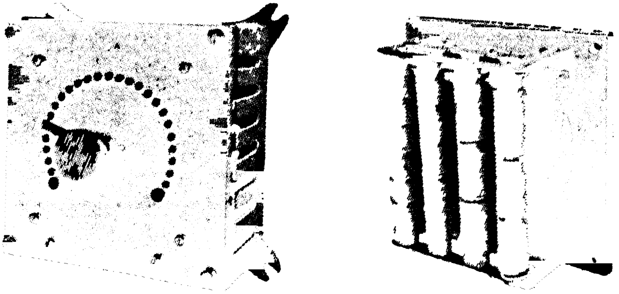


Fig. 28.—The Stud Type of Rheostat.

Any of the above types of rheostat may be used either as a variable series resistor as described in conjunction with Fig. 26b, or alternatively, when a variable voltage supply is required, as a *potentiometer rheostat* (alternative terms being *potential divider* or *potential slide*). Fig. 29a shows the usual connections for this method of control, the symbols A, B, D and S corresponding with those in Fig. 26b. The output voltage between the points X and Y may be varied from zero to the battery e.m.f. E volts by moving the sliding contact S from the end A to the end B of the resistance element, the point X always remaining *positive*. The alternative set of connections shown in Fig. 29b is employed when it is also desired to reverse the *polarity* of the output voltage. With X connected to the centre of the battery as shown, the output p.d. is zero when S is in contact with the mid-point of the resistance element. Moving S towards A makes Y positive and moving it towards B makes Y negative. It should be noted that, with the same total battery e.m.f. as in Fig. 29a, the output p.d. is now variable from zero to $\frac{E}{2}$ volts, i.e., over only one-half the numerical range available in the former case.

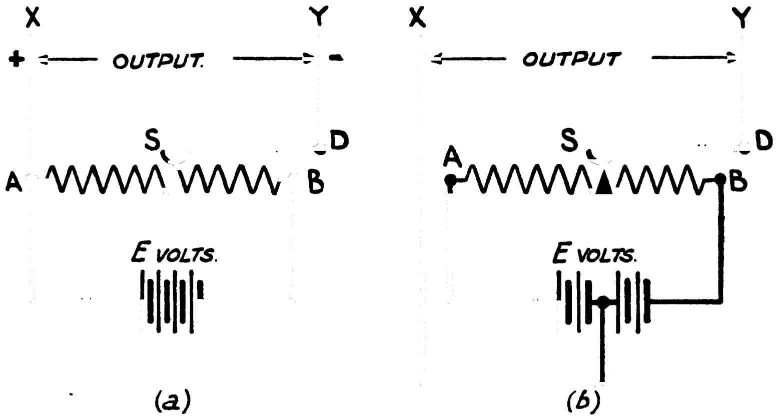


FIG. 29.—POTENTIOMETER RHEOSTAT CONNECTIONS.

Another form of rheostat, particularly useful as a variable series resistor when fairly large currents are being dealt with, is the *carbon-plate* type illustrated in Fig. 30. This consists of a number of carbon plates, clamped together in a frame as shown, the pressure between adjacent plates being adjusted by means of a screw and handwheel. A metal plate provided with a substantial terminal is placed at each end of the stack of carbon plates, thus enabling connection to be made to the circuit leads. An insulating plate is inserted between each of these terminal plates and the metal portion of the rheostat against which it bears. The current path is, therefore, through all the carbon plates in series, the total resistance being made up as follows:—

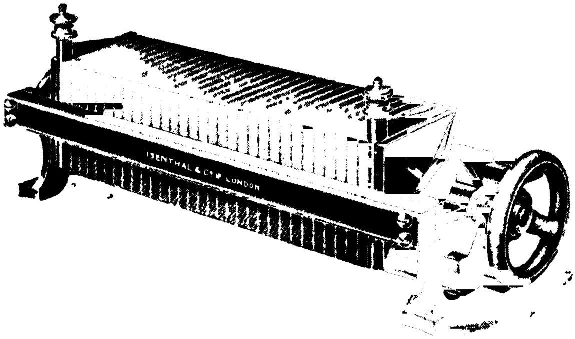


FIG. 30.—THE CARBON-PLATE TYPE OF RHEOSTAT.

(1) The sum of the resistances of the separate carbon plates. This value is quite low (of the order of 0·01 Ω), since the complete block of carbon is relatively short and has a large cross-sectional area;

(2) The sum of the *contact resistances* between the adjacent plates.

This latter part decreases as the clamping pressure is increased, part (1) remaining unaltered. Thus the resistance between the rheostat terminals cannot be decreased to zero, but the minimum value may be reduced by moving the terminal plates closer together, so as to include fewer carbon plates between them. It may also be noted that, for a definite setting of the hand-wheel, the resistance between the rheostat terminals *decreases* as the carbon plates become heated due to the passage of current, since carbon has a negative temperature coefficient of resistance. A typical carbon-plate rheostat, designed to carry 50 A, has a resistance range (utilising all the carbon plates) of approximately 0·1 ohm to 5 ohms.

(b) *Standard resistance boxes.* This type of variable resistor is employed (particularly in certain methods of measuring resistance) when a range of known values of resistance is required. A standard resistance box contains a number of resistance coils of precise ohmic value, joined in series, a typical box of the *plug type* being shown in Fig. 31. The insulating panel forming a cover for the box carries a number of brass blocks which can be joined together by well-fitting conical plugs as illustrated by Fig. 32. Each of these plugs short-circuits one of the standard resistance coils, and thus the total resistance between the terminals of the box can be determined by adding together (since the coils are in series) the resistances of all the coils from which the short-circuiting plugs have been removed. In winding these coils the wire (usually manganin) is doubled on itself as shown, in order to minimise the effects of self-induction (see Vol. II, Chapter III).

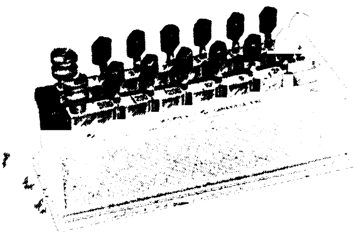


FIG. 31.—THE PLUG TYPE OF RESISTANCE BOX.

An alternative form of standard resistance box is illustrated in Fig. 33, this being termed the *dial type*. In this pattern each rotary dial switch is a miniature stud type of rheostat, each set of contact studs with its radial contact arm being enclosed beneath the insulating cover of the box. The right-hand dial switch (Fig. 33) selects resistance values from 0 to 10 Ω, in steps of 1 Ω, the central dial

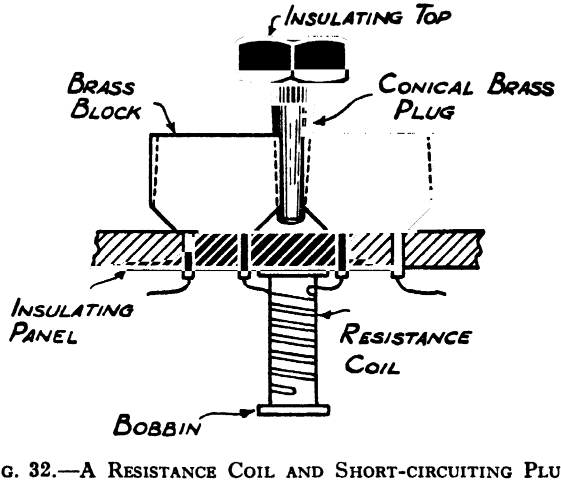


FIG. 32.—A RESISTANCE COIL AND SHORT-CIRCUITING PLUG.

switch gives values from 0 to 100 Ω in steps of 10 Ω, and the left-hand switch—0 to 1000 Ω in steps of 100 Ω. All the dial switches are connected in series, and thus the total resistance between the terminals of the box is obtained by taking the sum of the separate dial readings, e.g., the box as illustrated in Fig. 33 is set to give a resistance value of (8 × 100) + (4 × 10) + (10 × 1) = 850 Ω.

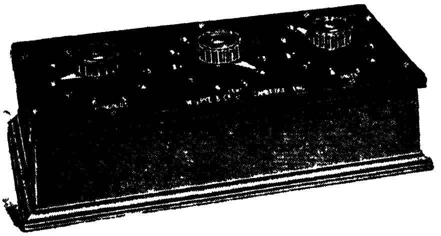


FIG. 33.—THE DIAL TYPE OF RESISTANCE BOX.

In all types of standard resistance box the coils are wound with wire of fairly fine gauge, and consequently the rate of heat dissipation, which depends on the *surface area* of the wire, is relatively low. Thus, the current employed should not exceed a small fraction of an ampere, particularly when the higher-resistance coils are introduced into the circuit, otherwise there is a serious risk of overheating the coils, with consequent damage to the insulation.

It may be stated that for all types of resistor, both fixed and variable, the physical size of the unit depends on the wattage to be dissipated, i.e., on the rate at which heat is to be dissipated from the external surface of the resistance element. A 3-watt fixed resistor, for example, would be considerably larger than one rated at, say, ½-watt.

24. The Measurement of Resistance

Before proceeding to a description of the most useful and important methods of resistance measurement, it is first necessary to outline briefly the chief characteristics of certain indicating instruments, as at least one of these instruments is employed in any method of resistance measurement. The construction and action of certain types of electrical measuring instrument are referred to briefly in Chapter VIII of Vol. I, but the reader will find a more comprehensive treatment in Chapter IX of Vol. II.

(a) Instruments used in resistance measurements

Ammeters and Voltmeters. The methods of connecting up and precautions to be observed when using these instruments have already been dealt with in Section 9 of Chapter II. *An ammeter* *should have a resistance as low as possible*; then when the ammeter is taking full-scale current the power absorbed is low ($P = I^2R$, and I is fixed). If an ammeter possessed a high resistance, its insertion into the circuit would cut down appreciably the current to be measured. On the other hand, *a voltmeter should have a resistance as high as possible*, so as to take a low current when registering the full-scale p.d. This ensures that the power absorbed by the instrument is low ($P = \frac{V^2}{R}$, and V is fixed). If a low-resistance voltmeter was employed for measuring the p.d. across one component of a series circuit (e.g., voltmeter V_1 in Fig. 6), it would cause a drop in the p.d. being measured; this is because the effective resistance between the terminals of this component (i.e., L and M in Fig. 6) would be reduced by connecting the low-resistance voltmeter across them.

The type of ammeter and voltmeter chiefly used in d.c. circuits is the *moving-coil* (M.C.) type, since it has an evenly-divided scale giving reasonable accuracy at quite low scale readings. A good M.C. ammeter is usually designed to have a voltage drop, when taking full-scale current, of about 0·075 volt (75 mV). From the Ohm's law formula we see that such an ammeter, with a full-scale reading of 1 amp., would have a resistance of 0·075 ohm, and with a full-scale reading of 10 amp., of only 0·0075 ohm. The higher the current range the smaller the resistance of the ammeter. Although low-range milliammeters (e.g., 0–10 mA) may have resistances of several ohms, ammeters with full-scale readings of 1 amp. or more have resistances of well under 1 ohm. A good M.C. voltmeter, on the other hand, is usually designed to have a resistance of at least 60 ohms for every volt of full-scale reading, so that a 0–5 voltmeter would have a resistance of several hundred ohms, and for a 0–300 voltmeter the resistance would be many thousands of ohms.

Moving-coil instruments cannot, however, be used with a.c. supplies. In a.c. circuits operating at the usual power frequencies —i.e., 25–100 c/s—*moving-iron* (M.I.) instruments are usually employed, these instruments also being capable of operating satisfactorily on d.c. supplies. Unfortunately M.I. voltmeters usually take much higher currents than M.C. voltmeters, on account of their lower sensitivity. This means that they have lower resistances than corresponding M.C. instruments, and thus are not so satisfactory for general use.

Galvanometers. A galvanometer is merely a very sensitive form of current-measuring (or -detecting) instrument. Galvanometers used in resistance measurements will easily detect a current of 1 mA, and for very sensitive measurements galvanometers may be obtained which will indicate a current considerably less than one-thousandth of a microampere, i.e., 10^{-9} ampere.

Moreover, galvanometers have usually a central zero position, so as to be able to indicate a current flowing in either direction. Thus there is no necessity for marking galvanometer terminals positive and negative, and it is immaterial which way round the galvanometer is connected in the circuit. Further, the galvanometer scale is divided into arbitrary divisions; it is not scaled in mA or any other sub-multiple unit, since in general a galvanometer is used merely for the purpose of indicating when there is no current flowing. Hence the need for extreme sensitivity.

(b) The ammeter and voltmeter method

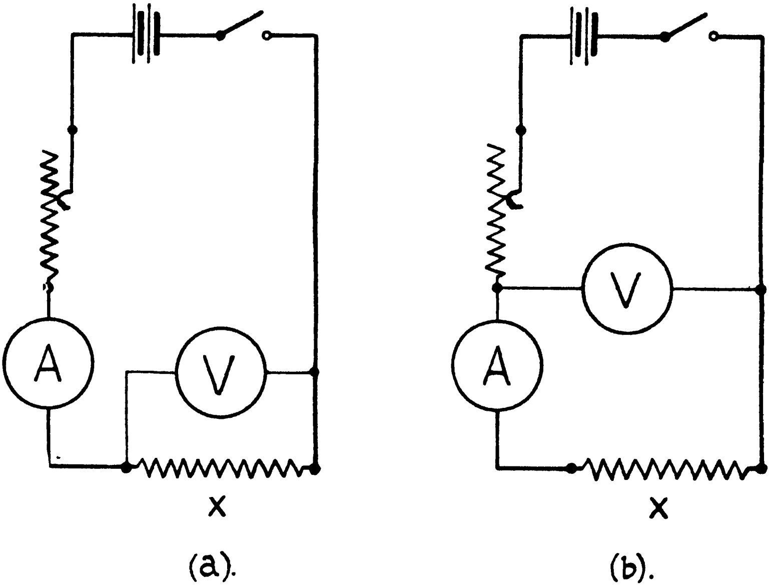
By definition, the resistance of a coil or component is the voltage drop across it divided by the current flowing through it. In this direct method of measuring resistance a current is passed through the component and measured by means of an ammeter, whilst the p.d. across the component is obtained by means of a voltmeter.

There are two ways in which the voltmeter may be connected. If the instrument is connected as in Fig. 34a, the ammeter reads the sum of the currents flowing through the component X and through the voltmeter; thus the ratio *voltmeter reading/ammeter reading* is *lower* than the resistance value required. If the connection of Fig. 34b is employed, the ammeter records the current flowing through the component X, but the voltmeter registers the sum of the p.d.'s across the component X and the ammeter, thus the above-mentioned ratio is now too *high*. The true value of the resistance being measured obviously lies between these two values of the ratio *voltmeter reading/ammeter reading*.

With a M.C. voltmeter the error involved in using the first circuit is usually quite small, since the current through the voltmeter is generally a small fraction of that through the component X. For components of high resistance, however, the main current is usually quite low, and in these cases the connection shown in Fig. 34b is preferable.

With a M.I. voltmeter the use of circuit (a) may introduce an appreciable error, because the resistance of such a voltmeter is usually low. In circuit (b) we are really measuring the resistance of the component X plus that of the ammeter, but the error will not be unduly large unless the resistance of the ammeter is appreciable compared with that of X.

In general, circuit (a) is preferable for measuring low values of resistance, and circuit (b) for high values, but if a M.I. voltmeter is being used it may be advisable to employ circuit (b) when the component has only a moderate resistance value. When using either circuit, a correction can always be applied, if desired, provided that the resistance of the voltmeter or ammeter is known. This method of resistance measurement is not so satisfactory as the Wheatstone bridge method described under (d), since ammeters and voltmeters frequently have errors of 2% or 3% each, but on occasions it is the most convenient method.



(a). (b).

Fig. 34.—The Measurement of Resistance by the Ammeter and Voltmeter Method.

When deciding on the number of cells or the supply voltage to use for this method of measurement, due regard must always be given to the *rating* (i.e., the current-carrying capacity) of the component concerned. It is always advisable to start with a *low* supply voltage and to increase this only if the instruments and the component being tested are capable of withstanding the additional p.d. and current. It should also be noted that the resistance of certain types of equipment depends on the current value employed for the measurement. This occurs with components for which the conducting material possesses a high value of temperature coefficient of resistance (see Section 22), the effect being due to changes in the temperature of the conducting material with the current flowing.

A d.c. supply is preferable for this method of resistance measurement, because for components possessing self-inductance (see Vol. II, Chapter III) the ratio *voltmeter reading/ammeter reading* on an a.c. supply is a measure not of the resistance of the component, but of what is known as the **impedance.** Certain types of component, however, possess negligible values of self-inductance (e.g., filament lamps, electric heating appliances and small fixed resistors), and in these cases it is possible to use an a.c. supply of the standard power frequency (50 c/s) for this method of resistance measurement.

Experiment 3. *To measure the resistance of a coil of eureka wire by the ammeter and voltmeter method.*

Connect up the coil, X, to a M.C. ammeter, M.C. voltmeter, rheostat and battery of accumulators as in Fig. 34a. Ensure that the full resistance of the rheostat is in the circuit, and close the switch. Note the ammeter and voltmeter readings and enter them in the table. Adjust the rheostat several times so as to obtain a wide range of instrument readings, in each case recording the readings in the table. Dividing the voltmeter readings by corresponding ammeter readings gives a series of results in the third column for the resistance of the coil. These values will be found to be constant within the limits of experimental error, and the resistance of the coil may be taken as the mean of these values.

Current (amp.).	P.D. (volts).	Resistance (ohms) $= \frac{\text{P.D.}}{\text{Current}}$.
Mean result: Resistance = ... ohms.		

(c) The substitution method

A simple but effective method of measuring a fairly high resistance is illustrated in Fig. 35. X is the component whose resistance is required, and R a resistance box. The resistance of R is adjusted until the milliammeter indicates a convenient value of current, and X is then replaced by a standard resistance box S. The resistance of S is now adjusted until the milliammeter registers exactly the same current as when the component X was in the circuit. The resistance value at which the standard box S is then set is obviously equal to that of the component X.

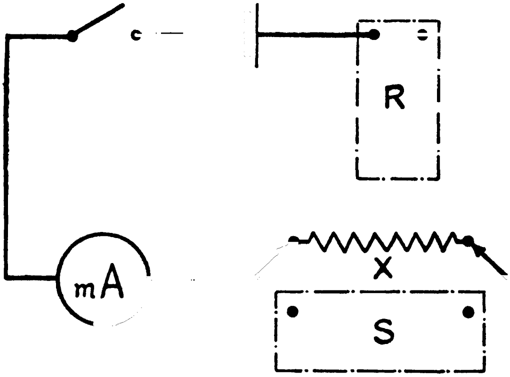


Fig. 35.—The Measurement of Resistance by the Substitution Method.

It is, of course, imperative that the setting of the resistance box R is not disturbed during the change-over from X to S. Also, it is important to ensure that the resistance boxes R and S are not overloaded. If desired, the resistance box R may be omitted, but in this case the final adjustment of S will probably be more difficult, as it may be necessary to set the pointer of the milliammeter to a position intermediate between adjacent scale divisions.

(d) The Wheatstone bridge

The Wheatstone bridge is an arrangement of resistors, a galvanometer and a cell as shown in Fig. 36. This arrangement is used in two forms of apparatus for the measurement of resistance, namely, the *metre bridge* and the practical form of resistance bridge usually known as the *Post Office box.* In connecting up the *network*, the 4 resistors should be wired up first to form a closed circuit. Note the 4 points A, B, C and D, each point lying at the junction between

2 resistors; a galvanometer should *bridge* a pair of *opposite* points such as C and D, and a cell the other pair, A and B.

Suppose the 4 resistors to have resistances R_1, R_2, R_3 and R_4 ohms respectively, and imagine the cell to supply a current I amp. which splits up at A into I_1 and I_2 amp. Whether current flows through the galvanometer from C to D or in the reverse direction depends upon whether C or D is at the higher potential. If the p.d. between the two points is zero, then the current flowing through the galvanometer will be zero. Suppose the 4 resistance values are such that the galvanometer current *is* zero. Then the current I_1 amp. after flowing from A to C continues unchanged from C to B and the current I_2 amp. will similarly flow from A to D and thence to B, these currents re-combining at B. Further, since there is no p.d. between C and D, the p.d. between A and C is equal to the p.d. between A and D. Let us denote this common p.d. by V_A volts.

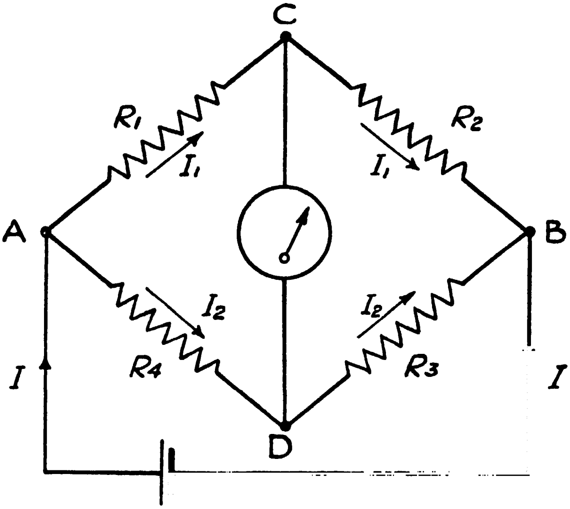


FIG. 36.—THE WHEATSTONE BRIDGE CIRCUIT.

Then, $V_A = I_1R_1$; also, $V_A = I_2R_4$

$$\therefore \quad I_1R_1 = I_2R_4$$

or,
$$\frac{I_1}{I_2} = \frac{R_4}{R_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

Similarly the p.d. between C and B is equal to the p.d. between D and B; let us denote *this* common p.d. by V_B volts.

Then, $V_B = I_1R_2$; also, $V_B = I_2R_3$

$$\therefore \quad I_1R_2 = I_2R_3$$

or, $$\frac{I_1}{I_2} = \frac{R_3}{R_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

From equations (a) and (b) it follows that $\frac{R_4}{R_1} = \frac{R_3}{R_2}$,

i.e., $$\frac{R_1}{R_2} = \frac{R_4}{R_3} \quad \text{or,} \quad R_4 = R_3 \times \frac{R_1}{R_2}$$

This relationship between the 4 resistance values has been arrived at solely on the assumption that no current flows through the galvanometer; so, if we can arrange for this condition to be brought about, then we know that this relationship between R_1, R_2, R_3 and R_4 holds good.

In using this principle for the measurement of a resistance R_4 ohms, we adjust the value of a variable standard resistor (R_3 ohms) and that of the ratio $\frac{R_1}{R_2}$ until no current flows through the galvanometer. We are then able to deduce the value of the unknown resistance (R_4 ohms) from the values R_3 ohms and the ratio $\frac{R_1}{R_2}$.

It should be noted that exactly the same relationship between R_1, R_2, R_3 and R_4 would have been obtained if the positions occupied in the circuit by the galvanometer and the cell had been interchanged; therefore, so long as they are each connected across a different pair of *opposite* points, which way round it is done is quite immaterial.

The metre bridge. This consists of three strips of copper, X, Y and Z, of negligible resistance, fixed to the surface of a board rather more than a metre in length and roughly 6 inches wide (Fig. 37). The copper strips are arranged with two gaps for the insertion of a standard resistance box (R_3 ohms) and the component whose resistance (R_4 ohms) is being determined. The copper strips X and Z are connected by a 1-metre length of hard, high-resistance wire (e.g., platinoid or nickel silver) of uniform cross-section, a millimetre scale being fixed alongside this wire. A sliding key K enables contact between one terminal of the galvanometer and the wire AB to be made at the point C, the wire AB being thus divided into two portions, AC and CB, having resistances R_1 and R_2 ohms respectively. The ratio $\frac{R_1}{R_2}$ is thus infinitely variable, its value being equal to the ratio of the lengths AC and CB. The other terminal of the galvanometer is connected to the mid-point D of the central copper strip Y. Finally, a Leclanché cell and switch S are connected in series between the points A and B.

A careful study of Figs. 36 and 37 will show that the electrical circuits are identical, components having resistances R_1, R_2, R_3 and R_4 ohms in each case forming a closed circuit, with a cell connected across one pair of opposite points A and B, and a galvanometer across the other pair, C and D. If, when the switch S and the sliding key K are closed, no current flows through the galvanometer, then:—

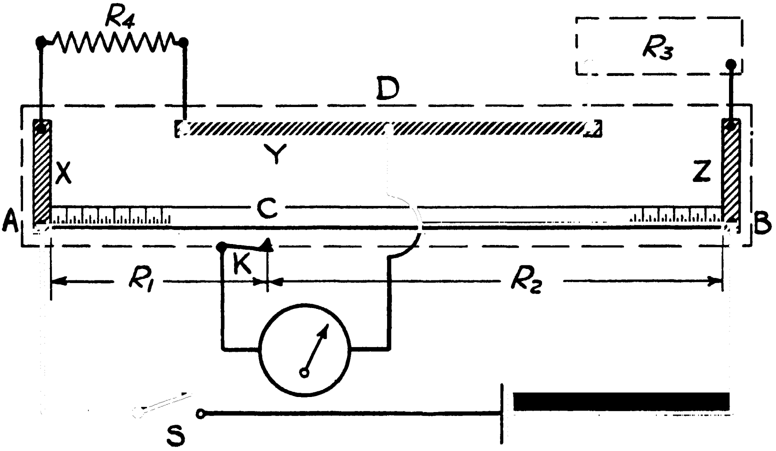


FIG. 37.—THE METRE BRIDGE.

$$R_4 = R_3 \times \frac{R_1}{R_2}$$

or,

$$R_4 = R_3 \times \frac{AC}{CB}$$

The *accuracy* with which measurements can be made with a metre bridge is largely determined by the extent to which the slide-wire is of uniform cross-section. In accurate bridge work it is essential to calibrate the wire so that the ratio $\frac{R_1}{R_2}$ is known precisely for every position of the variable point C. Apart from lack of uniformity in the wire, there is another factor responsible for producing a slight inequality between the ratios $\frac{R_1}{R_2}$ and $\frac{AC}{CB}$; this is the slight contact resistance between each end of the wire and the copper strip to which it is connected. But for measurements of resistance to an accuracy of 1% it can be assumed that the ratios $\frac{R_1}{R_2}$ and $\frac{\mathrm{AC}}{\mathrm{CB}}$ *are* equal.

The *sensitivity* of a metre bridge is largely determined by the sensitivity of the galvanometer. Unless the galvanometer used is very sensitive it may be found possible to move the point of contact C several millimetres along the wire without causing any appreciable movement of the galvanometer pointer. In such a case it is important to observe the range through which the point C can be moved without an appreciable deflection of the pointer, and to regard the point of balance as being at the middle of this small range.

The metre bridge, whilst serving admirably to illustrate the principle of the Wheatstone bridge, is not a convenient piece of equipment for industrial measurements of resistance, chiefly on account of its size, but also due to the limitations as regards accuracy mentioned above. Most practical *resistance bridges* are of the type usually referred to as the Post Office box.

Experiment 4. *To measure a resistance by means of a metre bridge.*

Set up the circuit shown in Fig. 37. The standard resistance box (assumed to be of the plug type) should have a maximum resistance at least equal to that of the component or resistor under investigation (R_4 ohms). Remove, say, a 10 Ω plug from the resistance box, ensuring that the remaining plugs are fitting tightly in their sockets. Close the switch S and by means of the sliding key K put the galvanometer in contact, first with the end A of the wire AB, and then with the other end, B. If the galvanometer is not deflected in opposite directions in the two cases, there is a fault in the circuit; if it is deflected in opposite directions, it is possible to discover some point on the wire where the galvanometer indicates zero current. Determine this *point of balance.* Calculate the value of the unknown resistance R_4 ohms from the formula:

$$R_4 = R_3 \times \frac{\mathrm{AC}}{\mathrm{CB}}$$

Having thus obtained a value for R_4, remove plugs from the standard resistance box to this value approximately and determine a new point of balance. Since R_3 has now been made approximately equal to R_4, the new point of balance will occur near the centre of the wire, since AC must be roughly equal to CB. (Each of these lengths can now be measured to an accuracy of 1 part in 500, whereas if AC is, say, 5 cm. in length, it can be measured to an accuracy of only 1 part in 50. The accuracy of the final result cannot, of course, be greater than that of any of the measurements leading to the result. Thus, the final measurements should always be taken with the point C approximately at the centre of the wire.) Re-calculate the value of the unknown resistance, R_4 ohms, using the above formula, and state the accuracy to which the result can be given, bearing in mind the range through which the contact point C can be moved without causing any movement of the galvanometer needle.

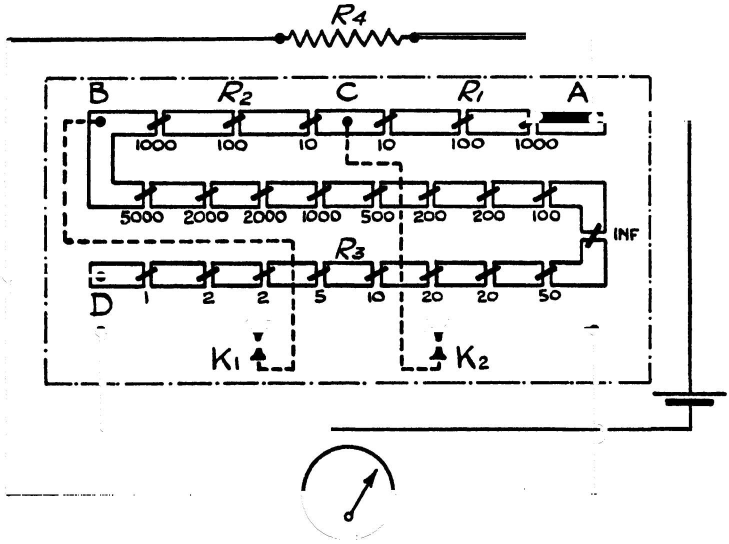


Fig. 38.—The Plug Type of Post Office Box.

The Post Office box. This form of resistance bridge is constructed on the same principle as the standard resistance box described in Section 23, being either of the plug or dial type. Fig. 38 illustrates the plug type. The box contains three sets of resistance coils, two of these sets being identical (known as the *ratio arms*) and forming the resistances R_1 and R_2 ohms of the bridge network shown in Fig. 36. The remaining set of resistance coils forms the standard resistance (R_3 ohms). By connecting the unknown resistance (R_4 ohms) as indicated in Fig. 38 we have components possessing resistances R_1, R_2, R_3 and R_4 ohms forming a closed circuit as in the conventional bridge network (Fig. 36), with the points A, B, C and D each between two arms of the network. The Leclanché cell and galvanometer, if connected as in Fig. 38, each bridge a pair of *opposite* points. A tapping key K_1 is included in series with the cell, and a second key K_2 in series with the galvanometer, the dotted lines in Fig. 38 indicating permanent connections of negligible resistance below the top panel of the box.

It may be noted that this type of resistance bridge is widely used for industrial measurements of resistance. The term " Post Office box " arises from the fact that this form of bridge was used at the time of its introduction by Post Office engineers for resistance measurements in connection with communication circuits and apparatus.

Experiment 5. *To measure a resistance by means of a Post Office box.*

Connect up the box as in Fig. 38, and then test the circuit by taking a 10 Ω plug out of each of the ratio arms, at the same time ensuring that the remaining plugs fit tightly in their sockets. Close first the battery key K_1 and then the galvanometer key K_2. Note the direction in which the galvanometer pointer is deflected. Then remove the infinity plug from the standard resistance arm (R_3 ohms), which has the effect of introducing a gap into this arm. Unless the galvanometer needle is deflected, on closing both keys, in the direction opposite to that in which it was deflected when R_3 was zero, there is a fault in the circuit. A frequent cause of trouble is that one or more plugs are loose in their sockets.

When this test indicates that the circuit is free from any fault, proceed to measure the unknown resistance by removing plugs from the arm R_3 until a balance is obtained. Whenever the galvanometer deflection is in the same direction as when R_3 was zero, more resistance must be put into this arm of the network. When balance has been obtained, the standard resistance R_3 ohms will be equal to the unknown resistance R_4 ohms, since

$$R_4 = R_3 \times \frac{R_1}{R_2} \quad \text{and} \quad R_1 = R_2$$

Actually it will be very much of a coincidence if an exact balance is obtained. Supposing, for example, that the value of the unknown resistance (R_4 ohms) = 57·34 Ω. Then, when R_3 = 57, the galvanometer pointer will be deflected in one direction, and when $R_3 = 58$, in the other direction. The only conclusion would be that the value of R_4 was between 57 and 58. Greater sensitivity can be obtained, however, as follows:—

Make $R_1 = 10$ and $R_2 = 100$, the ratio $\frac{R_1}{R_2}$ becoming $\frac{1}{10}$

A balance can now be obtained only when R_3 is 10 times R_4. Knowing R_4 approximately, give R_3 a value 10 times that of R_4 and adjust until a balance is obtained as before. Referring again to the above numerical example, the galvanometer pointer will show a slight deflection when R_3 is made equal to 573, and a slight deflection in the opposite direction when R_3 has the value 574. Hence it could be deduced that the value of R_4 was between 57·3 and 57·4.

The process should be repeated with $R_2 = 1000$, and R_1 still 10, a $\frac{1}{100}$ ratio being thus employed. In this latter case very small currents indeed pass through the galvanometer owing to the high resistances involved, so unless the galvanometer is very sensitive it may indicate zero current for a small range of values of R_3. The middle of this range should be used to calculate R_4 from the formula $R_4 = R_3 \times \frac{R_1}{R_2}$ and the percentage accuracy of the result stated.

In testing for zero deflection the battery key K_1 (Fig. 38) should always be depressed first; otherwise it is possible that the galvanometer pointer may be deflected initially in the wrong direction, due to the effects of self-induction (see Vol. II, Chapter III). This pitfall is avoided if a pause is made between the closing of the battery and galvanometer keys, since the p.d. between the points C and D will have then become steady when the galvanometer is put into the circuit.

Experiment 6. *To measure the resistivity of a sample of resistance wire.*

Connect the ends of a length of several metres of resistance wire (e.g., manganin, constantan or nichrome—see Table IV) to a Post Office box as described in connection with Experiment 5. Measure the resistance as accurately as possible. Then, allowing for the short lengths of the wire clamped under the P.O. box terminals, measure the length of wire lying between the terminals. Using a micrometer, measure the diameter of the wire at various points along its length and calculate the mean cross-sectional area. Substituting in the formula $\rho = \frac{Ra}{l}$, calculate the resistivity of the material, stating the percentage accuracy to which it has been determined.

Experiment 7. *To measure the temperature coefficient of a sample of copper wire.*

The sample, preferably in the form of fine-gauge enamelled wire, is closely wound as a single layer on a tube of glass, ceramic or other insulating material about $\frac{3}{4}$ in. diameter. The use of a fine-gauge wire enables a fairly high initial resistance to be obtained, and thus a change of resistance large enough to be measured with reasonable accuracy. The ends of the coil should be joined to heavy copper leads connected to fixed terminals at the top of the tube (see Fig. 39) or to flexible copper leads, and thence to a P.O. box circuit. The wound portion of the tube should be immersed in a boiling tube containing transformer oil, also a thermometer. After ensuring that the temperature of the oil is steady, the resistance of the coil should be measured and the corresponding temperature noted. The temperature of the oil should then be raised by about 25° C. by heating the surrounding water-bath, and when the temperature of the oil is again steady, the resistance should be determined and the temperature recorded. This should be repeated for two further values of tem-

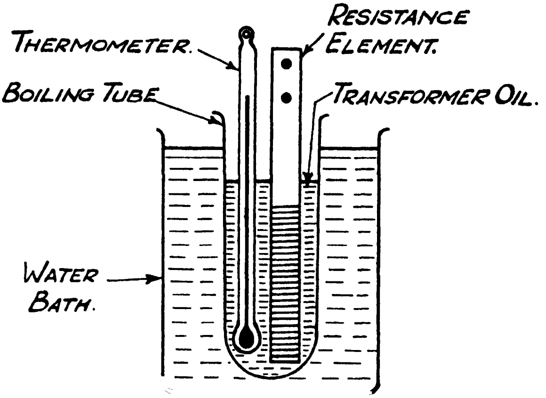


Fig. 39.—Apparatus for Expt. 7.

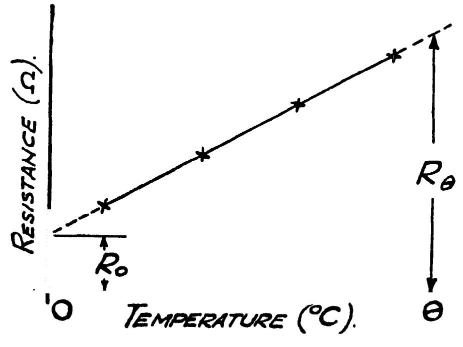


Fig. 40.—Resistance/Temperature Graph obtained in Expt. 7.

perature, the final measurement being made at about 90° C. The values of resistance should be plotted against those of temperature as in Fig. 40, and a straight line drawn so that the experimental points lie evenly about it. The values of R_0 and R_θ should be determined from this linear graph, θ being any convenient temperature (say 100° C.). The temperature coefficient of resistance may be calculated from Equation II of Section 22, viz.:—

$$\alpha = \frac{R_\theta - R_0}{R_0\theta}$$

Experiment 8. *To verify the formulæ for resistors in series and parallel.*

Using a P.O. box or metre bridge, measure the resistance of each of 3 resistors. Then connect them in series and measure the effective resistance, R_s ohms, comparing this value with that obtained from the formula $R_s = R_1 + R_2 + R_3$.

Similarly connect the 3 resistors in parallel and measure the combined resistance R_p ohms. Determine this value also from the measured values R_1, R_2 and R_3 ohms, for the separate resistors, making use of the formula

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Enter the results in a table as shown.

Individual resistances Measured values (ohms)			Combined resistances			
			Measured values (ohms).		Calculated values (ohms).	
R_1.	R_2	R_3	R_s	R_p	R_s	R_p.

25. The Internal Resistance of Cells

When a cell delivers current to an external circuit this current must flow through the cell itself. Since the cell is composed of materials which are not perfect conductors, the conducting path inside the cell will offer some resistance to the flow of current, this resistance being known as the **internal resistance** of the cell. It will be assumed in the following theoretical treatment that both the e.m.f. of the cell and its internal resistance are independent of current flowing through the cell, although in actual practice certain small changes in these quantities frequently occur.

Consider the circuit shown in Fig. 41. If the switch be closed, the voltmeter will indicate the voltage drop in the circuit from the point X, to which the positive terminal of the voltmeter is connected, to the point Y, which is joined to the negative terminal of the instrument. The voltmeter reading is referred to as the *terminal potential difference* (T.P.D.), since the voltmeter is connected across the terminals of the cell. This should not mislead the student into believing that the voltmeter will measure *the voltage drop in the cell.*

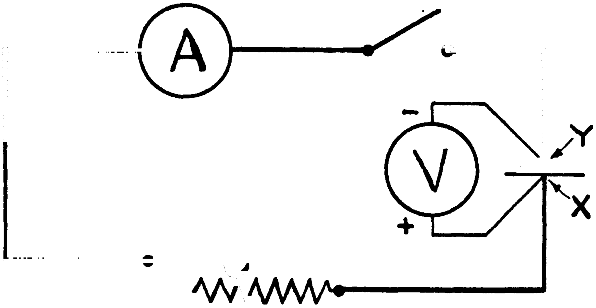


FIG. 41.—THE CIRCUIT EMPLOYED IN EXPT. 9.

The instrument certainly is connected across the terminals of the cell, but at the same time it is connected across the extreme ends of that part of the circuit external to the cell, and a voltmeter reading is a measure of the voltage drop from the point to which the positive terminal is connected, *round the circuit in the direction in which the current is flowing*, to the point connected to the negative terminal. Thus, the T.P.D. is a measure of the voltage drop in the external circuit, i.e., of the energy lost by each coulomb as it traverses the entire external circuit.

Let us denote the T.P.D. by V volts and the e.m.f. of the cell by E volts. Then E represents the number of joules given to a coulomb by the cell. The coulomb loses V joules in the external circuit, and therefore the remaining energy is lost in passing through the cell itself. Alternatively, in terms of p.d.,

$$\text{E.M.F. in volts} = (\text{Voltage drop in the external circuit}) + (\text{Voltage drop in the cell}).$$

If a current of I amp. is flowing and the internal resistance of the cell is r ohms, the voltage drop in the cell will be equal to Ir volts, so,

$$E = V + Ir, \quad \text{or} \quad V = E - Ir \quad . \quad . \quad . \quad \text{(a)}$$

It follows from this equation that if the current is increased, the terminal p.d. will fall. That this is the case may be verified, using the circuit of Fig. 41. The manner in which the T.P.D. falls with increasing current is shown in Fig. 42. If the current be reduced to zero the terminal p.d. will increase until it is numerically equal to the e.m.f. This is why it is possible to measure the e.m.f. of the cell by measuring the terminal p.d. *on open-circuit*, i.e., with the switch in Fig. 41 open; in such a case the only current flowing through the cell is that taken by the voltmeter. This latter current is very small owing to the high resistance of the voltmeter, and so the product of the current, I amp., flowing through the cell, and the internal resistance, r ohms, of the cell is quite negligible compared with the e.m.f. of the cell. Thus, with a cell on open-circuit, the e.m.f., E volts, is numerically equal to the terminal p.d., V volts, recorded on a high-resistance voltmeter connected across the terminals of the cell. If the switch is closed and the resistance of the circuit adjusted so that an appreciable current flows through the cell, the voltmeter reading will no longer be approximately equal to the e.m.f.

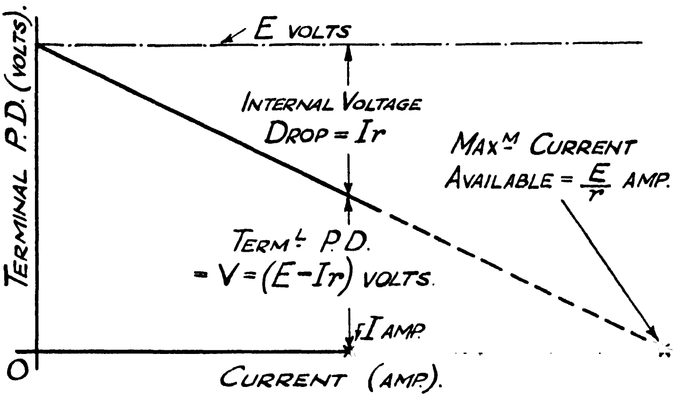


FIG. 42.—THE VARIATION OF T.P.D. WITH CURRENT TAKEN FROM A CELL.

Denoting the resistance of the external circuit by R ohms, and supposing a current I amp. to be taken from the cell, then the T.P.D. of V volts will be equal to IR volts, and the voltage drop in the cell itself will be equal to Ir volts. These two p.d's. will be together equal numerically to the e.m.f. E volts,

i.e.,
$$E = IR + Ir$$
$$= I(R + r)$$

i.e.,
$$\mathbf{I} = \mathbf{E}/(\mathbf{R} + \mathbf{r}) \quad . \quad . \quad . \quad . \quad . \quad \text{(b)}$$

Since $V = IR$ we have also,

$$I = \frac{V}{R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(c)}$$

Thus the current which flows through the circuit can be determined by dividing the e.m.f. by the total resistance of the circuit; this is because the e.m.f. is numerically equal to the total voltage drop in the circuit (see Experiment 1, Chapter II). Alternatively the current flowing may be calculated by dividing the terminal p.d. by the resistance of the external circuit only. Note the statement in Section 13 (Chapter II) that the Ohm's law formulæ can be applied to any part of a circuit or to a complete circuit; the full significance of this statement is now apparent.

The maximum current that can be taken from any cell is limited by the internal resistance of the cell. In the extreme case, when the terminals of the cell are *short-circuited* (i.e., joined by a short connection of negligible resistance), the internal resistance, r ohms, becomes the only resistance in the circuit, and so equation (b) reduces to $I = E/r$. So if the e.m.f. of a cell is 1·5 volts and the internal resistance 0·75 Ω, the maximum current obtainable from such a cell is 2 amp. (Note that when the terminals of a cell are short-circuited the terminal p.d., V, must be zero since $V = IR$ and R is zero.) If a resistance of 0·75 Ω be connected across such a cell, a current of 1 amp. will flow, but half the power will be wasted in the cell itself. Moreover, if currents varying from zero to 2 amp. are taken from the cell, the terminal p.d., i.e., the p.d. across the ends of the external circuit, will vary over the entire range from 1·5 volts to zero. For these reasons it is very desirable that a cell should have as low an internal resistance as possible. One case where it is absolutely essential that the internal resistance should be negligible is that of the motor-car starter battery. On cold mornings, owing to the stiffness of the oil, a current of hundreds of amperes may be necessary to turn the engine over. Supposing, for example, that a current of 240 amp. must be supplied from a 12-volt battery before the starter motor can overcome the load. For a 12-volt battery to supply 240 amp. the total resistance of the circuit must not exceed $12/240 = 0{\cdot}05\ \Omega$, and since this includes the resistance of the leads, the motor and the various electrical contacts, the internal resistance of the battery must be very much less than $0{\cdot}05\ \Omega$. Fortunately, accumulators can be designed to have negligibly small internal resistances. For this reason it is particularly important that their terminals should never be short-circuited. If a typical portable acid accumulator has an e.m.f. of about 2 volts, a maximum safe current of 2 or 3 amp. and an internal resistance of the order of $0{\cdot}1\ \Omega$, short-circuiting such an accumulator would cause a current of $2/0{\cdot}1 = 20$ amp. to flow, with consequent overheating which would ruin the cell if continued for an appreciable time.

The fact that the terminal p.d. is less than the e.m.f. when a current is taken from the source of e.m.f. is true not only of cells, but also of dynamos and, in fact, all sources of e.m.f. Since most electrical appliances are designed for operation on a particular p.d., it is essential that the e.m.f. of the supply be greater than the required p.d., in order to allow for the internal voltage drop. Since the e.m.f. is never *applied* to the equipment direct, but is the potential difference applied to the whole circuit, we frequently use the expression *applied p.d.* when we wish to refer to the voltage actually applied to the equipment. This will be equal to *the e.m.f. less the voltage drop* in the dynamo and leads. Thus, to use in the correct manner a lamp designed for *an applied p.d. of* 200 *volts*, the e.m.f. generated must be numerically equal to the voltage drop in the generator and leads plus the 200 volts drop across the lamp.

Experiment 9. *To determine how the terminal p.d. varies with the current taken from (a) a Leclanché cell, (b) a portable accumulator, and to measure the internal resistances of the cells.*

Connect up a circuit similar to that shown in Fig. 41, using either a wet Leclanché or a dry cell of the type used for operating electric bells and a milliammeter instead of the ammeter shown. Close the switch and adjust the rheostat to give a current of about 10 mA. Keep the current at 10 mA for a few minutes, during which time the terminal p.d. will possibly fall slightly. After 4 or 5 minutes, however, the voltmeter reading will remain steady. Record this steady reading of the voltmeter and also the milliammeter reading. Open the switch and take the voltmeter reading *immediately after opening the switch.* It is essential to take the voltmeter reading *immediately* after opening the switch, since after a momentary pause the e.m.f. commences to rise. This immediate reading gives the e.m.f., E volts, corresponding to the steady terminal p.d., V volts, recorded by the voltmeter before opening the switch. Calculate the internal resistance of the cell from the formula: $r = \frac{E - V}{I}$ deduced from equation (a) above.

Repeat the experiment with the same precautions and using currents of 20, 30, 40 and 50 milliamperes, allowing a few minutes after each test for the e.m.f. to rise again to its original value. Enter the results in a table as shown below and determine a mean value for the internal resistance of the cell.

I (amp.).	V (volts).	E (volts).	r (ohms).

Mean value: r(*ohms*) = . . .

Plot the values obtained for the terminal p.d. against corresponding current values. This graph will be a straight line as shown in Fig. 42 only if the e.m.f. and resistance of the cell remain appreciably constant throughout the experiment.

Repeat the experiment with a portable accumulator in place of the Leclanché cell and an ammeter instead of the milliammeter, taking readings for 3 or 4 current values up to the normal discharge current for the cell. It may be necessary in this case to allow 10 minutes or more for the terminal p.d. to settle down to its final steady value. On opening the switch the rise in the voltmeter reading will be minute, since the internal resistance of an accumulator is so very small. It will, therefore, be essential to observe the two readings of the voltmeter with exceptional care. Although, owing to this very small change in the voltmeter reading, there may be an appreciable percentage error in the values deduced for the internal resistance, the measurements will serve to indicate the difference between the magnitudes of the internal resistances of the two types of cell.

26. Cells Connected in Series and Parallel

As was explained in Section 9 of Chapter II, cells may be connected in series if a greater e.m.f. than that of a single cell is required. If each of the 4 cells shown connected in series in Fig. 43 possesses an e.m.f. of 1·5 volts and an internal resistance of 0·2 ohm, the e.m.f. of the battery as a whole will be 6 volts and the internal resistance 0·8 ohm. If the dimensions of each cell be such that it is undesirable to take a current of more than 1 amp. from each cell, then no more than 1 amp. must be taken from the battery of 4 cells, since the current supplied by the battery passes through each cell in turn. The e.m.f. of the battery, however, is 4 times that of a single cell, and thus the total power and energy available will be 4 times that available from each cell.

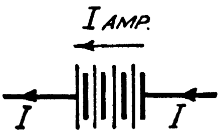


FIG. 43.—A BATTERY OF SERIES-CONNECTED CELLS.

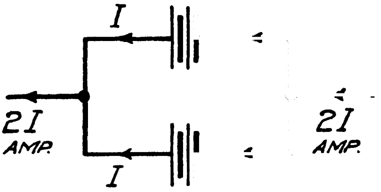


FIG. 45.—MIXED GROUPING OF CELLS.

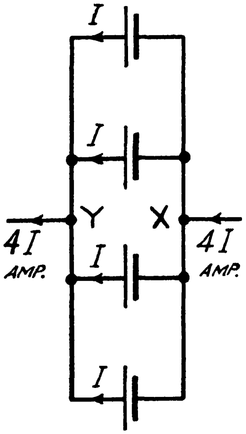


FIG. 44.—A BATTERY OF PARALLEL-CONNECTED CELLS.

In this case of 4 cells connected in series the e.m.f. of the battery is 4 times that of a single cell because a coulomb passes through each cell in turn, receiving 1·5 joules from each cell, and so 6 joules from the whole battery. But if the 4 cells are arranged in parallel, as in Fig. 44, the current splits up at X, one quarter of the total current passing through each cell; thus any one coulomb passes through one cell only and receives only 1·5 joules of energy. So the e.m.f. of the battery is 1·5 volts, and is equal to the e.m.f. of a single cell. As regards the internal resistance of the battery, the problem is simply one of resistors connected in parallel; thus, since each cell has an internal resistance of 0·2 ohm, the resistance of the 4 cells in parallel will be one quarter of this value, namely, 0·05 ohm. The advantage to be gained by connecting the 4 cells in this manner is that 4 times as much current may be taken from the whole battery as may safely be taken from a single cell.

Parallel arrangements of cells are very seldom used in practice, since the e.m.f's. of a number of cells are never precisely equal to one another, and as a result the total current is unevenly distributed between the various cells. It is even possible for current to flow through some of the cells in the reverse direction if their e.m.f.'s are appreciably lower than the e.m.f's. of other cells in parallel with them. (See Section 28.) But it is common practice to connect d.c. generators in parallel, adjusting the e.m.f's. so as to avoid such unequal sharing of the load current.

Fig. 45 shows how it is possible to combine series and parallel grouping. Thus if each of the cells has an e.m.f. of 1·5 volts, an internal resistance of 0·2 ohm and can safely supply a current of 1 amp., the battery of Fig. 45 possesses an e.m.f. of 3 volts, an internal resistance of 0·2 ohm and can safely supply a current of 2 amp. The internal resistance is 0·2 ohm, because there are two batteries in parallel, each of internal resistance 0·4 ohm.

Considering the three arrangements shown in Figs. 43, 44 and 45, it will be noticed that when the maximum safe current is taken from each battery, then each battery is developing a power of 6 watts, i.e., 4 times that which can be developed by a single cell supplying its rated current.

27. Kirchhoff's Laws

The circuit of Fig. 46 is so complicated that the connections can hardly be regarded as being a mixture of series and parallel arrangements, so such a circuit is referred to as a *network*. A practical example of such a network is the Wheatstone bridge circuit discussed in Section 24. In dealing with the theory of such networks Kirchhoff's two laws are almost indispensable. In view of what has already been said regarding the electric circuit, the truth of the laws is obvious; but it is frequently the case in scientific work that the simplest laws are the most useful and the most fundamental.

Kirchhoff's two laws are as follows:—

1. **In any network the sum of the currents flowing towards a point is equal to the sum of the currents flowing away from the point.**
2. **In any closed path in a network the algebraic sum of the e.m.f's. is equal to the algebraic sum of the potential differences (or products of current and resistance for each part of the closed path).**

The word *algebraic* is used to deal with the question of the sign. Thus, if one follows path ABK (Fig. 46) in a clockwise direction, terming the e.m.f. E_1 volts positive, then the e.m.f. E_3 volts must be regarded as negative, and the algebraic sum of the e.m.f's. in this path will then be $(E_1 - E_3)$ volts. Regarding the cells in the circuit of Fig. 46 as having negligible values of internal resistance, the algebraic sum of the p.d's. in path ABK—still going in a clockwise direction—will be $(R_2I_2 - R_3I_3)$ volts, since if I_2 amp. be considered a positive current, I_3 amp. must be regarded as negative; there will be a voltage drop in going from A to B, but voltage rise in going from B to H. Thus, according to Kirchhoff's second law:—

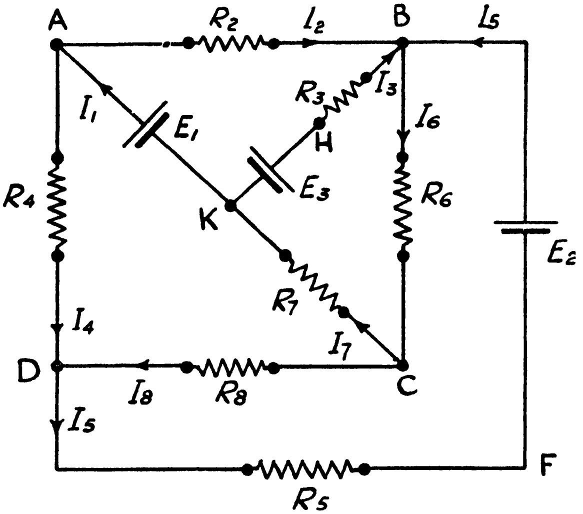


FIG. 46.—NETWORK USED TO ILLUSTRATE KIRCHHOFF'S LAWS—SECTION 27.

$$E_1 - E_3 = R_2I_2 - R_3I_3$$

Applying the first law to point B of the circuit:—

$$I_2 + I_3 + I_5 = I_6$$

This first law may be applied to every junction of wires in the circuit. The second law may be applied to every closed path in the circuit, even to such a complicated path as KBADCK.

The first law is obviously true, since the total number of coulombs leaving a point each second must be equal to the total number arriving at the point each second. The second law can perhaps be best appreciated by studying Fig. 47. The four horizontal lines represent, in a graphical form, the *potential levels* of points K, A, B and H respectively. (By ***potential level***, or merely ***potential***, is meant the number of joules of energy possessed by a coulomb at a particular point in the circuit.) In going from K to A there is a rise in potential of E_1 volts, from A to B a fall of R_2I_2 volts, from B to H a rise of R_3I_3 volts and from H to K a fall of E_3 volts. It is obvious from Fig. 47 that:—

$$E_1 - E_3 = R_2I_2 - R_3I_3$$

28. The Application of Kirchhoff's Laws

Although Kirchhoff's laws will be applied to various problems throughout this and the succeeding volumes, it is desirable that the manner in which they can be applied should be demonstrated at once, so three examples of their use are given below.

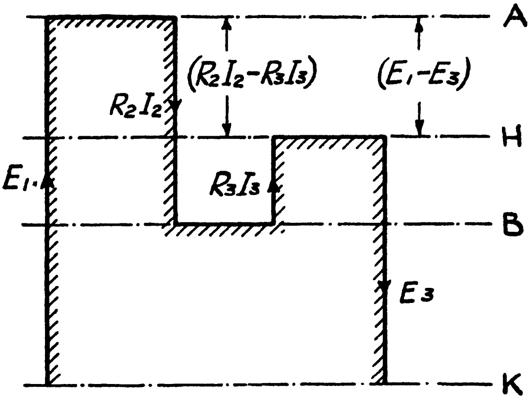


FIG. 47.—ILLUSTRATING KIRCHHOFF'S SECOND LAW.

Example 1. The formula for a balanced Wheatstone bridge circuit may be rapidly deduced with the aid of Kirchhoff's second law. Referring to Fig. 36, and assuming no current through the galvanometer, then, applying the second law (a) to the closed path ACD and (b) to the path CBD, we get:—

$$\left.\begin{aligned}&\text{(a)}\quad R_1I_1 - R_4I_2 = 0\\&\text{(b)}\quad R_2I_1 - R_3I_2 = 0\end{aligned}\right\}\ \text{since there is no source of e.m.f. in either of these closed circuits.}$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{R_4}{R_1} = \frac{R_3}{R_2} \qquad \text{Thus,} \quad R_4 = R_3 \times \frac{R_1}{R_2}$$

Example 2. *To calculate the currents flowing in the various parts of the circuit shown in Fig.* 48, *in which* 2 *batteries of e.m.f.* 20 *and* 21 *V. and internal resistance* 0·8 *and* 2·0 Ω *respectively are connected in parallel and supply current to a* 50 Ω *resistor.*

Applying Kirchhoff's first law to the point A, the currents in the 3 branches being denoted by I, I_1 and I_2 amp. respectively, we get:—

$$I = I_1 + I_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

Applying the second law to the closed path ACBDA and working in a counter-clockwise direction, we get:—

$$21 - 20 = 2I_1 - 0{\cdot}8I_2$$

or,

$$2I_1 - 0{\cdot}8I_2 = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

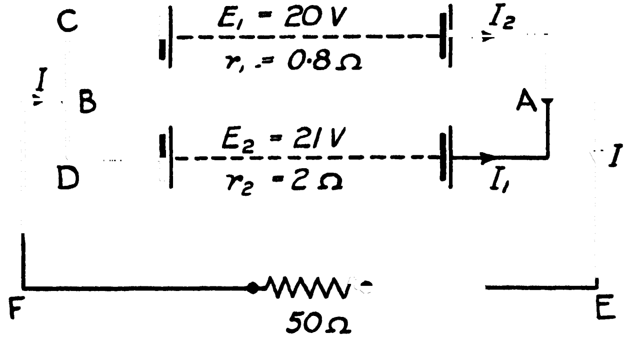


Fig. 48.—Circuit for Example 2 of Section 28.

Applying the second law to path AEFBDA and working in a clockwise direction, we get:—

$$21 = 2I_1 + 50I \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (c)$$

We thus have 3 unknown quantities which can be determined by solving the 3 equations, as follows:—

Eliminating I_2 from (a) and (b):—

$$2I_1 - 0{\cdot}8(I - I_1) = 1$$

i.e.,

$$2{\cdot}8I_1 - 0{\cdot}8I = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (d)$$

Eliminating I_1 from (c) and (d):—

$$2{\cdot}8\left(\frac{21 - 50I}{2}\right) - 0{\cdot}8I = 1$$

i.e.,

$$I = 0{\cdot}4011.$$

Substituting this value of I in (c), we get:—

$$21 = 2I_1 + 50 \times 0{\cdot}4011$$

i.e.,

$$I_1 = 0{\cdot}4725.$$

Substituting this value of I_1 in (a) we get:—

$$0{\cdot}4011 = 0{\cdot}4725 + I_2$$

i.e.,

$$I_2 = -\,0{\cdot}0714.$$

The negative sign obtained for I_2 means, of course, that this current flows in the direction opposite to that shown in Fig. 48.

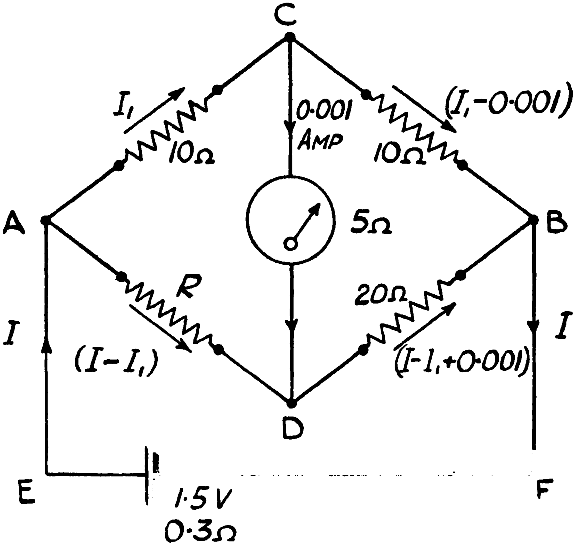


Fig. 49.—Circuit for Example 3 of Section 28.

Thus this example supports the statement made in Section 26 regarding the difficulty of using cells or batteries in parallel to supply an external load.

Example 3. *To calculate the unknown resistance R ohms in the bridge network shown in Fig.* 49 *assuming that a galvanometer current of* 1 *milliampere flows in the direction shown.*

Denoting the current through the cell by I amp. and that along the branch AC by I_1 amp., it follows from Kirchhoff's first law that the currents in the other branches are as shown.

Using Kirchhoff's second law we can derive the following 3 equations:—

(Path EACBFE)

$$1{\cdot}5 = 0{\cdot}3I + 10I_1 + 10(I_1 - 0{\cdot}001) \quad . \; . \; . \; . \; . \; . \; . \quad \text{(a)}$$

(Path EADBFE)

$$1{\cdot}5 = 0{\cdot}3I + R(I - I_1) + 20(I - I_1 + 0{\cdot}001) \; . \; . \; . \; . \quad \text{(b)}$$

(Path CBDC)

$$0 = (0{\cdot}001 \times 5) + 20(I - I_1 + 0{\cdot}001) - 10(I_1 - 0{\cdot}001) \quad \text{(c)}$$

From equation (a), $20I_1 = 1{\cdot}51 - 0{\cdot}3I$ . . . . . . (1)

„ „ (c), $30I_1 = 20I + 0{\cdot}035$ . . . . . . (2)

Multiplying equation (1) by 3 and equation (2) by 2 and equating:—

$$3(1{\cdot}51 - 0{\cdot}3I) = 2(20I + 0{\cdot}035)$$

i.e., $$4{\cdot}53 - 0{\cdot}9I = 40I + 0{\cdot}07$$

or, $$40{\cdot}9I = 4{\cdot}46$$

Thus, $$I = 0{\cdot}109.$$

Substituting this value of I in equation (1):—

$$20I_1 = 1{\cdot}51 - 0{\cdot}0327$$

So, $$I_1 = \frac{1{\cdot}4773}{20}$$

i.e., $$I_1 = 0{\cdot}07387.$$

Substituting for I and I_1 in equation (b):—

$$1{\cdot}5 = (0{\cdot}3 \times 0{\cdot}109) + 0{\cdot}03513R + 20(0{\cdot}03513 + 0{\cdot}001)$$

i.e., $$R = \frac{1{\cdot}5 - 0{\cdot}0327 - 0{\cdot}7226}{0{\cdot}03513}$$

Hence, $\boldsymbol{R = 21{\cdot}19}$.

One important feature of this result is that it indicates the importance of having a sensitive galvanometer in any Wheatstone bridge measurement. For a balanced bridge in the above case R should be equal to 20. When the value of R differs by as much as 1 ohm, the galvanometer current flowing is only 1 milliamp. If it differed by only 0·1 ohm the galvanometer current would be approximately 0·1 mA.

29. Other Forms of Resistance

The ideas regarding resistance and the laws relating to this property which have been developed in the preceding sections have assumed that the form of resistance being dealt with is that offered by solid materials possessing fairly good conductivities. This is the type of resistance most frequently encountered in dealing with electric circuits, but it is important for the student to recognise the existence of other forms of resistance, and to realise that these do not, in all cases, obey the laws applicable to solid conducting materials.

Direct reference will be made in the subsequent pages to the behaviour of these other forms of resistance, but it is advisable at this stage to classify them as follows.

(a) *Insulation resistances.* The term *insulation resistance* refers to the resistance offered by an insulating barrier, e.g., the paper insulation surrounding the core of a cable. Often the barrier includes several materials in series, whilst in other cases the problem is rendered more complex by the existence of several leakage paths in parallel. Insulation resistances are usually of a very high order, and are therefore expressed in *megohms* (MΩ).

(b) *Resistances at contact surfaces.* Practical examples of the importance of this particular form of resistance are: (i) the resistance between a carbon brush and the commutator or slip-ring in rotating electrical machinery, and (ii) the *rectifying* action displayed by certain pairs of materials in contact. This action, which involves the suppression of one-half of the cycle of an alternating current, is due to the enormous difference in the values of resistance at the contact surface corresponding to the two possible directions of current flow.

(c) *The resistance of certain liquid conductors.* The laws governing the flow of current and the mechanism of conduction in certain liquid conductors (known as *electrolytes*) are dealt with in Chapter V. It may be noted here that in certain cases of electrolytic conduction Ohm's Law cannot be applied directly, owing to the presence of a *back e.m.f.*

(d) *Gaseous conductors.* The subject of conduction in gases is one of ever-growing importance in modern electrical engineering. Certain fundamental aspects of this topic are dealt with in Vol. III.

APPENDIX

30. Dimensions of Resistivity

The student may at some time find the resistivity of hard-drawn copper expressed as 1·78 *microhms-cm.*, so a brief explanation is perhaps desirable. The formula $\rho = Ra/l$ holds good no matter whether the inch or the centimetre is employed as the unit of length. Dealing in terms of the centimetre, the formula may be regarded as an abbreviation for the statement:—

$$\rho \text{ units of resistivity} = \frac{R \text{ ohms} \times a \text{ sq. cm.}}{l \text{ cm.}}$$

Treated in this manner, ρ, R, a and l are merely the *numbers* of the various units involved. Treating the *units* in the same manner as the *numbers*, the formula may be written:—

$$\rho \text{ units of resistivity} = \frac{Ra}{l} \cdot \frac{\text{ohms} \times \text{cm.}^2}{\text{cm.}}$$
$$= \frac{Ra}{l} \text{ ohms} \times \text{cm.}$$

Thus, since $\rho = \frac{Ra}{l}$, 1 unit of resistivity = 1 ohm-cm., and in inch measure, 1 unit of resistivity = 1 ohm-inch. We may express these two facts by stating that the *dimensions* of resistivity are those of *resistance* × *length.*

A knowledge of the dimensions of the various quantities used in electrical technology can be of great assistance to lucid thought, particularly at a later stage of the work.

31. Temperature Coefficient at Various Temperatures

In dealing with resistance change due to temperature rise it is frequently more convenient to employ a temperature coefficient of resistance measured from a temperature other than 0° C. The *temperature coefficient at θ° C.* is defined as the increase in resistance of a conductor, having an original resistance of 1 ohm *at θ° C.*, when the temperature is raised through 1° C. To distinguish between temperature coefficients at 0° C. and θ° C. they may be written in symbolic form as α_0 and α_θ respectively.

If a conductor, composed of a material having temperature coefficients of α_0 and α_1 at temperatures of 0° C. and θ_1° C. respectively, possesses resistances of R_0, R_1 and R_2 ohms at temperatures 0° C., θ_1° C. and θ_2° C., then from Equation II of Section 22 we see that:—

$$\alpha_0 = \frac{R_1 - R_0}{R_0\theta_1} \quad . \; . \; . \; . \; . \; . \; . \quad \text{(a)}$$

and by similar reasoning it follows that:—

$$\alpha_1 = \frac{R_2 - R_1}{R_1(\theta_2 - \theta_1)} \quad . \; . \; . \; . \; . \quad \text{(b)}$$

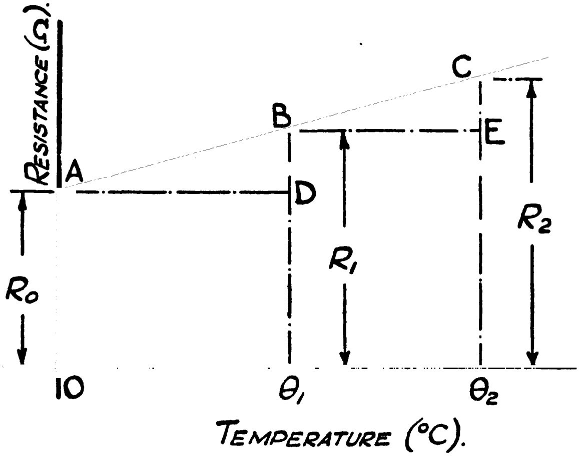


FIG. 50.—THE LINEAR RESISTANCE/TEMPERATURE RELATIONSHIP.

The manner in which the resistance of the conductor increases with temperature is shown in Fig. 50. From the similar triangles ABD, BCE it follows that:—

$$\frac{BD}{AD} = \frac{CE}{BE} \quad \text{i.e.,} \quad \frac{R_1 - R_0}{\theta_1} = \frac{R_2 - R_1}{\theta_2 - \theta_1} \quad . \; . \quad \text{(c)}$$

From Equations (a), (b) and (c) we get:—

$$\alpha_0 R_0 = \alpha_1 R_1$$

$$\therefore \quad \frac{\alpha_0}{\alpha_1} = \frac{R_1}{R_0} = \frac{R_0(1 + \alpha_0\theta_1)}{R_0} \quad \text{(See Equation I, Section 22.)}$$

$$\therefore \quad \frac{\alpha_0}{\alpha_1} = 1 + \alpha_0\theta_1$$

and, dividing throughout by α_0, we get:

$$\frac{1}{\alpha_1} = \frac{1}{\alpha_0} + \theta_1 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{(d)}$$

Thus if α_0 be known, α_1 can be calculated immediately.

For example, in the case of copper,

$$\frac{1}{\alpha_0} = 234{\cdot}5$$

so,

$$\frac{1}{\alpha_{20}} = 234{\cdot}5 + 20 = 254{\cdot}5.$$

i.e.,

$$\alpha_{20} = \frac{1}{254{\cdot}5}$$

So if the resistances of a copper winding at 20° C. and some higher temperature are 200 and 224 ohms respectively, the higher temperature, θ_2° C., can be rapidly calculated, using Equation (b) as follows:—

$$(\theta_2 - 20) = \frac{224 - 200}{200 \times \frac{1}{254{\cdot}5}}$$

$$\therefore \quad \theta_2 = 20 + \frac{24 \times 254{\cdot}5}{200}$$

or,

$$\theta_2 = 50{\cdot}54.$$

32. Variation of Resistance over a Large Temperature Range

In Section 22 it was pointed out that the relationship between resistivity and temperature may be considered to be linear only for a limited range of temperature. For most conductors this assumption holds good over the temperature range 0–100° C., and as the temperatures attained by the windings of electrical machinery and apparatus do not usually extend beyond this range, the formulæ given in Section 22 may be employed in these cases without serious error.

For conductors operating at higher temperatures, e.g., in electric heating appliances, etc., the assumption referred to above is no longer valid, and reference must be made to the appropriate graph connecting resistance with temperature. In the case of platinum the resistance/temperature relationship obeys the law—$R_\theta = R_0(1 + \alpha\theta + \beta\theta^2)$, which should be compared with Equation I of Section 22. Fig. 51 shows this relationship for platinum in graphical form (curve A), whilst the corresponding linear relationship (i.e., omitting the $\beta\theta^2$ term) is included for the purpose of comparison (curve B). It should be noted that the coefficient β in the above expression has a negative value. The resistance/temperature graphs for most of the other metals employed as conductors at high temperatures do not obey such simple mathematical laws, the graph for pure nickel being included in Fig. 51 (curve C).

The actual current-carrying conductor in most electric heating appliances (see Chapter IV) consists of a wire or strip composed of an alloy of nickel and chromium. It has already been noted (Section 22) that the change in resistivity for this type of material is relatively low, but the actual resistance/temperature relationship is often of some importance, and graphs showing this relationship for typical nickel-chromium alloys are illustrated in Fig. 52. It will be noted that the law is practically linear up to about 500° C., but beyond this temperature the resistance at first decreases and then rises again.

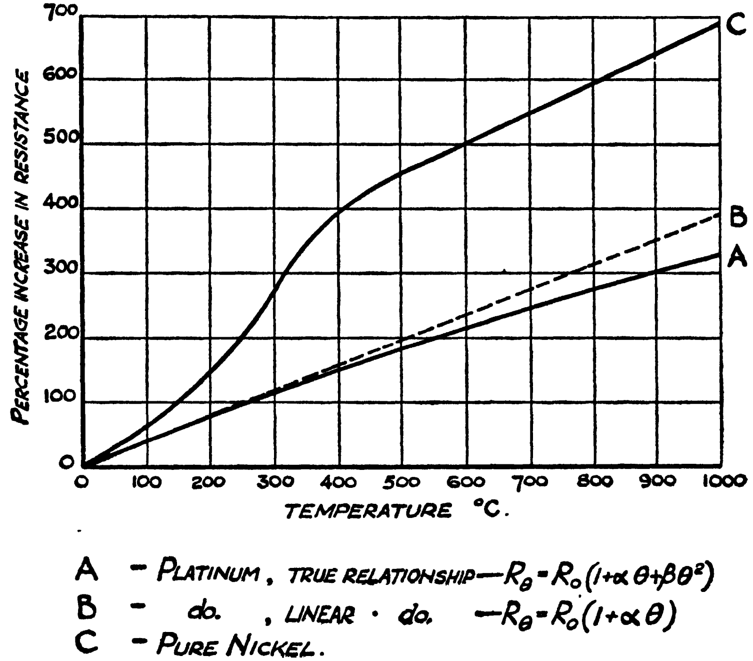


FIG. 51.—RESISTANCE/TEMPERATURE RELATIONSHIPS OVER A LARGE TEMPERATURE RANGE.

The graphs in Figs. 51 and 52 emphasise the necessity for caution when attempting to calculate resistance values at high temperatures. It should be borne in mind that any stated value of temperature coefficient is applicable only to a limited temperature range, or, alternatively, represents an average value over a wider range.

33. Resistance Thermometers

The basis of this method of temperature measurement is the change in the electrical resistance of a conductor with temperature. The conductor is usually in the form of a suitably-protected coil or spiral of wire and is placed at the point where the temperature is to be measured, e.g., in a furnace, cold storage chamber, or fluid. If the resistance/temperature relationship for the spiral is known accurately, and can be relied upon to remain unaltered over a reasonably long period, then a measurement of the resistance of the spiral enables the corresponding temperature to be obtained.

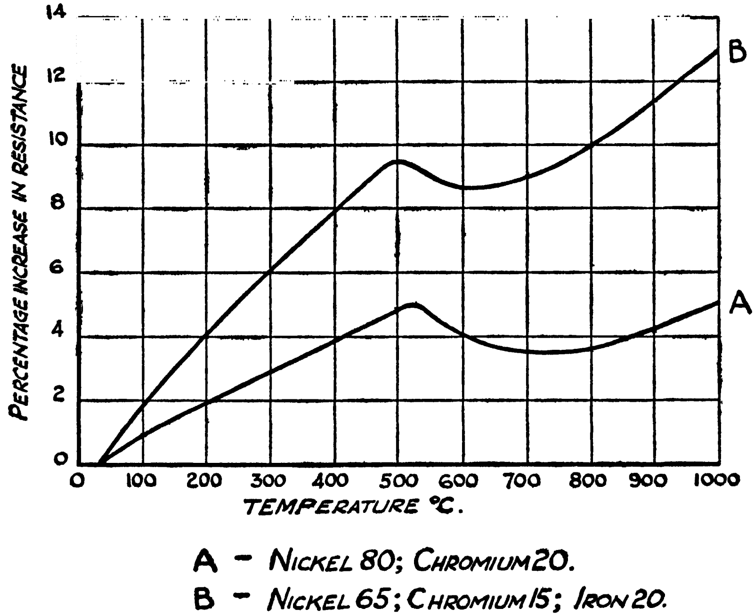


FIG. 52.—RESISTANCE/TEMPERATURE RELATIONSHIPS FOR ALLOYS.

The material usually employed for the spiral is either nickel or platinum, these metals being chosen because (i) they do not corrode readily at high temperatures, and (ii) they can be refined so that different batches of the same material have, within practical limits, identical resistance/temperature characteristics. The resistance of

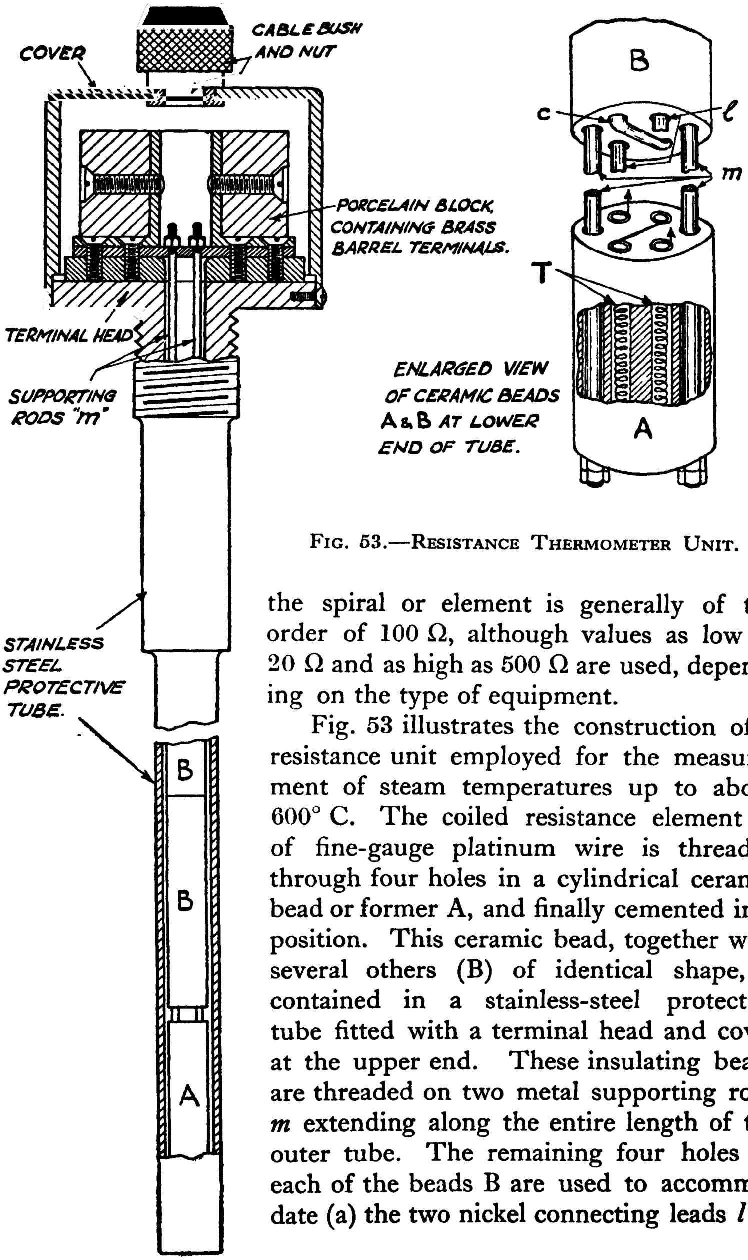

CABLE BUSH AND NUT
COVER
PORCELAIN BLOCK CONTAINING BRASS BARREL TERMINALS.
TERMINAL HEAD
SUPPORTING RODS "m"
STAINLESS STEEL PROTECTIVE TUBE.
B
B
A
B
c
l
m
T
ENLARGED VIEW OF CERAMIC BEADS A & B AT LOWER END OF TUBE.
A

Fig. 53.—Resistance Thermometer Unit.

the spiral or element is generally of the order of 100 Ω, although values as low as 20 Ω and as high as 500 Ω are used, depending on the type of equipment.

Fig. 53 illustrates the construction of a resistance unit employed for the measurement of steam temperatures up to about 600° C. The coiled resistance element T of fine-gauge platinum wire is threaded through four holes in a cylindrical ceramic bead or former A, and finally cemented into position. This ceramic bead, together with several others (B) of identical shape, is contained in a stainless-steel protective tube fitted with a terminal head and cover at the upper end. These insulating beads are threaded on two metal supporting rods *m* extending along the entire length of the outer tube. The remaining four holes in each of the beads B are used to accommodate (a) the two nickel connecting leads *l* to which the ends of the resistance element are silver-soldered, and (b) the compensating leads *c*, identical with the connecting leads, but consisting of a single length of nickel wire bent into U-shape. The element leads and the compensating leads are connected to separate terminal blocks mounted at the upper end of the protective outer tube, thus enabling copper cables to be employed for connecting the resistance unit to the measuring circuit.

Two examples of this circuit are illustrated in Fig. 54, both employing the Wheatstone bridge principle. In circuit (a) a direct indication of temperature is obtained on the indicator M as follows.

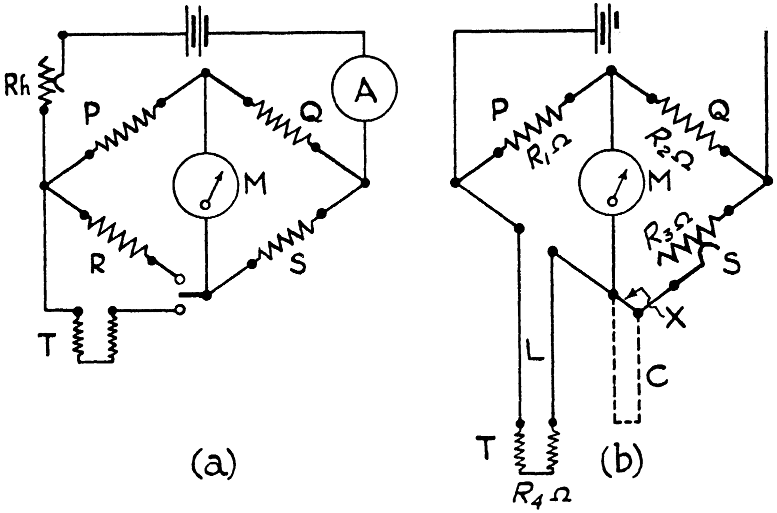


FIG. 54.—RESISTANCE THERMOMETER CIRCUITS.

P, Q, R and S are standard resistors, the value of R being made equal to the resistance of the element T at, say, 0° C. The values of P, Q and S are chosen so that, with R in the bridge circuit, balance is obtained on M (the usual type of moving-coil galvanometer), and this instrument reads 0° C. Consequently, with T in the circuit instead of R, M will read 0° C. when the resistance element is actually at this temperature. For any temperature above 0° C. the bridge will be out of balance, and M will indicate an out-of-balance current, the strength of which will increase as the resistance of T (and hence the temperature) increases. For temperatures below 0° C., M will be deflected *in the reverse direction* by an amount also depending on the actual temperature value. Thus, M can be calibrated directly as a temperature indicator. To avoid errors, it is necessary to keep the total current flowing to the bridge network at a constant value, and this may be effected by means of the rheostat Rh and ammeter A. Alternatively, the indicator M may be used instead of A by means of suitable switching arrangements. By switching the standard resistor R into the bridge circuit in place of T, the circuit can be checked periodically.

When circuit (b) is employed the bridge is adjusted to the condition of balance for each temperature measurement, and as the indicator reading is independent of the total bridge current, means for adjusting the latter are unnecessary. Balance is obtained with circuit (b) by adjusting the standard resistor S until M reads zero; then, using the resistance values given in the diagram, $R_3 = R_4$ (assuming $R_1 = R_2$). The temperature corresponding to this value R_3 may be obtained from the calibration graph for the resistance unit or, alternatively, the standard resistor S may be calibrated directly as a temperature indicator.

When compensating leads C are employed they are arranged alongside the element leads L and connected in the bridge circuit as shown in Fig. 54b, the link X being omitted. It should be noted that the leads L and C in this circuit diagram include not only the internal leads *l* and *c* illustrated in Fig. 53, but also the copper cables connecting these internal leads to the measuring circuit. The compensating leads C are made identical with the element leads L, and since they are alongside each other, their resistances are equal under all operating conditions. If this common resistance value be denoted by r ohms, then, when balance has been obtained, $R_3 + r = R_4 + r$, or $R_3 = R_4$. Thus the effect of changes in the resistance of the element leads L is nullified.

The resistance type of thermometer is preferable to the liquid expansion type when the temperatures at a number of different points are required. The individual resistance units are then located at these test-points (e.g., the various rooms of a cold-storage plant) and connected to a common measuring circuit through a selector switch. The resistance thermometer suffers, however, from the disadvantage that a low-voltage source of direct current is required. This type of thermometer is not usually employed for commercial temperature measurements above about 550° C., as the *thermocouple* (see Chapter IX, Vol. II) possesses many advantages for the measurement of temperatures above this value.

Exercises on Chapter III

Any additional data required for the solution of these exercises may be obtained from the tables at the end of this book.

1. Three resistors of 10, 15 and 25 ohms respectively are joined in series across a 200-volt supply. Determine (a) the current flowing, (b) the p.d. across each of the resistors and (c) the power loss in each resistor.

[(a) 4 A; (b) 40, 60 and 100 V; (c) 160, 240 and 400 W.]

2. A coil of resistance 120 Ω is connected in series with a rheostat of range 0–80 Ω. The supply voltage is 240. Calculate the maximum and minimum values between which the current may be varied, also the maximum p.d. across the rheostat.

[2·0 to 1·2 A; 96 V.]

3. Two resistors, each of 25 kΩ, are joined in parallel. It is desired to connect a third resistor in parallel with these so that the total resistance of the combination is 10 kΩ. Find the value of this third resistor.

[50 kΩ.]

4. An electric hotplate has two windings of resistance 40 and 60 ohms respectively. Determine the combined resistance, the current taken from the supply and the total power dissipated (a) when the windings are connected in series and (b) when they are in parallel. The supply is at 240 volts.

[(a) 100 Ω, 2·4 A, 576 W; (b) 24 Ω, 10 A, 2400 W.]

5. In a wireless receiver there are four valve heaters joined in parallel, each requiring a p.d. of 4 volts. Three of the heaters each have a working resistance of 4 ohms and the fourth 2 ohms. What resistance must be connected in series with this combination if the only available supply is one of 6 volts?

[0·4 Ω.]

6. Three coils P, Q and R, of resistance 10, 12 and 15 ohms respectively, are joined in parallel. In series with this combination is another coil S, of resistance 3·5 ohms, the four coils being connected in this manner to a supply of 30 volts. Find (a) the total current taken from the supply, (b) the p.d. across coil S and (c) the current

in each of the coils P, Q and R. (d) If coil P becomes disconnected, what is the new p.d. across coil S?

[(a) 4 A; (b) 14 V; (c) P—1·6 A, Q—1·33 A, R—1·07 A; (d) 10·3 V.]

7. Two coils A and B, when connected in parallel across a 200-volt supply, take a total current of 8 A, the power absorbed by coil A being 1 kW. Calculate the resistance of each coil.

[A—40 Ω; B—66·7 Ω.]

8. Four 200-volt lamps, rated at 100, 60, 40 and 25 watts respectively, are connected in parallel across a 200-volt supply. Calculate the total conductance of the combination, also the conductance of the additional lamp required to bring the total supply current up to 1·5 amp.

[$5{\cdot}625 \times 10^{-3}$ mhos; $1{\cdot}875 \times 10^{-3}$ mhos.]

9. Calculate the resistance, at 20° C., of 1000 yd. of 7/·064″ hard-drawn copper conductor, assuming an average increase of 2% in the length of the wires due to stranding.

[1·142 Ω.]

10. Given that 1000 yd. of 7/·029″ copper conductor has a resistance of 5·281 Ω, find the approximate resistance per mile, at the same temperature, of (a) 3/·036″ and (b) 19/·064″ copper conductor.

[(a) 14·07 Ω; (b) 0·703 Ω.]

11. A nichrome heater spiral, rated at 1000 watts, 230 volts, consists of 180 turns, each having a mean diameter of 0·6 in., the wire diameter being 0·0180 in. Calculate the resistivity (at the operating temperature) of the material of which the spiral is made.

[39·7 $\mu\Omega$/in. cube.]

12. A coil of 2400 turns is wound with 22 S.W.G. annealed copper wire, the mean diameter of a turn being 8 in. Find the power dissipated by the coil when connected to a 50-volt supply, the mean temperature of the winding being taken as 20° C.

[37·5 W.]

13. A 1000-yd. length of 7/·144″ hard-drawn aluminium conductor has a resistance of 0·3544 ohm at 60° F. Assuming that the effect of the stranding is to increase the length of each of the 6 outer wires by 2%, calculate the resistivity of hard-drawn aluminium at 60° F.

[1·104 $\mu\Omega$/in. cube.]

14. An electric bell requires 0·7 amp. at a p.d. of 4·5 volts and is fed by a pair of galvanised-iron wires from an 8-volt supply situated 50 yd. away. What is the most suitable gauge of iron wire for this purpose? Take the conductivity of the wire as $1 \cdot 6 \times 10^5$ mhos. per inch cube.

[0·076″ dia.; nearest 14 S.W.G.]

15. A building is provided with 250 lamps, 100 of these taking 100 watts each and the remainder 60 watts each. If the building is supplied with current from a 250-volt source situated 100 yd. away, and the p.d. across the lamps must not be less than 245 volts, find the minimum size of the copper feeder cable.

[0·0758 in.^2.]

16. A portable floodlamp is supplied with a current of 12 amp. from a battery by means of a 100-yd. length of flexible cable having a sectional area of 0·02 sq. in. per core (copper). The battery consists of 12 cells in series, each having an e.m.f. of 2·0 volts and an internal resistance of 0·0025 ohm. Determine (a) the terminal p.d. of the battery, (b) the voltage drop in the cable and (c) the power dissipated in the lamp.

[(a) 23·64 V; (b) 2·93 V; (c) 248·5 W.]

17. The resistance of a coil of copper wire is 80 ohms when the mean temperature of the wire is 15° C. What is the resistance when the mean temperature rises to (a) 55° C. and (b) 70° C.?

[(a) 92·8 Ω; (b) 97·6 Ω.]

18. A length of nickel wire is found to have a resistance of 50 Ω at 10° C. When the temperature is raised to 60° C. the resistance of the wire is 65 Ω. Find the temperature coefficient of resistance for nickel from 0° C.

[0·00638 per deg. C.]

19. The field current of a motor at 20° C. was observed to be 2·5 amp., and after 6 hours on load it had fallen to 2·08 amp. If the p.d. across the field coils (wound with copper wire) was 230 volts in each case, what was their final mean temperature?

[71·5° C.]

20. The circuit of Fig. 34a is employed for measuring the resistance of a component X. The meter readings were: A—1·5 amp.; V—120 volts. If the resistance of V is 3000 ohms, determine (a) the percentage error involved in the assumption that the

resistance of X = reading of V ÷ reading of A, and (b) the true resistance of X.

[(a) 2·67%; (b) 82·2 Ω.]

21. A Wheatstone bridge network ABCD is balanced with a) AB = BC = 100 Ω; CD = 165 Ω; (b) AB = 1000 Ω; BC = 100 Ω; CD = 5755 Ω; (c) AB = 10 Ω; BC = 1000 Ω; CD = 8075 Ω. Find the value of the unknown resistor DA in each of the three cases.

[(a) 165 Ω; (b) 57,550 Ω; (c) 80·75 Ω.]

22. A cell having an e.m.f. of 1·5 volts has a 10 Ω resistor connected across it. If the current flowing is 125 mA, determine the internal resistance of the cell and the terminal p.d.

[2 Ω; 1·25 V.]

23. A battery has an e.m.f. of 6·3 volts and supplies 36 watts to a lamp, at a terminal p.d. of 5·9 volts. What is the internal resistance of the battery and what would be the terminal p.d. when supplying a current of 3 amp.?

[0·0656 Ω; 6·103 V.]

24. How many cells (joined in series), each of e.m.f. 1·5 volts and internal resistance 3 ohms, would be required to send a current of 100 mA through a resistance of 240 ohms? What is the terminal p.d. of each cell?

[20; 1·2 V.]

25. A battery of 18 cells consists of 3 rows, connected in parallel, each row containing 6 cells in series. The e.m.f. of each cell is 1·4 volts, and the battery supplies a total current of 0·45 amp. to a circuit of resistance 12 ohms. Calculate the terminal p.d. and internal resistance of each cell.

[0·9 V; 3·33 Ω.]

26. A motor-car battery consists of 6 cells in series, each cell having an e.m.f. of 2·15 volts and an internal resistance of 0·0035 Ω. Find the current taken from the battery and the terminal p.d. when the following are switched on: (a) 3 lamps, each rated at 12 volts, 6 watts, (b) an additional pair of headlamps, each rated at 12 volts, 36 watts. All the lamps are wired in parallel, and their resistances may be assumed to be unaffected by changes in the terminal p.d.

[(a) 1·61 A, 12·87 V; (b) 7·96 A, 12·73 V.]

27. A battery B of e.m.f. 6 volts and internal resistance 0·5 Ω is joined in parallel with another battery C of e.m.f. 10 volts and internal resistance 1 Ω. A 12 Ω resistor is connected across this combination. Determine the magnitude and direction of the current through each battery.

[B—2·27 A charge; C—2·87 A discharge.]

28. A battery B of e.m.f. 180 volts and internal resistance 1·5 ohms is joined in parallel with a second battery C of e.m.f. 120 volts and internal resistance 6 ohms. In series with this combination is a 3-ohm resistor, the complete circuit being connected across 200 volt mains, with the negative terminal of the mains joined to the negative terminal of each battery. Determine the magnitude and direction of the current in each battery.

[B—1·90 A discharge; C—9·52 A charge.]

29. A Wheatstone bridge network ABCD has the following details: AB = 1000 Ω; BC = 100 Ω; CD = 450 Ω; DA = 5000 Ω. A galvanometer of resistance 500 Ω is connected between B and D and a battery of e.m.f. 4·5 volts between A and C, with A positive. Find the magnitude and direction of the galvanometer current. The internal resistance of the battery may be neglected.

[Approx. 37 μA, B to D.]

30. A network PQRS consists of the following resistors: PQ—5 kΩ; QR—10 kΩ; RS—15 kΩ; SP—20 kΩ. A fifth resistor of 10 kΩ is connected between the points P and R, and a dry battery of e.m.f. 120 volts and internal resistance 500 ohms across the resistor SP. Determine (a) the total current supplied by the battery, (b) the p.d. between the points R and S, and (c) the magnitude and direction of the current through the 10 kΩ resistor PR.

[(a) 11·17 mA; (b) 81·72 V; (c) 3·27 mA, P to R.]

* 31. British standard high-conductivity annealed copper has a resistance of 0·02401 Ω per 1000 yd. of 1 sq. in. section at 60° F., and the temperature coefficient of resistance is 0·002222 per deg. F. at this temperature. Find the temperature coefficient per deg. C. and the resistivity, in microhms per inch cube, at 60° C.

[0·003395 per °C.; 0·7857 μΩ/in. cube.]

* *This exercise should be attempted only after reading Section* 31, *included in the Appendix to Chapter III.*

CHAPTER IV

ELECTRICAL ENERGY AND ITS CONVERSION INTO HEAT

34. Units of Power and Energy

When a current of I amperes flows through a conducting path of R ohms resistance with a voltage drop of V volts, the power absorbed, P watts, is given (as we have seen in Section 14) by the formulæ:—

$$P = VI = I^2R = \frac{V^2}{R}$$

Similarly the energy taken, W joules, in t seconds is given by the formulæ:—

$$W = VIt = I^2Rt = \frac{V^2}{R} . t$$

a joule being a **watt-second** (see Section 14).

A larger and more convenient unit of energy than the joule is the **Board of Trade Unit (B.O.T. unit).** Just as a joule is a watt-second, so the B.O.T. unit is a **kilowatt-hour (kWh).** Since 1 kWh is equivalent to 1000 × 3600 watt-seconds it follows that

1 B.O.T. unit = $3{\cdot}6 \times 10^6$ joules

The B.O.T. unit is the unit of electrical energy used for commercial purposes, being the unit referred to by the public in such expressions as " electricity at 1d. a unit." Electricity supply meters record the consumption of electrical energy in kWh, or " units."

35. Energy Costs

Since it is not practicable to store electrical energy to an appreciable extent, the energy must be generated as it is consumed. Thus, if the maximum power demand from the public supply in a residential area is likely to be, say, 1100 kW, the supply undertaking must install plant capable of generating and distributing 1100 kW, although the average load may be only 500 kW (see Fig. 55). During a large portion of a day of 24 hours much of the plant of a supply company is idle or is performing a task which could be done by smaller equipment. This tends to make capital costs form a large proportion of the total cost of producing and distributing electrical energy, the remaining costs consisting of running charges, which include the cost of fuel used and maintenance, these being roughly proportional to the amount of energy used.

For this reason electrical energy is usually sold on the basis of a two-part tariff, one part being a fixed charge dependent upon the maximum load the consumer takes (or is likely to take), and the other a running charge directly proportional to the amount of energy the consumer uses. In the case of a large industrial consumer it is usual to install a maximum demand indicator, the consumer paying a certain sum per annum per kW of maximum load taken, a typical rate being £5 per kW of maximum demand. In addition, a certain price (e.g., 1d. per unit) is paid for each B.O.T. unit taken, the number of units consumed being recorded on an energy meter. Thus on a tariff of £5 per kW of maximum demand plus 1d. per unit consumed, a user of 960,000 units per annum with a maximum demand of 1000 kW would pay $£\frac{960{,}000}{240} + £1000 \times 5 = £4000 + £5000 = £9000$, whereas with a maximum demand of 1500 kW and the same energy consumption he would pay £11,500.

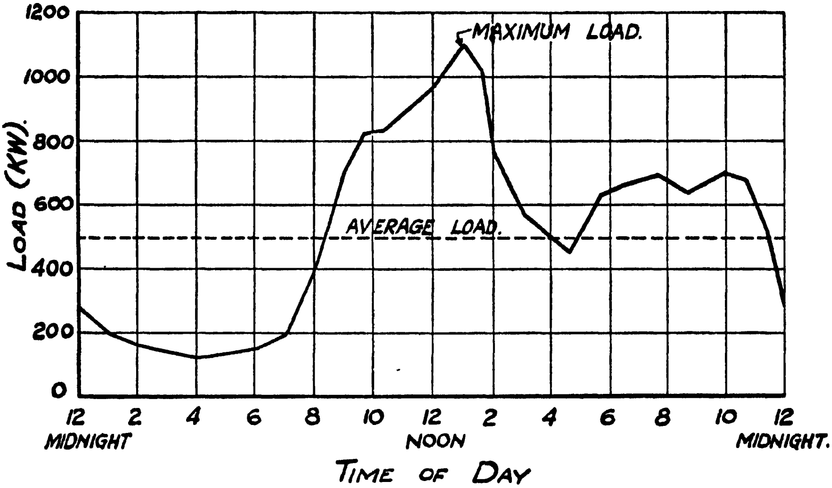


FIG. 55.—LOAD CURVE FOR A RESIDENTIAL AREA ON A WEEKDAY IN MID-WINTER.

Actually, the reader may have discovered already that the maximum demand is usually expressed in **kVA (kilovolt-amperes)** (see Chapter VIII, Vol. II) instead of kW. This is because the public supply is usually a.c., but we must suppose for the present that a d.c. supply is being used, in which case the maximum demand *would* be expressed in kW.

Consumers can frequently reduce energy costs by arranging that processes using large amounts of power are not carried out simultaneously. Some supply companies offer special low tariffs for energy used during off-peak periods; such processes, for example, as pumping, refrigeration and water-heating can often be carried out at night, when there is little demand for electrical energy. It should be appreciated that it is to the advantage of both supplier and consumer that the ratio of maximum load to average load should be as low as possible.

A two-part tariff is also offered to domestic consumers, a running charge at a low rate per unit consumed being made and also a fixed charge per annum roughly proportional to the size of the house, the fixed charge being based upon the rateable value of the house or, alternatively, upon the number of rooms in the house. Domestic consumers using very little energy frequently pay on a flat-rate system, a charge of 4d. to 6d. per unit being made for energy used for lighting purposes, reduced rates being allowed for other purposes.

36. Transformation of Electrical Energy into Heat

One of the most widely-known scientific laws is **the law of the conservation of energy,** which states that **energy can neither be created nor destroyed.** Thus, when coal is burned in the boiler of a steam locomotive, the chemical energy of the coal is changed to the heat energy of the steam, and this heat energy in turn is changed to the mechanical energy of movement in the engine cylinders. This energy of movement, or kinetic energy as it is called, is finally changed to heat because of friction, heat being produced whenever work is done against frictional forces.

When electrons move through a wire there is considerable friction between each electron and the molecules of the material forming the wire. For this reason a current of I amperes can flow through a wire of resistance R ohms for t seconds only if an amount of energy I^2Rt joules is available. This energy is not destroyed, the whole of the I^2Rt joules being changed into heat, since the whole of this energy is necessary to overcome the molecular frictional forces we refer to as the electrical resistance. This transformation of electrical energy into heat which occurs whenever a current flows through a conductor is often referred to as the **heating effect** of the current.

The reader should note that electrical energy can be converted into heat by processes other than the conduction process outlined above. Reference is made to these other processes, namely, magnetic hysteresis and dielectric heating, in Vols. II and III, the former being dealt with in Chapter II of Vol. II. In the remainder of the present chapter the only method of heating considered is that in which the heat is produced by causing a current to flow through a conducting material.

Heat energy is not usually measured in joules, but in calories, C.H.U. or B.Th.U. The number of joules equivalent to 1 calorie has been very carefully measured on many occasions, using many different methods, with the result that we now know that **1 calorie is equivalent to 4·185 joules.** It therefore follows that 1 joule is equivalent to 0·24 calorie approximately. So the heat, H calories, produced when a current of I amp. flows through a resistance of R ohms for t seconds, is given by the formula $\mathbf{H} = 0{\cdot}24\, I^2Rt$.

Similarly when 1 B.O.T. unit of electrical energy is wholly converted into heat, 3414 B.Th.U. are produced, i.e.,

1 kWh = 3414 B.Th.U. = 1896 C.H.U.

These relationships are particularly useful when estimating the temperature rise that can be produced in a given time in a furnace or in water-heating equipment. Such estimates are not easy to make, since many factors are involved. It is necessary to know not only how much heat is being supplied, but also how much heat is being lost per second. Heat which escapes must in the first place come through to the surface by means of conduction; it is then dissipated primarily by means of convection, although some of the heat is lost by radiation. Where lagging (i.e., heat-insulating material) is used, as in furnaces, the chief factors upon which the rate of loss of heat depends are the thickness and nature of the lagging, the internal and external temperatures and the surface area. In the case of equipment such as electric kettles and hot plates, where lagging is not used, a greater proportion of the heat supplied is lost due to the higher temperature of the outer surface. The rate of heat loss, due to convection and radiation, for surface temperatures up to nearly 100° C. and with normal air temperatures, is roughly proportional to the temperature difference.

But the loss of heat from the surface of electrical heating equipment is not the only factor contributing to its inefficiency. A considerable portion of the heat supplied goes to raise the temperature of the equipment itself. In this connection it should be noted that, if an oven is used again immediately after one charge has been heated up and while the oven is still hot, the efficiency of the process of heating-up a second charge will be considerably higher than for the first charge.

Summarising, we see that:—

(Heat supplied) = (Heat used in raising the temperature of the charge) + (Heat used in raising the temperature of the heating equipment) + (Heat dissipated by radiation and convection).

So a reasonable estimate of the time necessary to produce any desired temperature rise in a given quantity of material is possible only if the efficiency of the heating process is known, so accounting for the heat losses. This efficiency can be defined as follows:—
Efficiency

$$= \frac{\text{Heat used in raising the temperature of the charge}}{\text{Heat supplied in the form of electrical energy}} \times 100\%$$

Example. An electric furnace takes 10 kW. Assuming that the heat losses amount to 20% of the total energy input, find the time required to raise the temperature of a charge consisting of 30 lb. of steel bars from 40° C. to 850° C. Take the specific heat of steel as 0·120.

Amount of heat necessary to raise the temperature of the steel $= 30 \times 0{\cdot}12 \times 810$ C.H.U.

$\therefore$ Energy input to the furnace $= \dfrac{30 \times 0{\cdot}12 \times 810}{1896} \times \dfrac{100}{80}$ kWh

$\therefore$ Time taken by the process $= \dfrac{30 \times 0{\cdot}12 \times 810 \times 100}{1896 \times 80 \times 10}$ hours

$= 0{\cdot}1923$ hours

$=$ **11·54 minutes.**

Owing to the difficulty of controlling and measuring heat losses, it is extremely difficult to measure precisely the number of joules equivalent to 1 calorie. The experimental method described below will not give the relationship with accuracy, but the method is good enough to show that the approximate relationship is 4·2 joules per calorie.

Experiment 10. *To measure the number of joules equivalent to 1 calorie.*

Determine the mass of a small copper calorimeter fitted up with a heater coil, thermometer and stirrer as shown in Fig. 56a. Regard the thermal capacity of the calorimeter and fittings as 1/10th of this mass (i.e., an average specific heat of 0·1). Then add water to the calorimeter to one-third or half its volume. Place this calorimeter inside a larger vessel with cork or slag-wool lagging placed in the intervening space. The heater coil should consist of sufficient eureka or nichrome wire to give a resistance of about 1 ohm, and should be connected to terminals A and B by thick copper leads. Connect a voltmeter across these terminals, and an ammeter, battery, rheostat and switch in series with the heater coil, as shown in Fig. 56b. Stir and take the temperature of the water. Repeat at 3-minute

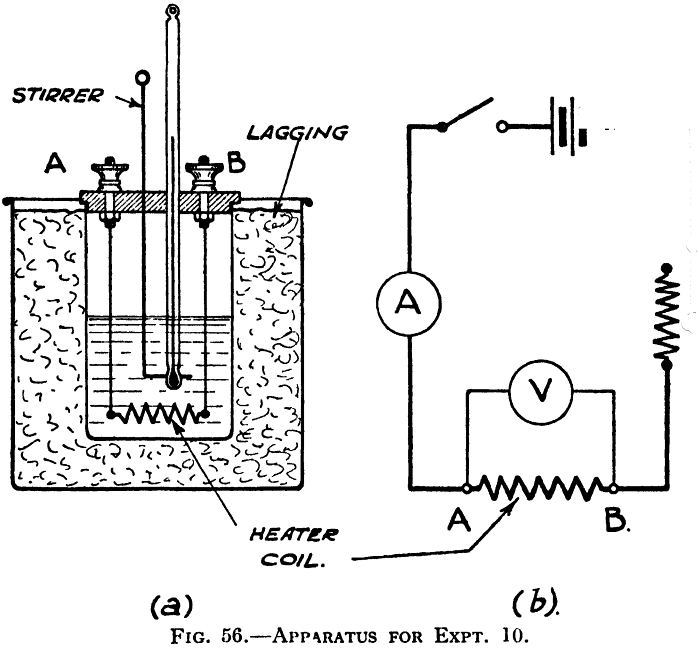


FIG. 56.—APPARATUS FOR EXPT. 10.

intervals until it is ascertained that the temperature is remaining constant, then pass a current of, say, 3 amp. through the coil for 10 minutes, adjusting the rheostat, if necessary, to keep the current constant. At the end of this period switch off, stir and take the temperature of the water. Allow the calorimeter to cool for 5 minutes and again stir and take the temperature.

Owing to the heat lost due to convection and radiation, the rise in temperature will not be so great as it would be if there were no such heat loss, but an allowance can be made for this loss by adding to the rise in temperature in the first 10 minutes an amount equal to the slight drop in temperature in the next 5 minutes. (This approximate correction is based on the assumption that the cooling in 5 minutes at the maximum temperature is equal to the cooling in 10 minutes at the mean temperature of the heating process.)

Amount of electrical energy supplied = (Voltmeter reading) × (Ammeter reading) × (Time)

Amount of heat produced = [(Mass of water) × (Corrected rise in temperature)] + [(Thermal capacity of calorimeter and fittings) × (Corrected rise in temperature)]

The no. of joules equivalent to 1 calorie

$$= \frac{\text{Electrical energy supplied (in joules)}}{\text{Amount of heat produced (in calories)}}$$

37. Applications of the Heating Effect

The heating effect of the electric current has been utilised for many purposes, and these applications have resulted in an immense variety of appliances and equipment. In this section, however, it would be neither practicable nor desirable to give a complete account of all the equipment whose operation is based on the transformation of electrical energy into heat. The present aim, therefore, is merely to classify the various applications of the heating effect and to describe the construction and characteristics of typical equipment.

Heating Appliances

Under this heading are included all types of equipment *designed* to *produce heat* by the passage of an electric current. The current may flow in a *heating-resistor* under the influence of the supply p.d., this principle being known as **resistance heating.** Alternatively, the current giving rise to the heating effect may be produced by electro-magnetic induction in a secondary circuit, the alternating current from the source of supply flowing in a separate primary circuit (see Chapter VIII, Vol. I, and Chapter IV, Vol. II). This method is known as **induction heating.** Finally, the transformation of electrical energy into heat may take place in an electric arc, giving rise to **electric arc heating.** These three methods of heating will now be dealt with separately.

(A) *Resistance heating.* Appliances operating on this principle may be divided into two types, viz.:—

(1) the *direct-resistance* type, in which the heating-resistor is the body or substance to be heated, and
(2) the *indirect-resistance* type, in which a separate heating-resistor of wire, strip or rod is wound or supported on an *element-former*, the complete unit being termed a *heating-element*; the heat from this element is transmitted by radiation, convection or conduction (or a combination of these methods) to the body to be heated.

Heating appliances of the direct-resistance type include electrode water-heaters and steam-raisers, in which water is heated by the passage through it of current between immersed electrodes (rods or plates), electrode salt baths for the heat treatment of steel, and equipment for the manufacture of graphite and carborundum. In this latter equipment the heat resulting from the passage of current through the substance to be heated produces a sufficiently high temperature to cause the desired chemical changes.

Fig. 57 illustrates an electrode type of salt-bath suitable for the heat-treatment processes known as cyanide hardening, liquid carburising, tempering and annealing. The salt-bath illustrated contains only one pair of electrodes, but multi-electrode baths are in use. The supply to the electrodes is at a very low voltage, hence there is no danger of electric shock due to the exposed electrodes. This low-voltage supply is obtained by stepping-down the a.c. mains voltage through the medium of a transformer (see Chapter IV, Vol. II), this portion of the equipment being visible on the extreme left of the illustration in Fig. 57.

The bulk of the common domestic and industrial heating appliances, however, are of the indirect-resistance type, these including convection heaters, electric irons, ovens, immersion heaters, boiling-plates, radiant fires and furnaces.

The material employed for the heating-resistor in the indirect-resistance type of equipment depends on the temperature at which this resistor is required to operate. The usual resistor material used in modern appliances of this type is an alloy of nickel with chromium, known as *nichrome*, the most suitable grade of this alloy having approximately 80% of nickel and 20% of chromium and being capable of operation at temperatures up to 1000° C. For lower temperatures a cheaper alloy, containing some iron, is generally employed. Nichrome possesses the following characteristics, which render it eminently suitable for use as a heating-resistor material:—

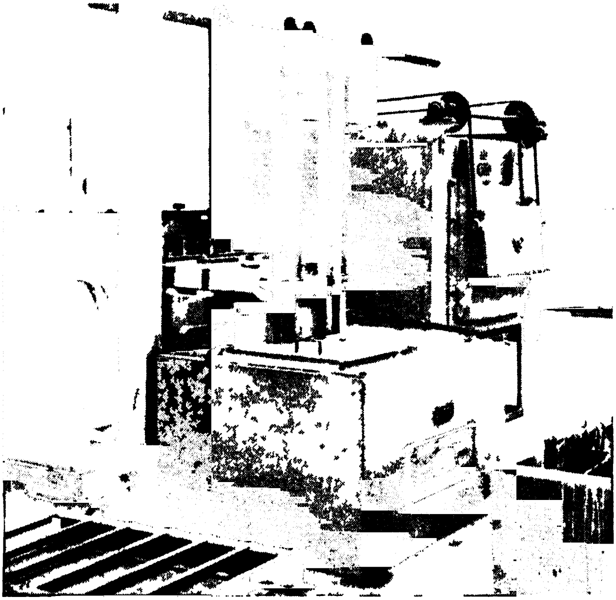


FIG. 57.—AN ELECTRODE TYPE SALT-BATH.

(a) It has a high melting point, so that a high operating temperature may be employed (the melting point is approximately 1400° C. for 80/20 nichrome).

(b) The resistivity is high (see Table V), hence the necessary value of resistance may be obtained with a relatively short length of wire or strip.

(c) It has a relatively small change in resistance, from cold to the normal working temperature (see Section 32 and Fig. 52). This ensures that heating-elements, when switched on from cold, do not impose unnecessarily high power demands on the supply mains.

(d) It does not deteriorate rapidly, due to oxidation, when used at temperatures up to 1000° C. This is due to the fact that the material acquires a thin but tough and adherent coating of oxide which does not subsequently flake off due to repeated heating and cooling.

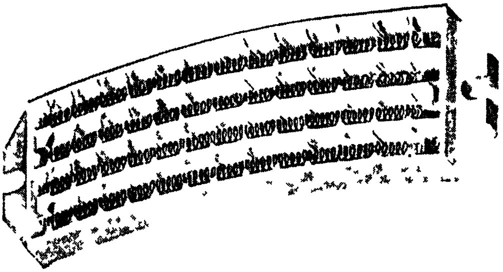


FIG 58.—TYPICAL FIREBAR FOR RADIANT ELECTRIC FIRE.

For heating-resistors required to operate at temperatures between 1000° C. and 1300° C. (usually in furnaces) rods of silicon carbide are used. This material possesses a negative temperature coefficient of resistance, and thus some form of control is desirable in order that the maximum safe temperature for the heating-resistor shall not be exceeded. This control may be effected manually by adjusting the p.d. applied to the furnace, or alternatively some form of automatic temperature control may be employed, any description of which falls outside the scope of this book. It is possible to use graphite heating-resistors for operating temperatures considerably in excess of 1300° C., although such high-temperature furnaces are usually of the induction or arc type.

Fig. 58 shows an example of an indirect-resistance type of heating element, as used in domestic electric fires of the radiant type. The heating resistor is in the form of a spiral of nichrome wire contained in grooves in a moulded ceramic base, this being convex in form so as to radiate the heat over a wide area. Such *firebars* are usually rated at 1 kW each.

In one type of electric boiling-plate the heating-resistor is in the

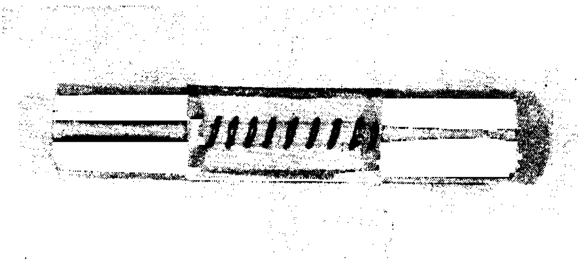


FIG. 59.—CUT-AWAY SECTION OF A BOILING-PLATE ELEMENT.

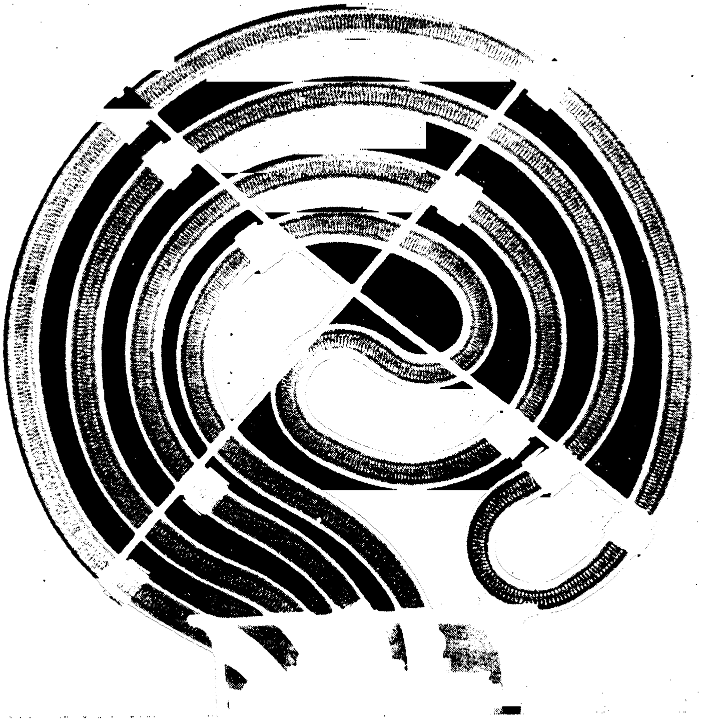


FIG. 60.—X-RAY PHOTOGRAPH OF BOILING-PLATE ELEMENTS.

form of a nichrome spiral tightly embedded by a special process in magnesium oxide (an insulator), the whole being enclosed solidly in an outer sheath consisting of a 5/16 in. diameter tube of a non-scaling and non-corrodible alloy. Fig. 59 shows a cut-away section of this type of element, while the X-ray photograph of the complete boiling plate, reproduced in Fig. 60, demonstrates the even thickness of insulation between the heating-resistor and the outer metal sheath

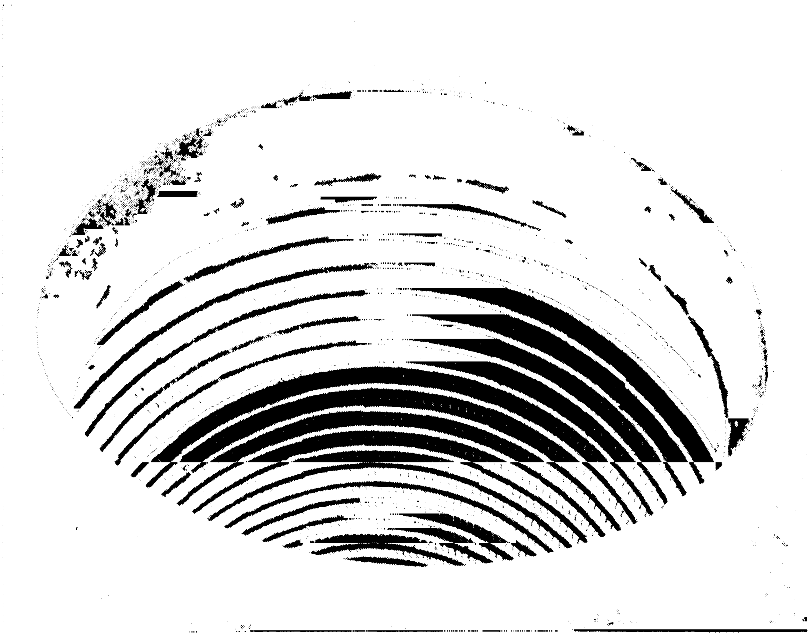


FIG. 61.—150-kW BRIGHT-ANNEALING FURNACE

An industrial application of the indirect-resistance method of heating is illustrated in Fig. 61. This is a view looking down into the interior of a 150-kW furnace used for the bright-annealing of steel strip. In the illustration the spirals of heavy-gauge nichrome wire, contained in fireclay supports round the well-shaped interior of the furnace, are clearly visible.

(B) *Induction heating.* Important developments have taken place during the past 20 years in connection with this method of heating, resulting in two types of equipment comparable with the types referred to under resistance heating. These types of inductively-heated appliances, for which an alternating-current supply is essential, are as follows:—

(1) *direct-induction* appliances, in which a current is induced directly into the charge or body to be heated, and
(2) *indirect-induction* appliances, in which the current is induced in a separate heating-resistor, from which the heat is transferred to the charge by radiation, etc., as in the indirect-resistance type of appliance.

Induction-heating equipment is usually employed only (a) in cases where the temperature required is so high that no suitable heating-resistor material is available, or (b) where the location of the body to be heated renders resistance heating difficult. Heating appliances of the direct-induction type include furnaces for the melting and refining of metals and portable equipment for purposes such as heating thermionic valve electrodes, mounted inside a glass envelope, during the manufacture of the valves.

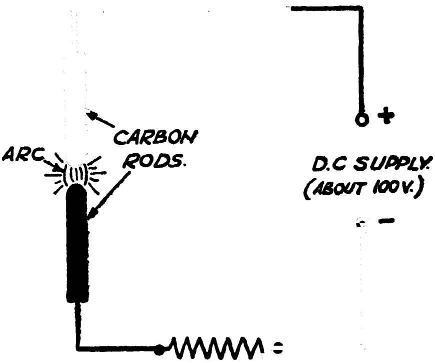


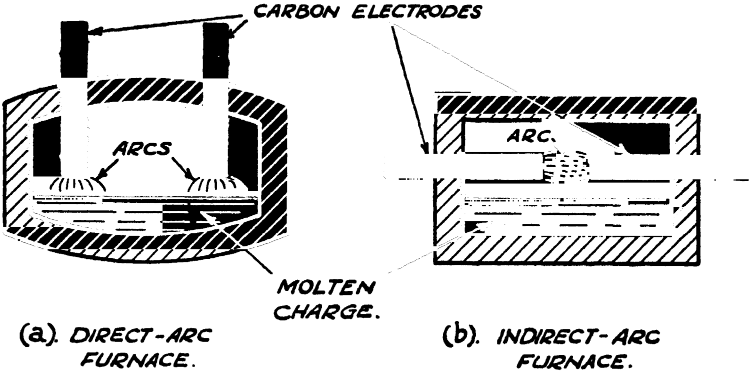
FIG. 62.—CIRCUIT FOR DEMONSTRATING THE CARBON ARC.

It is not proposed to deal, in this section, with the construction of typical induction-heating appliances, as certain features of the construction of these appliances depend on electromagnetic principles referred to in subsequent chapters.

(C) *Electric arc heating.* This principle might be described as a special method of resistance heating, the heating-resistor being in the form of an *electric arc.*

The reader has probably observed the spark or " flash " which occurs when a current-carrying plug, connected to an electric fire, is removed from its socket, or when a circuit is broken by means of a switch. This sparking occurs, to a greater or lesser degree, whenever a current-carrying circuit is interrupted, but the actual spark usually persists for only a fraction of a second.

With the circuit shown in Fig. 62, however, it will be found that when the carbon rods are brought together momentarily and then separated by a short distance (about 1/8 in.), a continuous spark or flame persists between them, and current continues to flow. This continuous flame is referred to as an *electric arc*, and the above procedure is termed " striking the arc." The electric arc is obviously a conductor, and consists of a stream of hot gases and volatilised carbon. If the distance between the carbon rods (termed *electrodes*) is now increased, it will be found that the arc is eventually extinguished. The temperature of the arc on a d.c. supply approaches 4000° C., but is somewhat lower when an a.c. supply is used, as is generally the case with electric arc furnaces. These furnaces may be classified as follows:—



(a). DIRECT-ARC FURNACE.

(b). INDIRECT-ARC FURNACE.

FIG. 63.—THE TWO TYPES OF ARC FURNACE.

(1) the *direct-arc* type, shown diagrammatically in Fig. 63a, in which the arc is formed between the carbon or graphite electrode and the charge, and
(2) the *indirect-arc* furnace, illustrated diagrammatically in Fig. 63b, the arc in this type of furnace being maintained between a pair of carbon or graphite electrodes situated above the charge.

In the direct-arc type of furnace there are usually at least two electrodes, as shown in Fig. 63a, and hence two arcs in series. This avoids the necessity of making an electrical connection to the charge itself. In this type of furnace some heat is produced due to the current flowing through the charge itself and, owing to the electromagnetic forces thus set up in the molten metal, a stirring action results (see Chapter VIII); this eliminates the necessity for mechanical

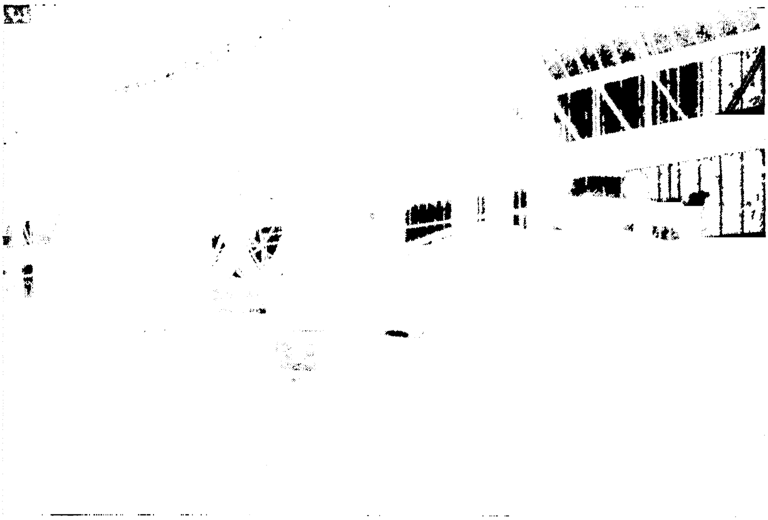


FIG 64 —A 30-TON DIRECT-ARC FURNACE

stirring or mixing of the charge. The direct-arc type of furnace is employed for the production of steel, a typical furnace designed for this work being illustrated in Fig. 64. In this case it will be noted that there are three electrodes passing through the top of the furnace. This is because the equipment is designed to operate on a 3-phase a.c. supply (see Vol. III), an arc being maintained between each of these electrodes and the charge. The power supplied to these

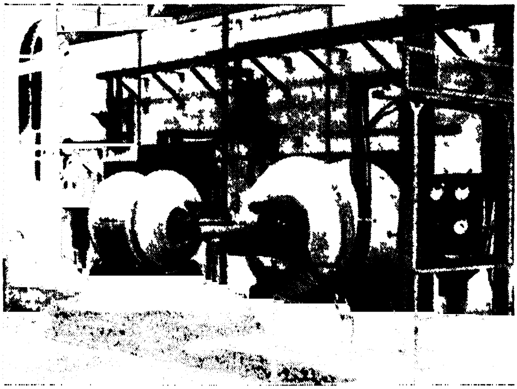


FIG 65.—A PAIR OF 10-CWT. ROCKING INDIRECT-ARC FURNACES

furnaces is controlled by adjusting the applied voltage and by raising or lowering the electrodes, thus varying the length of the arc and the arc resistance.

The indirect-arc furnace is generally cylindrical in shape, with the axis horizontal, so that a rocking motion may be imparted to the furnace for the purpose of mixing the charge, there being no inherent stirring action, as in the direct-arc type. The temperature of the charge is lower than with the direct-arc type, since the heat is transmitted to the charge solely by radiation. The chief application of the indirect-arc type of furnace is in the melting and refining of non-ferrous metals, a pair of typical furnaces being illustrated in Fig. 65.

Electric Welding

The process by which metal parts are joined together by heating them to a temperature near the melting point is known as *welding*. The welding of iron has been carried out for centuries by the blacksmith, who employed an open fire as the heating medium and obtained a satisfactory weld by hammering the joint on the anvil.

The introduction of electrical methods of heating has resulted in an enormous improvement in welding technique, and this process is now commonly used for the jointing of constructional steelwork (e.g., the frames of ships and buildings; bridges and tanks), the fabrication of machine parts from steel plates and rolled-steel sections (thus replacing castings) and also for the repair of broken castings. There are two general methods of electric welding—namely, *resistance welding* and *arc welding*. In the former method the necessary heat is obtained by passing a current through the joint to be welded, whereas in the latter the heat is obtained from an electric arc struck between an electrode and the metal being welded.

(A) *Resistance welding*. The essential conditions for satisfactory resistance welding are: (i) a sufficiently high current value to cause melting of the metals at the joint, and (ii) sufficient mechanical pressure to ensure a satisfactory weld. In all methods of resistance welding the contact resistance at the joint is high compared with the remaining resistances in the series circuit, and hence the bulk of the energy supplied is dissipated as heat *at the joint*.

Resistance welding may be effected in several ways, depending on the form of the metal parts being dealt with. For welding rods, wires and small tubes the process known as *butt welding* is employed, the two ends to be joined being forced together under mechanical pressure while current is passed across the joint (see Fig. 66a). For joining thin sections of metal together (e.g., a lap joint between two metal sheets, or the attachment of thin metal parts to a sheet of metal) a process known as *spot welding* is commonly used. As illustrated in Fig. 66b, the sheets are held under mechanical pressure between a pair of pointed copper electrodes connected to a low-voltage source capable of supplying a heavy current. This current, passing across a relatively small area of contact between the two metal sheets, causes local fusion and gives rise to a "spot" weld. *Seam welding* is employed where a continuous joint is required between two overlapping metal sheets, as in the manufacture of metal containers. In this process the pointed electrodes used for spot

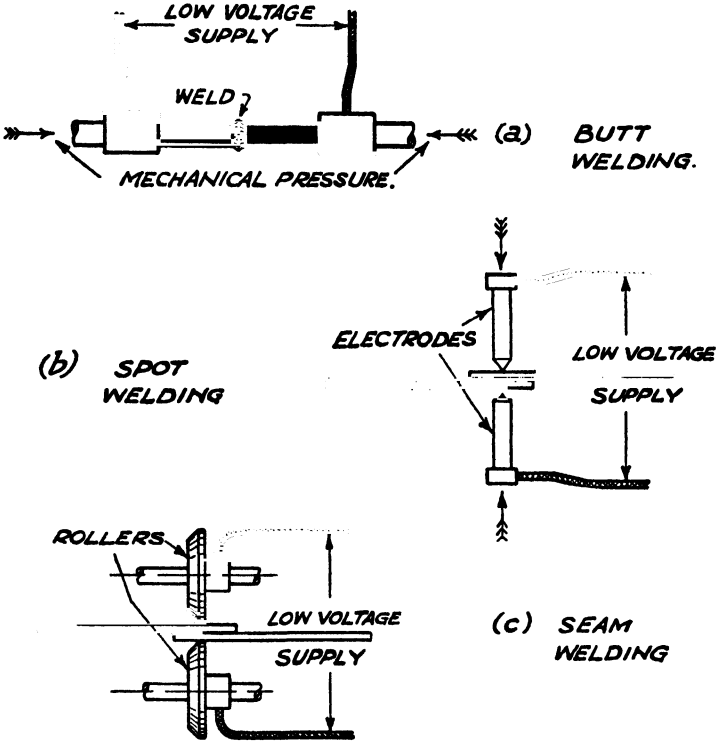


FIG. 66.—RESISTANCE WELDING PROCESSES.

welding are replaced by a pair of rollers between which the lap joint is passed, these rollers having narrow contact edges (see Fig. 66c). It is found preferable to use an intermittent current for this process, and thus a seam weld actually consists of a series of overlapping spot welds. A typical seam-welding machine is shown in Fig. 67. Resistance welding may be used for the jointing of iron and steel, aluminium, copper, brass and various other alloys.

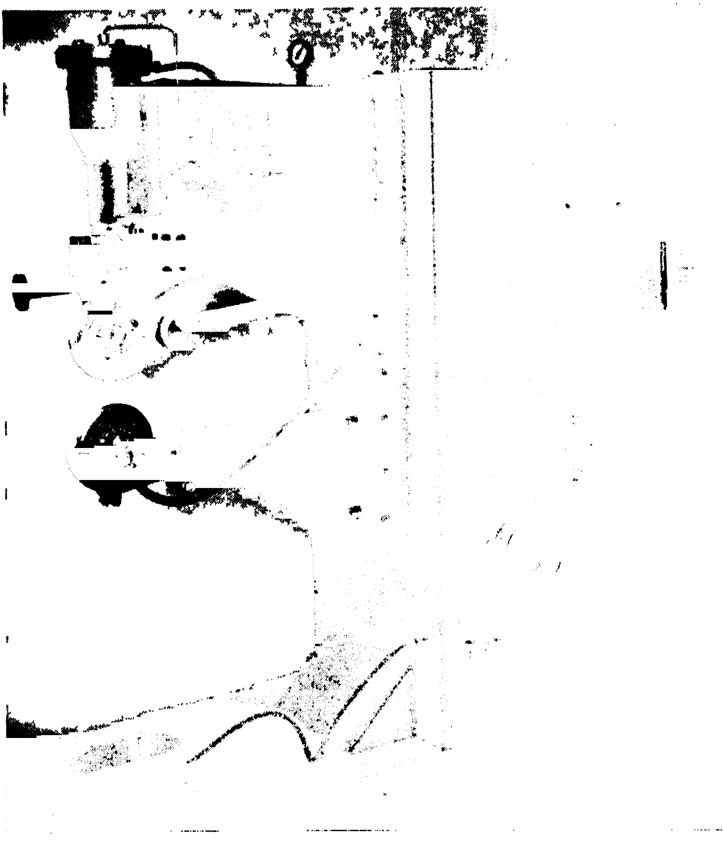


FIG 67.—A 60-kVA GENERAL PURPOSE SEAM-WELDING MACHINE.

An alternating-current supply is used in all these methods of resistance welding, since the necessary heavy current at a low voltage can be obtained very conveniently by means of a transformer.

(B) *Arc welding* The principle of this method of welding is illustrated in Fig. 68. In the arc-welding process the metals in the vicinity of the joint must be heated to the appropriate temperature and, in addition, a filler metal must be supplied and fused at the joint.

The electric arc does not possess a constant resistance, but one which decreases as the current is increased. This effect would render the maintenance of a steady arc impossible, and so the special supply to the arc is designed so that the voltage falls off rapidly as the current rises. This characteristic also helps to maintain the arc, as any tendency for the arc to become extinguished results in a drop in the current, and the consequent rise in voltage helps to maintain it. When two or more welding operators are using a common source of supply, the above method of stabilising the arc is no longer practicable. In such cases a constant voltage supply is used and a resistor connected in series with each welding electrode. (In a.c. circuits a *reactor* may replace the resistor—see Chapter VIII, Vol. II.) Thus, if the current tends to increase, the voltage drop across the resistor also rises, and the arc p.d. is therefore reduced. As we have seen, the source of supply for the electric arc may be either d.c. or a.c. In the former case a separate d.c. generator is installed, and in the latter a transformer, since the voltage required is lower than the usual mains voltage.

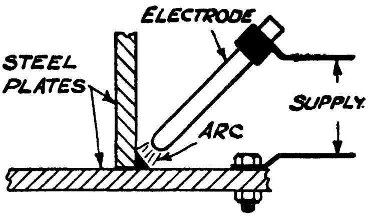


FIG. 68.—THE PRINCIPLE OF ARC WELDING.

The electrode for arc welding may be either of carbon or of metal, thus giving rise to the *carbon-arc* and *metal-arc* processes. In the former process it is found that the electrode must be made negative, otherwise an inferior weld is produced due to the particles of carbon carried across to the joint. Thus a d.c. supply must be used for carbon-arc welding, currents up to about 800 A being employed, with electrodes up to 1 in. in diameter. This process is commonly used for the repair of castings and for non-ferrous metals.

In metal-arc welding the electrode itself supplies the filler metal, and for joints in iron or mild steel a steel electrode containing not more than 0·2% of carbon is used. These electrodes are often covered with a layer of a substance which forms a slag on the metal, preventing oxidation and undue chilling of the joint. The necessity for ensuring satisfactory transference of the filler metal from the electrode to the weld limits the size of the electrode for metal-arc welding to about $\frac{3}{8}$ in. diameter, and the current to about 300 A, the supply being either d.c. or a.c.

Electric Arc and Filament Lamps

There are three main types of electric lamp—namely, the arc, incandescent filament and discharge lamps. The first two of these types behave as *thermal radiators*—i.e., their operation as light sources is dependent upon raising a body to a high temperature, when it is said to be in a state of *incandescence.* The heating effect of the electric current is utilised in these types of electric lamp in order to obtain the necessary high temperature. In the electric discharge lamp an entirely different principle is employed, and this type of lamp will not be dealt with at the present stage.

The higher the temperature at which a thermal radiator operates, the more efficient it becomes as a light-producing source, up to a temperature of about 6200° C., at which the efficiency of light production is a maximum. In order to express this *luminous efficiency* for electric lamps, reference must be made here to the term used for "unit rate of emission of luminous radiant energy"—viz., the **lumen.** Thus, the efficiency of a lamp is expressed in **lumens per watt,** this value being merely the ratio of the useful power output, in the form of light, to the electrical power input. It may be mentioned here that the maximum efficiency of light production in a thermal radiator, corresponding to the temperature of 6200° C. mentioned above, would be about 87 lumens per watt, but this figure is not approached in practical light sources.

(A) *The electric arc lamp.* This was the first type of electric lamp to be employed, but except for special applications, such as cinema projection lamps and searchlights, it has now been replaced by the incandescent filament lamp. For these special applications the open-type of *carbon-arc lamp* is employed, and this is the only type of arc lamp with which it is proposed to deal in this section.

A d.c. supply is essential if the carbon-arc is to be used as an efficient light source for projection work. The positive carbon rod supplies the particles of carbon necessary for maintaining the arc, and it thus develops a crater at the tip, this crater attaining a temperature of about 3500° C. and supplying approximately 90% of the total light emitted. Although, as previously mentioned, the temperature of the arc itself approaches 4000° C., and that of the negative tip is about 2700° C., these emit comparatively little light. The efficiency of the d.c. open-type of carbon-arc lamp is about 20 lumens per watt, this relatively high value being due to the high operating temperature of the positive crater.

The positive carbon is consumed at a much greater rate than the negative rod, and is therefore made thicker. It is also provided with a *core* of softer carbon, having a lower resistivity than the surrounding portion of the rod. This tends to keep the arc steady, since with both carbons solid the arc wanders over the end surface of the positive carbon, thus causing a flickering effect. Owing to the relatively small size of the positive crater, from which most of the light is emitted, the carbon-arc is particularly suitable for cinema projection and searchlight equipment, as focusing is rendered easier.

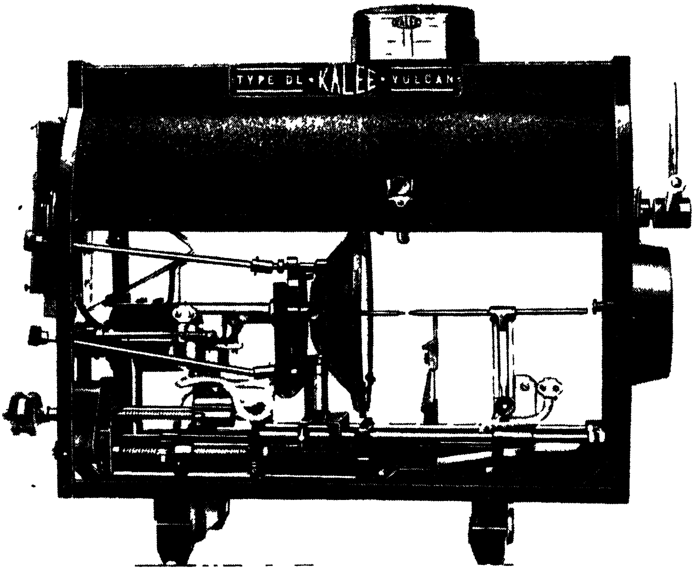


FIG. 69.—A CINEMA PROJECTION ARC LAMP.

The p.d. required to strike the arc is about 40 V, but a higher voltage, depending on the length of the arc, is required to maintain it. As mentioned in connection with electric arc welding, the resistance/current characteristic of the electric arc is such that some form of stabilisation is essential—e.g., a series resistance with a constant-voltage supply, or a supply p.d. decreasing rapidly with increase of current. The current taken by open carbon-arc lamps varies from about 5 A for a small projection lamp, up to about 100 A for a large cinema projection arc, whilst for powerful searchlights currents considerably in excess of this value are employed. A typical cinema projection arc lamp is illustrated in Fig. 69, the mirror being placed around the negative carbon rod to reflect the light from the positive crater.

(B) *The electric filament lamp.* This type of electric lamp is also a light-source of the thermal radiator group. The *filament* is a fine thread of conducting material which is raised to a high temperature by the passage of current through it and is contained in a sealed glass bulb. In order to prevent rapid oxidation and volatilisation of the filament material at this high temperature, it is necessary to remove all traces of air from the glass bulb before it is sealed; this gives the *vacuum* type of filament lamp.

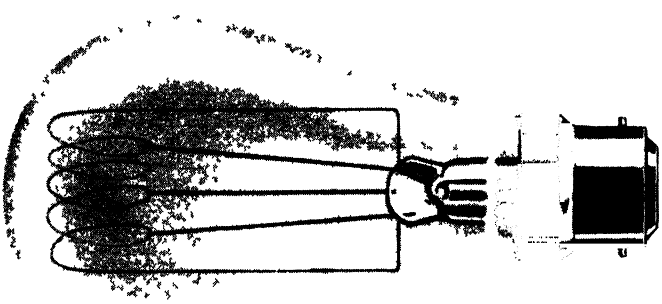


FIG. 70.—A CARBON FILAMENT LAMP.

(1) *Carbon filament lamps.* The first practical incandescent filament lamps were of this type, and were placed on the market in about 1880 as a result of independent work by Swan in this country and by Edison in America. The filaments of these early lamps were of carbonised fibre (wood splints or cotton-thread), but were soon replaced by "squirted" filaments prepared by forcing a cellulose solution through a jet and subsequently carbonising the resulting gelatinous thread by heating it to about 1100° C., having previously wound it to the required spiral form. These

carbonised filaments are quite hard, but flexible, and are thus easily handled during the subsequent stages of manufacture.

A carbon filament can be operated in an evacuated bulb at about 2000° C. with a reasonable life, and gives an efficiency of about 2·5 lumens per watt. The carbon filament lamp is now employed for special purposes only, a typical lamp being illustrated in Fig. 70, in which the relatively thick filament, formed into a large, open spiral consisting of a few turns only, is clearly visible.

(2) *Metal filament lamps.* In an attempt to find a more satisfactory material for the filaments of electric lamps (i.e., a substance capable of operating continuously at a higher temperature than carbon) various metals such as osmium and tantalum were tried out, and eventually it was found possible to prepare satisfactory filaments from tungsten, a metal having a melting point of about 3300° C. All modern metal-filament lamps have tungsten filaments, and the term *tungsten-filament lamp* has become synonymous with *metal-filament lamp*. Owing to its high melting point, a tungsten filament can be run at about 2400° C. in a vacuum, giving an average efficiency during its useful life of between 7·7 and 8·8 lumens per watt, according to the voltage and wattage of the lamp.

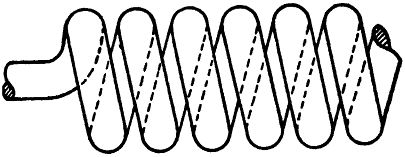


FIG. 71.—DIAGRAMMATIC VIEW OF A SINGLE-COIL TUNGSTEN FILAMENT (HIGHLY MAGNIFIED).

The *vacuum* type of tungsten-filament lamp, however, is not now used except in the smaller sizes, nearly all modern tungsten-filament lamps being of the *gas-filled* type. A small quantity of an inert gas, usually a mixture of argon and nitrogen, is introduced into the bulbs of these lamps after evacuation. The effect of the gas molecules is to reduce the rate of *evaporation* or *volatilisation* of the filament, and thus, for the same life, it may be run at a higher temperature. Further, the disadvantage of additional heat loss from the filament due to convection is overcome by coiling the filament wire into a close spiral (see Fig. 71), thus giving, in effect, a relatively short and thick cylinder of incandescent metal. This principle is further extended in the modern *coiled-coil* tungsten-filament lamp, in which the close spiral of fine tungsten wire is coiled again on itself (see Fig. 72). It should be noted that a coiled-coil filament does *not* operate at a higher temperature than a single-coil filament, but owing to the reduced convection loss per unit length of the filament, a longer and thicker filament wire can be employed, thus resulting in an increased light output from the greater volume of incandescent metal. The operating temperature in modern gas-filled tungsten filament lamps is about 2800° C., the corresponding efficiencies for single-coil and coiled-coil lamps being as follows:—

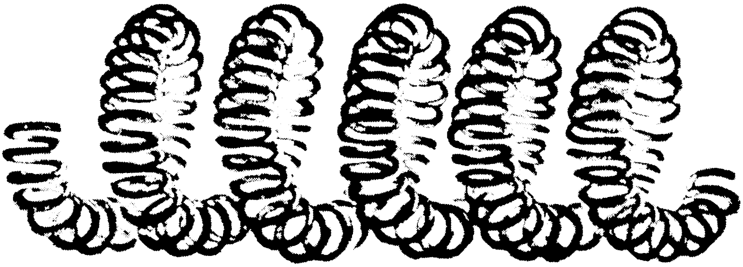


FIG. 72.—ACTUAL PHOTOGRAPH OF A COILED-COIL TUNGSTEN FILAMENT (HIGHLY MAGNIFIED).

Type.	Nominal efficiency in lumens per watt (average throughout life) for 240 V lamps.						
	40 W.	60 W.	75 W.	100 W.	150 W.	500 W.	1500 W.
Coiled-coil .	9·75	11·08	11·73	12·60	—	—	—
Single-coil .	8·12	9·58	10·40	11·60	13·07	15·40	18·33

The above values are obtained from data given in B.S. 161/1956.

A typical general-service tungsten filament lamp is illustrated in Fig. 73, from which the method of mounting and supporting the filament will be appreciated. The lead-in wires connected to the ends of the filament spiral are usually of nickel, each of these being welded to a short length of *copper-clad* wire (a core of nickel-iron alloy, coated with copper) having expansion characteristics similar to those of glass. Thus a good vacuum joint is maintained when the copper-clad wire is sealed into the glass " pinch." Each of these short lengths of copper-clad wire is welded to a copper wire for connection to one of the lamp contacts. There are two types of contact and lamp cap in use in this country—viz., (a) the *bayonet cap* (abbreviated to B.C. and illustrated in Fig. 73), with which two contact plates are used in general-service lamps, these plates being insulated from the lamp cap, and (b) the screw type of cap (see Fig. 74), usually employed for general-service lamps of 200 W and upwards in size. In this case there is one central contact plate, the other connection being made to the brass cap, on which is formed a coarse screw-thread. This type of lamp cap is used in two sizes for general service lamps—the *Edison screw cap* (E.S.) and the *Goliath Edison screw cap* (G.E.S.), the latter being fitted only to the largest sizes of lamp.

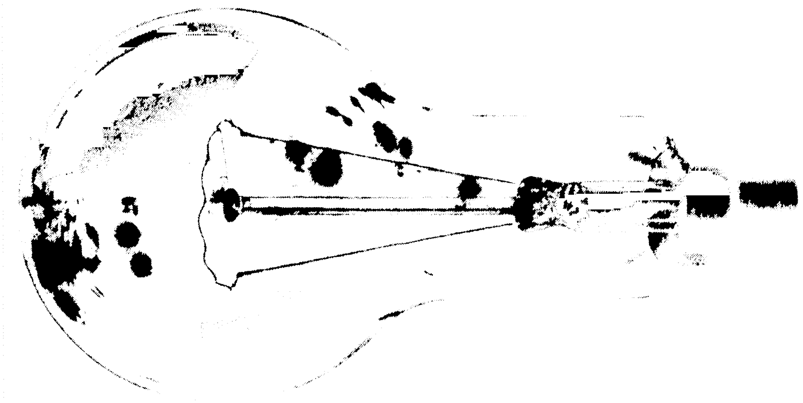


FIG. 73.—A GENERAL PURPOSE TUNGSTEN FILAMENT LAMP.

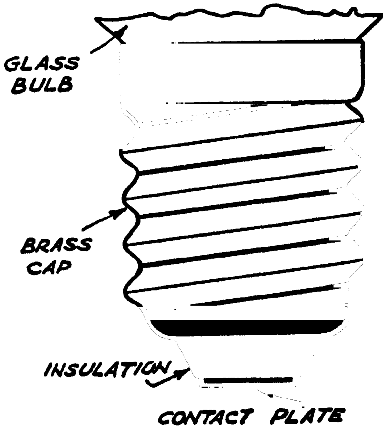


FIG. 74.—THE SCREW TYPE OF LAMP CAP.

The glass bulbs of modern tungsten filament lamps may be clear, internally frosted ("pearl") or opal. The latter type has a thin skin of opal glass on the outside, this resulting in a reduction in light output of about 7%.

In the larger sizes of tungsten filament lamp, such as those employed in cinema-studio projectors, the internal construction differs considerably from that used for general service lamps. Fig. 75 illustrates the construction of a 5-kW lamp of a special type, developed for use in high-intensity projection apparatus. The two main and substantial filament supports are of nickel, and these are connected by upper and lower cross-members of *steatite*—a ceramic insulating material. A special hard glass bulb is necessary to withstand the high temperatures attained in lamps of this wattage loading. The relatively high current is fed to the lamp by means of the two pins shown, which are of the *copper-thimble* type to overcome difficulties due to the differences in expansion between the glass and metal.

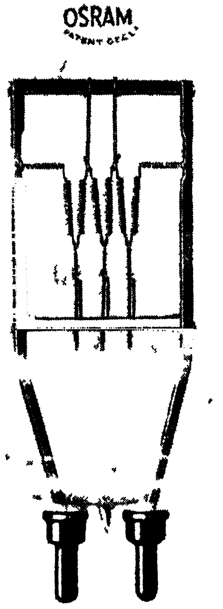


FIG 75.—A 5-kW PROJECTION LAMP

The Electric Fuse

If we consider the simple lighting circuit illustrated in Fig. 76 it is obvious that if the filament of the lamp L became short-circuited (due to a "fault" occurring in either the lamp, lamp-holder or flexible cable supporting the lamp and fitting), an excessive current would flow when the switch S was closed, since the total resistance of the circuit has now been reduced to a relatively low value. This excessive current would very quickly result in serious overheating of the connecting cables, with consequent damage to the insulation of these cables. It should be noted that if the current increases to *ten times* its original value, the power dissipated as heat per unit length of conductor increases *at least one hundred times*, since $P = I^2R$.

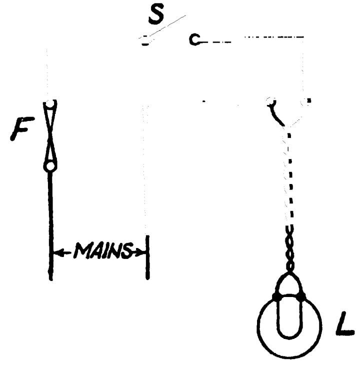


FIG 76—A SIMPLE LIGHTING CIRCUIT, PROTECTED BY A FUSE.

To prevent such damage a protective device known as *the electric fuse* is employed. This is indicated at F in the above-mentioned circuit, and consists of a " weak link," termed the **fuse-element,** designed to melt when the current reaches an excessive value, thus interrupting the circuit. The lowest current capable of melting a given fuse-element is termed the *minimum fusing current.* With this value of current flowing a relatively long period elapses before the temperature of the fuse-element reaches the melting point; with higher current values this melting temperature is attained more rapidly.

The minimum value of fusing-current for any given fuse-element depends on (i) the cross-sectional area of the wire or strip from which the element is made, (ii) the material employed and (iii) the manner in which the fuse-element is mounted. (i) and (ii) affect the resistance per unit length of the fuse-element and hence, for a given current value, the watts dissipated per unit length. (iii) governs the rate of heat dissipation from the fuse-element and hence the final steady temperature for a given current value, whilst on (ii) depends the melting point of the fuse-element. Usually, under fault conditions, the current attains a value greatly in excess of this minimum fusing-current before the circuit is finally interrupted. However, the minimum fusing-current is of importance when fuses are relied upon to give protection against sustained overloads. The ratio of the minimum fusing-current to the rated normal current of the fuse is termed the *fusing-factor.* There is clearly a risk in designing a fuse so that the fusing-factor is only a little in excess of unity. The normal operating temperature of the fuse-element would be high, probably resulting in gradual deterioration and eventual premature failure. Fusing-factors varying between 1·5 and 2·0 are fairly common and may be employed with safety in good-quality fuses.

Fuses may be divided into the following types, according to the manner in which the fuse-element is mounted:—

(1) the open type, in which, apart from any external protective cover, the fuse-element is in free air,
(2) the semi-enclosed type, in which the fuse-element is neither in free air nor totally enclosed, and
(3) the cartridge type, in which the fuse-element is totally enclosed. The enclosing cartridge may be filled with a solid substance, usually in granular form, or with a liquid, to assist in extinguishing the arc.

The *open type of fuse* is not very frequently employed nowadays, except for the supply authority's main service fuses in domestic installations.

Most of the fuses employed for the protection of relatively low-power consumers' circuits, at voltages up to 440, are of the *semi-enclosed re-wirable type.* A typical re-wirable porcelain fuse is illustrated in Fig. 77. This fuse consists of two main parts—the removable fuse-carrier, between the contacts of which the fuse-wire is attached, and the fixed base provided with terminals for connection in the circuit. These terminals are connected to, or form part of, fixed contacts into which fit the fuse-carrier contacts. The material now generally employed for the fuse-wire is *tinned copper*—i.e., copper wire provided with a coating of tin, which minimises oxidation under normal operating conditions. The table on p. 133 gives the approximate values of minimum fusing-current for various sizes of tinned copper wire, *supported in free air.*

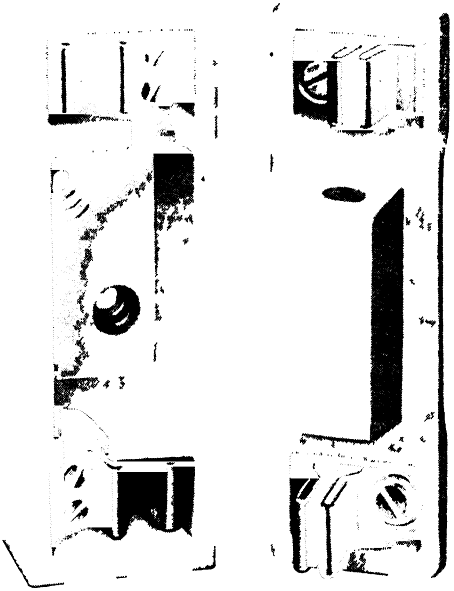


FIG. 77.—A TYPICAI PORCELAIN RE-WIRABLE FUSE

Cartridge fuses are used for the protection of circuits in which a considerable amount of power can be fed into the arc formed when the fuse "blows" under short-circuit conditions. Such a fuse, known as an H.R.C. (high rupturing capacity) fuse, for use in circuits up to 440 V, is shown in Fig. 78. The fuse-element in this case is of silver, but copper, aluminium and zinc are also employed. It will be noted that there are two separate fuse-elements in parallel in this fuse, and that each of these elements has a "neck" of minimum cross-section near the centre of the cartridge. These factors tend to assist in reducing the normal operating temperature of the fuse. External evidence of the "blowing" of the fuse is provided by means of the indicator wire, a fine wire in parallel with the main fuse-elements. When this indicator wire fuses, a small quantity of mildly explosive powder causes charring of an external label. The outer body of this fuse is of a ceramic material, whilst the filler is an inert substance such as silica. An alternative type of filler (e.g., calcium carbonate) gives off a gas when heated by the arc, this gas assisting in arc extinction, but necessitating the provision of a pressure-relief vent in the body of the fuse.

Nominal size of wire (S.W.G.) .	47	43	38	33	27/28	22	18
Diameter of wire (in.) . .	0·0020	0·0036	0·0060	0·0100	0·0156	0·028	0·048
Approx. minimum fusing-current (amp.) . .	1	2	5	10	20	50	100

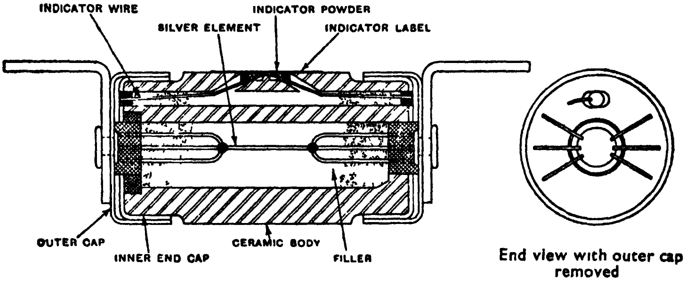


FIG 78.—H.R.C. CARTRIDGE FUSE (440 V).

H.R.C. cartridge fuses are also used on high-voltage circuits (up to 33 kV), and are, in general, similar to those for use on lower-voltage circuits, except that the fuse-element is longer and a liquid arc-extinguishing medium (e.g., carbon tetrachloride, or oil) is used in certain types.

The Hot-Wire Ammeter

In this application the heating effect of the electric current is utilised for the measurement of current strength. The current is passed through a stretched length of fine-gauge wire of high resistivity material (e.g., an alloy of platinum with iridium), the extension of this wire due to its rise in temperature being magnified and indicated on a scale graduated in terms of current.

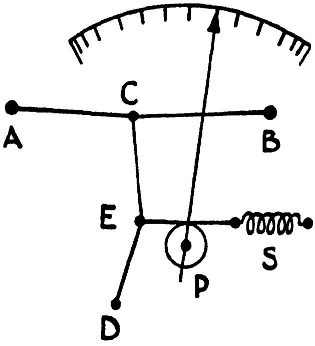


Fig. 79.—The Hot-wire Type of Ammeter.

In Fig. 79 AB represents the resistance wire, having a second wire CD attached to its mid-point C. This second wire is used merely to amplify the sag of the hot wire, and does not carry current. A silk thread has one end attached to a point E on the second wire and passes round a pulley P. The other end of this thread is joined to a light spring S, which keeps the wires taut. The instrument pointer is connected to the pulley P and moves over a graduated scale as the extension of the hot wire AB is taken up by the spring S. Although the rate at which heat is produced from electrical energy is proportional to the square of the current, and so the extension of AB is also very roughly proportional to the square of the current, the system for magnifying this extension is such that the angular deflection of the pointer is more nearly proportional to I. Thus the scale of a hot-wire ammeter is not so unevenly divided as might be expected.

One important advantage of the hot-wire ammeter, however, must be referred to here—namely, its ability to measure both alternating and direct currents. This type of ammeter is, in fact, used for measuring the high-frequency alternating currents used in radio work. The instrument does suffer from the disadvantage that the zero setting of the pointer tends to alter with changes in room temperature, although compensating devices can be fitted to reduce this source of error.

Experiment 11. *To determine the current/voltage characteristic for (a) a nichrome heating-element, (b) a carbon filament lamp and (c) a tungsten filament lamp.*

Connect first the nichrome heating-element (e.g., a low-wattage clamp-on element, or convection heater) in circuit with the ammeter, voltmeter and rheostat Rh, as shown in Fig. 80. The voltage of the supply should preferably be capable of variation from a low value to at least the rated p.d. of the appliance. The supply may be either d.c. or a.c., provided that suitable instruments are used in the latter case (see Section 24, Chapter III). Commencing with the applied p.d. at about one-tenth of the normal rated voltage of the heating-element, note the corresponding current value when the ammeter reading has become steady. Record the voltmeter and ammeter readings, and repeat for higher voltages up to the normal rated p.d. of the heating-element (use Rh to adjust the voltmeter reading in each case to an exact scale marking). Plot the current/voltage relationship as in Fig. 81, the axes being scaled in terms of " voltage expressed as a percentage of normal rated voltage " and " current expressed as a percentage of current at normal rated voltage " respectively. Repeat this procedure with the carbon filament lamp and then the tungsten filament lamp connected in the circuit of Fig. 80 in place of the heating-element, plotting the current/voltage curves on the same pair of axes as were used for the first set of results (see Fig. 81).

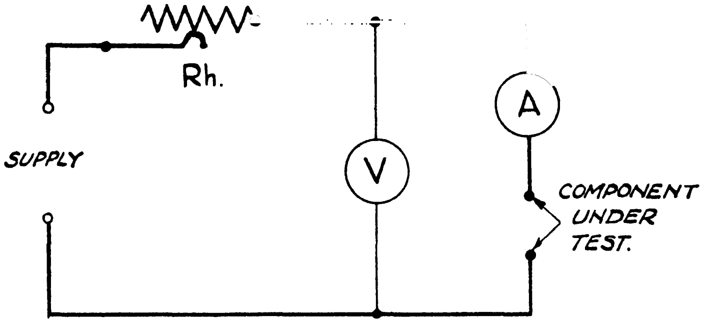


FIG. 80.—CIRCUIT FOR EXPT. 11.

The reader has probably realised by now that each of the voltmeter readings recorded above includes the voltage drop across the ammeter in addition to the p.d. across the appliance under test. For a good-quality ammeter, however, this voltage drop should not exceed 0·1 V at full-scale deflection, so the error is negligible.

The curves obtained will be of the general shape indicated in Fig. 81. Thus, for the nichrome heating-element the current/voltage characteristic is practically linear, because the resistance of the heating-resistor does not change appreciably with temperature (see Fig. 52 and Section 32; appendix to Chapter III). A component possessing this straight-line current/voltage characteristic is frequently referred to as a **linear resistor.** For the carbon-filament lamp the graph curves upwards, because the resistance of the carbon filament *decreases* with increase of temperature (see Section 22, Chapter III), and hence the current increases *more rapidly* than the voltage. The resistance of the tungsten filament, however, *increases* with increase of temperature (in common with most pure metals), and the voltage therefore increases more rapidly than the current, giving rise to a characteristic of the shape indicated. Components

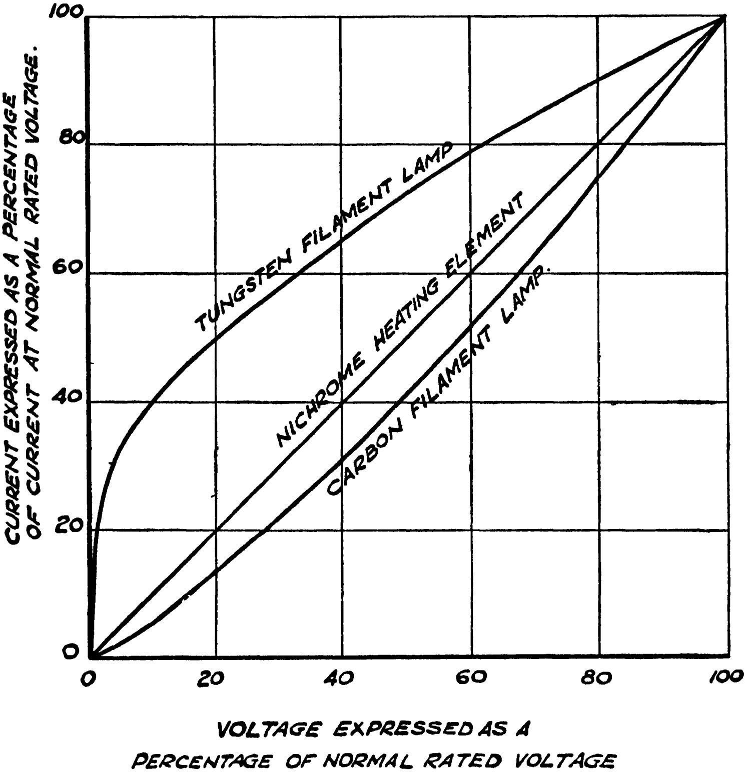


FIG. 81.—CURRENT/VOLTAGE GRAPHS OBTAINED IN EXPT. 11.

giving such non-linear current/voltage characteristics are often referred to as **non-linear resistors.**

It will now be appreciated that a *precisely linear* current/voltage characteristic is merely an ideal, since all substances undergo *some* change in resistance, however slight, with change in temperature. This ideal linear characteristic is, however, approached very closely by practical resistors in which either (a) the conducting material possesses a very low temperature coefficient of resistance, as in the case of manganin, eureka or nichrome, or (b) the temperature rise when carrying the normal current is very low, due to the facility with which the heat is dissipated from the conductor.

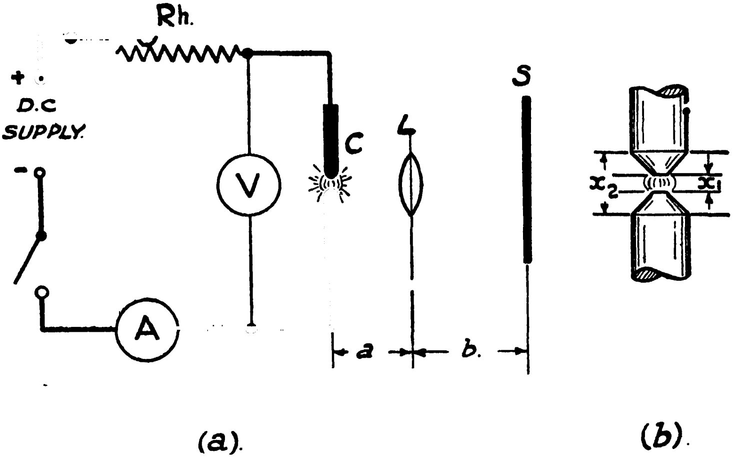


FIG. 82.—APPARATUS FOR EXPT. 12.

Experiment 12. *To determine, for a carbon-arc, the relationship between (a) arc voltage and arc length and (b) arc voltage and current.*

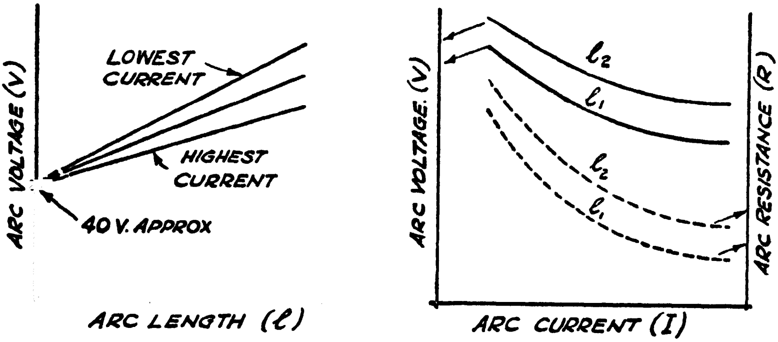
Connect the arc C (preferably an open-type projection arc) in series with a rheostat Rh, ammeter and switch as indicated in Fig. 82a, with a voltmeter directly across the arc. The arc is struck by bringing the carbons together and then separating them slightly. The current flowing in the circuit is controlled by the rheostat Rh. A lens L of long focal length is arranged to throw an image of the arc on to a paper screen S. If x mm. is the length of the image, then the actual length of the arc, l mm., is given by:—

$$l = x \times \frac{a}{b}$$

In Fig. 82a the distance *b* has been reduced for convenience, actually *b* should be at least ten times *a* to give an image of size suitable for accurate measurement.

With the carbons separated by the smallest convenient distance, mark the extremities of the image length on the screen S and rule a pair of parallel lines as shown in Fig. 82b. Keeping the arc image constantly adjusted to this fixed length, take a number of corresponding readings of arc voltage (V volts) and current (I amp.) from the minimum current at which the arc can be maintained to the value at which the arc commences to hiss. Repeat this procedure for three or four increased values of arc length. Tabulate the results as shown below and calculate the corresponding values of arc resistance, R ohms $\left(= \frac{V}{I}\right)$. The distances a cm. and b cm. (Fig. 82a) should be measured and recorded, also the lengths of the arc image (x mm.). Thus the actual arc lengths, l mm., may be calculated and entered in the table below.

$x_1 = \ldots.$ (mm.)	I						
$l_1 = \frac{x_1 a}{b}$	V						
$= \ldots.$ (mm.)	R						
$x_2 = \ldots.$ (mm.)	I						
$l_2 = \frac{x_2 a}{b}$	V						
$= \ldots.$ (mm.)	R						



(a). (b).

Fig. 83.—Results for Expt. 12 in Graphical Form.

Plot the V/l relationship, as indicated in Fig. 83a, for each of three widely-spaced current values. Note that this relationship is approximately linear in each case, and that each of the three graphs, when produced, cuts the voltage axis at about 40 V, indicating that this is the minimum voltage required to maintain the arc.

A second set of curves should be plotted as shown in Fig. 83b, the V/I and R/I graphs being drawn for two values of arc length ($l_2 > l_1$). It will be noted that the arc voltage decreases as the current is increased, for a given arc length. Thus the arc resistance also decreases with an increase in arc current, and does so more rapidly than the arc voltage, since $R = \frac{V}{I}$. The carbon arc, therefore, behaves as a *non-linear resistor* (see Experiment 11).

APPENDIX

38. The Heating Effect with Alternating Current

The formula giving the total amount of heat produced, H calories, when a steady current of I amp. flows through a resistance of R ohms for t sec., has already been derived (Section 36), and is as follows:—

$$\mathrm{H} = 0{\cdot}24\, I^2Rt$$

From this result we may express the *rate*, h calories per sec., at which electrical energy is being transformed into heat, viz.:—

$$\mathrm{h} = \frac{\mathrm{H}}{t} = 0{\cdot}24\, I^2R$$

Thus, for a fixed value of the resistance, R ohms, the rate of production of heat is directly proportional to the *square* of the current, or $\mathrm{h} \propto I^2$.

When the current has a steady value, heat is produced in the resistor at a constant rate, these conditions being illustrated by the graphs A and B in Fig. 84. If the current is varying continuously, as, for example, in graph C (Fig. 84), between 0 and $\frac{T}{2}$ sec., then h also varies from instant to instant. If the current at a particular instant is i amp., then the rate at which heat is being produced at this instant is given by: $\mathrm{h} = 0{\cdot}24\, i^2R$. The manner in which h varies with time, from 0 to $\frac{T}{2}$ sec., is shown by curve D in Fig. 84.

If the *direction* of the current reverses, this does not affect the rate at which electrical energy is transformed into heat because, for solid metallic conductors, the electrical resistance, R ohms, is independent of the direction of current flow. Thus, if the current/time graph (C) in Fig. 84 is of precisely the same shape between $\frac{T}{2}$ and T sec. as from 0 to $\frac{T}{2}$ sec., but merely reversed in sign, the two portions of the h/time graph (D) will also be identical.

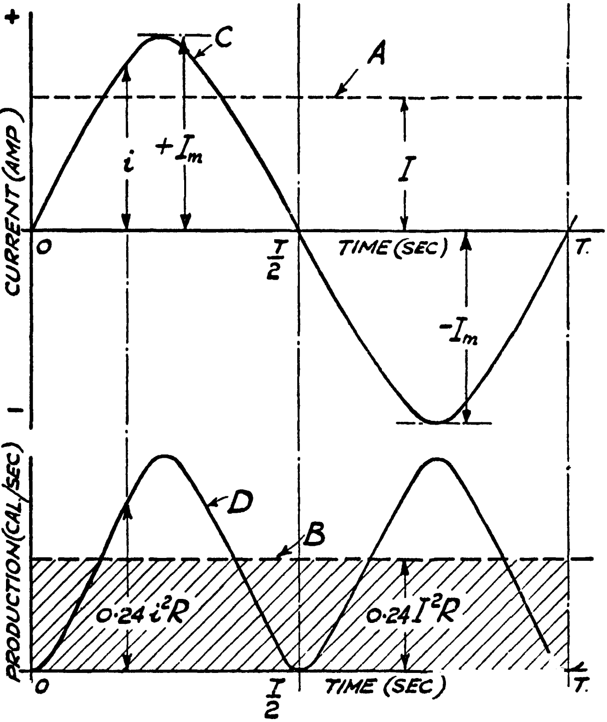


FIG. 84.—THE HEATING EFFECT WITH D.C. AND A.C.

The current/time graph C represents the waveform of an *alternating current* (see Section 16, Chapter II) and illustrates the variations of current during one complete cycle, the time occupied by this cycle being T sec., and the frequency $\frac{1}{T}$ cycles/sec. The value of the alternating current, i amp., at any instant of time during the cycle is referred to as the *instantaneous value* of the current.

Returning now to the steady current I amp., flowing through a resistance R ohms; the total amount of heat produced in T sec. is $0{\cdot}24\ I^2R \times T$ calories—i.e., *it is represented by the shaded area under graph B* in Fig. 84. Similarly, when the rate of heat production varies from instant to instant, as, for example, in curve D, the total amount of heat produced in a given time is represented by the corresponding area under the h/time graph. Suppose the value of the steady current, I amp., has been chosen so that the total amount of heat produced in T sec. is the same as when the alternating current represented by curve C is employed, the same resistance, R ohms, being assumed in both cases. Then the area of the shaded rectangle (Fig. 84) must be equal to the area under the curve D, from 0 to T sec. Hence,

$$\text{average ordinate to curve D} = \text{constant ordinate to graph B}$$

$$\therefore \text{ the average value of } (0{\cdot}24\ i^2R) = 0{\cdot}24\ I^2R$$

$$\text{i.e.,} \quad \text{average value of } i^2 = I^2$$

$$\text{Whence,} \quad I = \sqrt{\text{average value of } i^2}$$

The above reasoning applies to a period of time, T sec., equal to the duration of one cycle of the alternating current. Clearly, the same result would be obtained for any period of time representing a multiple of T sec. Since the frequency of practical alternating currents is usually at least 50 cycles/sec., there is little error in assuming that any practical time interval *does* represent a multiple of T sec., and hence, in all cases, the above result may be regarded as sufficiently accurate for practical purposes.

The value of the alternating current referred to above—viz., $\sqrt{\text{average value of } i^2}$—is termed the *root-mean-square* (abbreviated to R.M.S.) value, or alternatively the *virtual* or *effective* value of the alternating current.

The relationship between this value and the *maximum* or *crest* value of the a.c. (I_m in curve C, Fig. 84) depends on the precise waveform, but for the waveform most frequently encountered in electrical engineering (the *sine* curve, see Section 89, Chapter VIII) the relationship is:—

$$\textbf{R.M.S. value} = \frac{1}{\sqrt{2}} \times I_m = 0{\cdot}707\ I_m$$

This matter will be dealt with more fully in Chapter VII of Vol. II, but the result obtained above is sufficiently important to justify the following summarised statement:—

> An alternating current gives rise to the same heating effect (i.e., produces the same amount of heat in a given time) as a steady current of *I* amp. if the R.M.S. value of the alternating current is *I* amp.

It will now be appreciated that the hot-wire ammeter (Section 37) records the true R.M.S. value of any alternating current passed through it, regardless of the waveform of this current. This is because the temperature rise of the hot wire, and hence the deflection of the pointer, depends solely on the *average* rate at which heat is produced in the wire, i.e., on the average value of the square of the instantaneous current.

Exercises on Chapter IV

Any additional data required for the solution of these exercises may be obtained from the tables at the end of this book.

1. Two electric radiators, taking 750 watts and 2 kilowatts respectively, are connected in parallel across 200-volt mains. Calculate the total current taken from the supply and the number of B.O.T. units used in 24 hours.
[13·75 A; 66.]

2. If the current taken by a d.c. motor developing 10 H.P. be 40 amp. at a p.d. of 220 volts, determine the efficiency of the motor and the number of B.O.T. units it will use per hour.
[84·8%; 8·8.]

3. In a power-station 2500 tons of coal, having an average calorific value of 12,600 B.Th.U. per lb., are burnt in a week, the corresponding output of electrical energy during this period being $5{\cdot}36 \times 10^6$ kWh. Calculate the overall thermal efficiency of the station.
[25·94%.]

4. The rate of heat loss from the walls, etc., of a closed room, maintained at 65° F., is 2500 B.Th.U. per hour, whilst an additional loss of 3500 B.Th.U. per hour is introduced by ventilating the room. Find the power, in watts, necessary to maintain this room at 65° F., and the cost, per hour, of the electrical energy used, if the charge for heating purposes is 0·875 d. per B.O.T. unit.
[1756 W; 1·54 d.]

5. An office is illuminated by twelve 200-watt ceiling fittings and six 100-watt desk standards, the cost of electrical energy for lighting

purposes being 3½d. per unit. Calculate the annual cost of the electrical energy used for lighting this office if all the lamps are in use for an average of 22 hours during each of the 50 working weeks in the year.

[£48 2s. 6d.]

6. Calculate the total current taken by eight 60-watt lamps, three 100-watt lamps and a 2-kW fire, also the cost per hour of running them if the charge per B.O.T. unit is 4d. for lighting purposes and 1d. for heating. The supply is at 230 volts.

[12·09 A; 5·12 pence.]

7. A current of 780 amp. at 600 volts is *delivered* through a cable with a voltage drop of 35. What percentage of the power *generated* is lost in the cable and what is the cost per hour of the wasted energy at ½d. per unit?

[5·51%; 13·65 pence.]

8. A factory power load consists of:—

One 100-h.p. motor having an efficiency of 90%.
Two 50 „ motors „ „ „ „ 87%.
Five 20 „ „ „ „ „ „ 80%.
Heating apparatus taking 200 amp. at 250 volts.
Lighting equipment taking 50 amp. at 250 volts.

Estimate the total annual charge for electrical energy, the tariff being £4 per kW of maximum demand plus 0·4d. per unit consumed. The maximum demand occurs when all the equipment but the 100-h.p. motor is in use at full-load. The 100-h.p. motor runs for 1000 hours per annum, and the remaining motors for 2000 hours per annum (all at full-load). The heating apparatus is in use for 1200 hours and the lighting equipment for 500 hours in the year.

[About £1810 10s.]

9. A domestic consumer uses 450 units per annum for lighting purposes and a further 3500 units per annum for heating and other power purposes. Calculate the total annual charge for electrical energy on each of the following alternative tariffs:—

(a) 3d. per unit for lighting and 1d. per unit for power purposes,
(b) a fixed annual charge of £6 5s. (12½% of the rateable value of the property) plus ½d. per unit consumed.

Assuming that the annual consumption of energy for lighting purposes remains fixed, what annual consumption for other purposes would make tariffs (a) and (b) equally attractive?

[(a) £20 4s. 2d.; (b) £14 9s. 7d.; 750 kWh.]

10. A vessel containing 1 litre of water, initially at 5° C., is heated by an immersion element taking 500 watts. Determine (a) the number of calories produced every minute, (b) the rise in temperature of the water per minute and (c) the time taken for the water to reach 80° C. Neglect heat losses.

[(a) 7168; (b) 7·168° C.; (c) 10·47 min.]

11. A 20-gallon electric water-heater is fitted with two heating-elements each taking 4·5 amp. at 230 volts. Find the time required to raise the temperature of the water from 45° F. to 190° F., assuming that the total heat loss amounts to 12% of the energy supplied.

[4·662 hr.]

12. Find the cost, with electrical energy at 1d. per B.O.T. unit, of boiling 3 pints of water in 7 minutes, using an electric kettle for which the efficiency may be taken as 85% and assuming an initial temperature of 8° C. Determine also the power required and the resistance of the heating-element, if the supply is at 230 volts.

[0·214d.; 1835 W; 28·82 Ω.]

13. An electric immersion-heater is required to raise the temperature of 15 gallons of water from 50° F. to 200° F. in 3 hours. The heat losses from the tank during this period may be taken as 2025 B.Th.U., and the thermal capacity of the tank, heater, etc., as 2·5 B.Th.U. per deg. F. Estimate the loading of the heater and the efficiency of the heating operation.

[2·431 kW; 90·36%.]

14. A vessel contains 1 kgm. of oil (specific heat 0·9) and a heating coil of resistance 8 ohms. If the heater be connected to a 100-volt supply, find how long it will take for the oil to rise 50° C. in temperature. Assume that the heat losses from the vessel amount to 10% of the total heat produced and that the thermal capacity of the vessel and coil is 200 cal. per deg. C.

[3 min. 24 sec.]

15. An electric arc furnace takes 2500 amp., the p.d. between the electrodes being 105 volts. The charge consists of 0·5 ton of

brass, and this is to be heated from 40° C. to 900° C. (below the melting point). Determine the time taken, assuming that the efficiency of the heating operation is 72·5%.

[14·41 min.]

16. In making a butt-welded joint between two 1-inch-diameter steel bars it may be assumed that a length of ⅜ inch on each bar is to be raised to a mean temperature of 870° C. from an initial temperature of 20° C. Assuming that this heating operation is effected with an efficiency of 60%, estimate the cost of heating, per thousand welds, with electrical energy at 0·875d. per unit.

[About 11d.]

17. In a certain d.c. arc-welding operation the mean arc p.d. was 50 volts and the current 150 amp., a resistance of 0·1 ohm being connected in series with the welding electrode. Determine the cost of the electrical energy used for welding, per minute, if the charge per B.O.T. unit taken from the a.c. mains is 1·28d. and the efficiency of the a.c./d.c. conversion is 65%. What percentage of the total energy supplied is wasted in the series resistance?

[0·32d.; 15%.]

18. A d.c. arc lamp has the following current/voltage characteristics, for a fixed separation of the electrodes.

Arc current (amp.) . .	2	4	6	8	10
Arc p.d. (volts) . . .	73	61·5	53·8	50·4	49·5

Estimate (a) the series resistance necessary to stabilise the arc current at 8 amp. on an 80-volt supply, (b) the supply p.d. necessary to give an arc current of 5 amp. with a series resistance of 6 ohms, and (c) the arc current when the supply p.d. is at 90 volts and the series resistance is 5 ohms.

[(a) 3·7 Ω; (b) 87 V; (c) 7·9 A.]

19. A tungsten-filament lamp has the following current/resistance characteristic:—

Current (amp.) . .	0·1	0·14	0·18	0·22	0·26	0·30
Resistance (ohms) . .	110	243	372	482	578	690

Determine (a) the p.d. across the lamp when joined in series with a resistor of 250 ohms on a 200-volt supply, and (b) the series resistance necessary to limit the power dissipation in the lamp to 25 watts on the same supply as in (a).

[(a) 138 V; (b) 395 Ω.]

*20. A current flowing in a 5-ohm resistor rises uniformly from zero to 15 amp. in 3 milli-seconds, remains constant for a further 4 milli-seconds and falls uniformly to zero in the next 3 milli-seconds, this cycle occurring periodically. Plot the power/time graph from 0 to 10 milli-seconds and determine (a) the energy, in joules, dissipated in this time, (b) the average power, in watts, and (c) the steady current necessary to give the same reading on a hot-wire ammeter.

[(a) 6·75 J; (b) 675 W; (c) 11·62 A.]

* *This exercise should be attempted only after reading Section* 38—*Appendix to Chapter IV.*

CHAPTER V

ELECTRO-CHEMISTRY

39. Electrolytes

FROM the point of view of electrical conductivity, liquids may be divided into three groups. Firstly, there are liquids with resistivities so high that they may be regarded as non-conductors. Oils of various types belong to this class. Secondly, there are mercury and molten metals which behave just like solid metallic materials, being excellent conductors; when a p.d. is applied across such a metallic liquid a current flows, heat being produced due to the resistance offered, but no chemical change takes place, since the liquid consists of a single metallic element. Finally, there are liquids, consisting of compounds in the fused state or in solution, which behave as conductors allowing a current to flow—accompanied by a chemical change—when they are subjected to an applied p.d.

Such compounds are known as **electrolytes,** and consist of either **acids, bases** or **salts.** An *acid* may be defined as a substance which turns blue litmus paper red, and a *base*, if soluble in water, as one which turns red litmus paper blue. (In the Appendix more fundamental definitions are given.) A *salt* is a substance produced when an acid and a base are mixed. The usual result of allowing an acid to react with a base is the production of a salt and water; thus if hydrochloric acid and sodium hydroxide (a base) are mixed, sodium chloride and water are formed. Sodium chloride is such a common *salt* that it is called *common salt*, but it is only one of very many salts, other examples of which are ammonium chloride (sal ammoniac) and sodium sulphate.

40. Chemical Change

A chemical reaction always involves two types of change—namely, a change in the form of the matter, and a change in the energy of the system. Although in some chemical changes there is an absorption of energy, in most cases energy is liberated, usually in the form of heat. Sometimes, however, energy is liberated in other forms, as for example during an explosive reaction when heat, sound, light and kinetic energy may be produced simultaneously. In this chapter we are particularly concerned with the case where the change in energy is from the chemical form to the electrical form, or vice-versa.

When, for example, a piece of common zinc is placed in a dilute sulphuric acid solution, heat is generated and there is a simultaneous change in the chemical form of the materials, the zinc slowly disappearing, the acid being gradually changed to zinc sulphate and hydrogen gas being evolved. This reaction can, however, be made to occur in such a manner that some of the chemical energy is converted into electrical energy instead of heat, as described in the next section.

41. The Simple Voltaic Cell

If a plate of really pure zinc be placed in dilute sulphuric acid the reaction just described does not occur. Similarly, if a plate of commercial zinc be amalgamated and placed in acid, there is no reaction. (To amalgamate a zinc plate, clean its surface thoroughly and then rub mercury into the surface with a damp cloth.) If, however, a copper plate be also placed in the acid, and this copper plate connected externally by means of a conductor to the pure or amalgamated zinc plate, the reaction occurs as described in the last section.

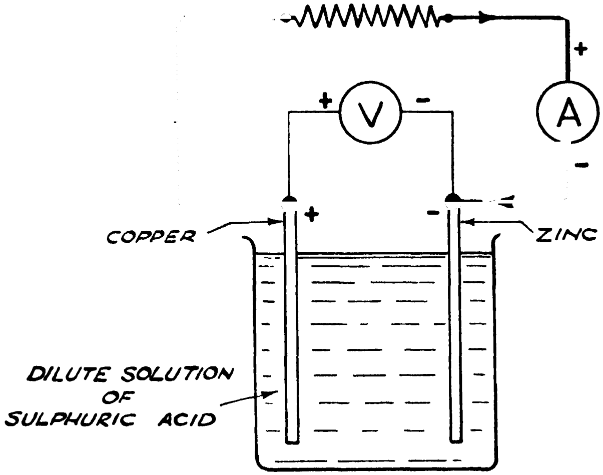


FIG. 85.—A SIMPLE VOLTAIC CELL.

It should be noted, however, that in this case the hydrogen is liberated on the copper plate. If an ammeter be inserted into the external circuit it will be found that a current flows, and a voltmeter connected across the two plates demonstrates the existence of a p.d. between them, the copper being positive and the zinc negative (see Fig. 85). Such an arrangement, whereby two plates of different metals placed in a liquid electrolyte give rise to an e.m.f., is known as a *simple galvanic, or voltaic cell,* since Galvani and Volta were the first to produce an e.m.f. by this method.

A simple voltaic cell need not necessarily consist of copper and zinc plates immersed in dilute sulphuric acid. The plates may consist of any two dissimilar metals or of carbon and some metal; carbon is a very suitable material for the positive plate, being inert in most liquids which may be used as the electrolyte. Carbon positive and zinc negative plates are the most satisfactory, since they give a comparatively large e.m.f. For the electrolyte most acids and certain strong salt solutions give satisfactory results. Taking into account the e.m.f., internal resistance and the " life " of the negative plate, perhaps the most suitable electrolyte is a strong solution of sal-ammoniac (ammonium chloride).

42. Local Action and Polarisation

It seems very probable that the reaction of a metal with an acid is an electro-chemical process, and that for a reaction to occur some other metal or carbon must be present. Common zinc contains an appreciable amount of metallic impurities, iron, for example, being usually present; so, when immersed in acid, the zinc forms a number of voltaic cells with the microscopic particles of iron or other metallic impurity. Each of these particles acts as the positive pole of a cell which is short-circuited due to the contact between the impurity and the zinc. Hydrogen is evolved at these particles of iron, and local currents flow from them to the zinc. The electrical energy of these local currents is converted into, and dissipated, as heat, so it appears, from superficial observation, that heat energy is produced direct from chemical energy. This phenomenon is known as *local action,* and must be prevented in a cell designed for commercial use, otherwise even when a cell is on open-circuit the zinc will continue to react with the acid. Thus all zinc rods or plates used in cells are amalgamated, the zinc on the surface dissolving in the mercury and the impurities being covered up.

A simple voltaic cell is not used in practice, since it suffers from *polarisation.* When the plates are connected through an external circuit, the current that flows fairly soon dies away. There are two reasons for this. Firstly, the positive plate becoming partially covered with a thin film of hydrogen, the internal resistance of the cell increases, since a gas is a poor conductor. Secondly, the e.m.f. of the cell tends to decrease. The primary reason for this is that the hydrogen film usurps the function of the positive plate, the p.d. between the hydrogen and the electrolyte being less than that between the non-polarised positive plate and the electrolyte.

To reduce polarisation the positive plate must be placed in some substance which will either react with the hydrogen and remove it or prevent its formation. Such substances are known as *depolarisers* or *depolarising agents.* Practical forms of the voltaic cell differ from the simple cell in so far as they contain a depolarising agent.

43. Primary Cells

A **primary cell** supplies a current as a result of a primary chemical action. As the cell supplies electrical energy certain chemical changes take place, and ultimately the active materials of the cell become used up. In the case of a **secondary cell,** or **accumulator,** the materials used up in supplying a current are restored by passing a charging current through the cell in a direction opposite to that in which the current flows during discharge (see Chapter VI). With a primary cell it is usually impossible to restore the active materials by this method; in certain cases where it is possible it is not an economic proposition, since it is cheaper to purchase fresh materials. Thus the distinction between primary and secondary cells is that whereas the latter possess the characteristic of reversibility, the former are usually irreversible and, in practice, primary cells are never recharged by passing a reverse current through them.

Accumulators have displaced primary cells in many cases, but the latter are still widely used in telephony, in torch batteries, for operating electric bells, for the H.T. supply to radio sets, and even for the L.T. supply in the case of portable sets, and, in laboratories, for general intermittent use; also certain special primary cells, owing to their constant e.m.f's., are used for standardisation purposes.

Practical forms of primary cells always contain two fluids, one or both of which may exist in the form of a moist paste. The one known as the *excitant*, in contact with the negative zinc plate, is the one in which the fundamental chemical change takes place—the change that liberates electrical energy. The other is the depolariser. The excitant and the depolariser are frequently prevented from mixing too freely with each other by means of a porous pot or canvas sack. Many primary cells employing various plates, excitants and depolarisers have been devised in the past, but, apart from standard cells, the only primary cell used to any extent nowadays is the Leclanché cell.

44. The Leclanché Cell—Wet and Dry Types

Fig. 86 illustrates typical forms of the wet and dry types of Leclanché cell. In each case the positive and negative poles are of carbon and zinc respectively. In the dry cell a moist paste replaces the liquid of the wet cell. It should be appreciated that, if really dry, a cell would supply no current, since electrolytic action demands the presence of moisture. The excitant in the Leclanché cell is sal-ammoniac, existing in the wet cell as a solution and in the dry cell mixed with plaster of Paris and flour to form a thin white paste, zinc chloride being added to assist in retaining the moisture. In each case the depolarising agent is manganese dioxide mixed with crushed carbon in order to provide a fairly good conducting path. In the wet cell this mixture is contained in a porous pot which keeps the manganese dioxide packed round the carbon plate and prevents the liquid sal-ammoniac from mixing too freely with the manganese dioxide. The carbon plate, depolarising agent and porous pot are made up into a single unit. In the dry cell the zinc negative is made in the form of a canister, which acts also as the container. This is given a lining of the white paste containing the excitant, and the remaining space is filled with the depolarising agent with the positive carbon rod in the centre. Owing to the absence of a porous pot and the larger area of the zinc negative, a dry cell usually possesses an internal resistance considerably less than that of a wet cell of similar size, and has the further advantage of portability due to lightness and the absence of liquids. Wet Leclanché cells are, however, now available in which the porous pot is dispensed with and in which the zinc is in the form of a cylindrical sheet surrounding the central unit which may consist either of a canvas bag containing the carbon rod and depolariser or of the depolarising mixture compressed round the carbon rod to form an agglomerate block. Owing to the small amount of moisture present, the dry cell tends to deteriorate rapidly in store. Because of this, and owing to the fact that dry cells are usually smaller than wet cells, the dry cell possesses a rather short life as compared with the wet type. In each case the e.m.f. is approximately 1·5 volts.

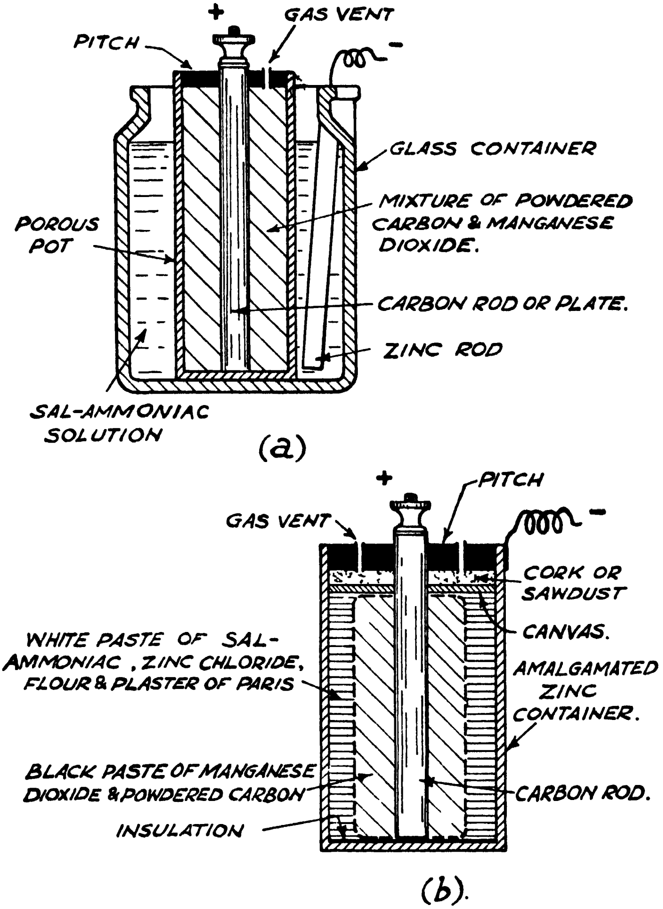

+
GAS VENT
PITCH
-
GLASS CONTAINER
MIXTURE OF POWDERED CARBON & MANGANESE DIOXIDE.
POROUS POT
CARBON ROD OR PLATE.
ZINC ROD
SAL-AMMONIAC SOLUTION
(a)
+
PITCH
GAS VENT
-
CORK OR SAWDUST
CANVAS.
WHITE PASTE OF SAL-AMMONIAC, ZINC CHLORIDE, FLOUR & PLASTER OF PARIS
AMALGAMATED ZINC CONTAINER.
BLACK PASTE OF MANGANESE DIOXIDE & POWDERED CARBON
CARBON ROD.
INSULATION
(b).

FIG. 86.—TYPES OF LECLANCHÉ CELL.
(*a*) wet type, (*b*) dry type

The chemical action in all types of Leclanché cell consists in the zinc of the negative plate combining with the sal-ammoniac to form zinc chloride, ammonia and hydrogen. The ammonia dissolves in the excitant, and the hydrogen collects on the carbon plate or rod, where it is removed by the manganese dioxide attacking it to produce another oxide of manganese and water. This latter reaction takes place rather slowly, chiefly owing to the fact that manganese dioxide is a solid; were it a liquid, closer contact would be made with the hydrogen. If an appreciable current is taken from a Leclanché cell, then, owing to this comparatively slow action, hydrogen is formed more rapidly than it is removed by the manganese dioxide. In other words, polarisation is not completely avoided. If, however, the cell is allowed to rest, it recuperates.

45. The Weston Standard Cadmium Cell

Since the e.m.f. of a cell depends on the materials used, it is possible, by selecting materials which do not appreciably change over a period of many years, to obtain a cell which possesses a remarkably constant e.m.f. Providing the e.m.f. of such a cell varies very little with change in temperature, the cell will be highly satisfactory as a standard of e.m.f. or p.d. The standard cell almost exclusively employed nowadays is the Weston cell. At 20° C. the e.m.f. of a Weston cell is 1·0187 volts, the e.m.f. having a maximum value of 1·0191 volts at 4° C. and falling to 1·0183 volts at about 28° C. A detailed account of the variation of the e.m.f. with temperature change is given in " Principles of Electric and Magnetic Measurements " by Vigoureux and Webb, but for most purposes it

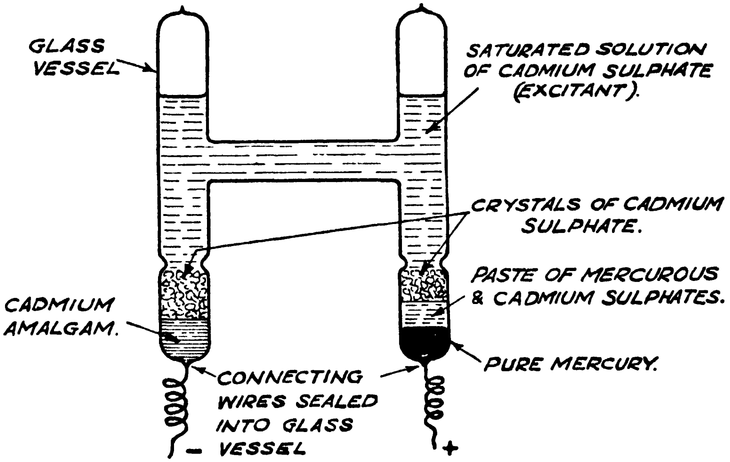

GLASS VESSEL
SATURATED SOLUTION OF CADMIUM SULPHATE (EXCITANT).
CRYSTALS OF CADMIUM SULPHATE.
PASTE OF MERCUROUS & CADMIUM SULPHATES.
CADMIUM AMALGAM.
PURE MERCURY.
CONNECTING WIRES SEALED INTO GLASS VESSEL
−
+

FIG. 87.—A WESTON STANDARD CADMIUM CELL.

is sufficient to regard the e.m.f. as 1·019 volts, since this figure is correct to 1 part in 1000 at normal room temperatures. It is very essential, if the e.m.f. of the cell is to remain constant, that the currents taken during standardisation measurements should not exceed a very small fraction of a micro-ampere.

The constituents of a Weston cell are arranged in an H-shaped glass vessel as shown in Fig. 87. When a current is taken from the cell, cadmium goes into solution from the negative electrode of cadmium amalgam (consisting of pure mercury and cadmium in the ratio of 8 parts by weight of mercury to 1 part of cadmium), and mercury is produced at the positive pole which consists of a pool of mercury.

46. The Potentiometer

The potentiometer is a device for comparing p.d's. or e.m.f's., one of which may be a standard. Fig. 88 illustrates a simple potentiometer circuit as used for comparing the e.m.f's., E_1 and E_2 volts of two cells. CD represents a resistance wire (e.g., of constantan, platinoid or nickel silver) of uniform cross-section, and of such a length and gauge as to possess a resistance of at least several ohms, A an accumulator having an e.m.f. greater than that of either of the cells under test and G a sensitive galvanometer. It is essential that the positive terminal of the accumulator A and the positive terminals of each of the two cells under test should be connected to the same end of the potentiometer wire CD. The galvanometer G is connected between the negative terminal of the cell being tested and a sliding contact P, which is able to connect the galvanometer to any point on the potentiometer wire CD.

In using the potentiometer to compare the e.m.f's. of two cells, one of the cells is put in circuit (as shown by the full line connection in Fig. 88), and a position P_1 for the sliding contact P is found on the wire CD such that the galvanometer indicates zero current through the branch $CFGHP_1$. The distance CP_1 should then be recorded, and the process repeated with the second cell in circuit (dotted line connection—Fig. 88), a position P_2 being found which again gives a balance.

Let the distances CP_1 and CP_2 be denoted by d_1 cm. and d_2 cm. respectively. Assuming that the terminal p.d. of the accumulator A remains constant, then the voltage drop per cm. length of the potentiometer wire will also remain constant; let us denote it by v volts per cm. Considering firstly the conditions when a balance is obtained with the sliding contact P at P_1, the e.m.f. of the cell (E_1 volts) is balanced by the voltage drop from C to P_1 (d_1v volts), thus:—

$$E_1 = d_1v$$

Similarly, when the second cell is in the circuit and a balance is obtained with P at P_2:—

$$E_2 = d_2v$$

Hence,
$$\frac{E_1}{E_2} = \frac{d_1}{d_2}$$

If one of the cells is a Weston standard cell, the potentiometer wire may be calibrated. That is, since the e.m.f. of this cell is known with accuracy, it is possible to determine precisely the voltage drop per cm. length of the potentiometer wire. In the case of the direct-reading potentiometer described in Chapter V of Vol. II the equivalent of a very long potentiometer wire is obtained, and it is possible to adjust the p.d. applied to this wire so that the voltage drop per scale division has some convenient size such as 1 millivolt. It is then possible to read off directly from a length such as CP_1 (Fig. 88), the e.m.f. or p.d. being measured.

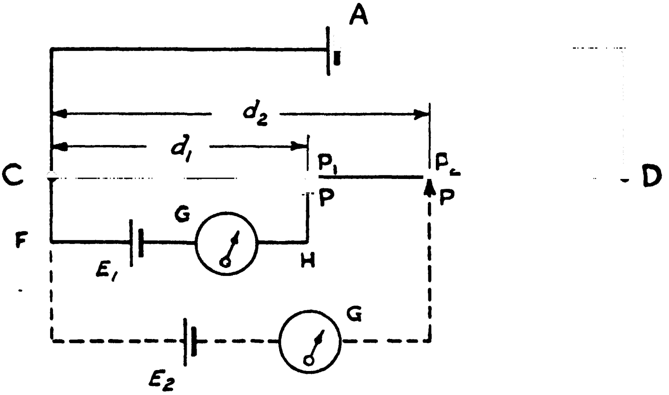


FIG. 88.—THE SIMPLE POTENTIOMETER.

When a Weston standard cell is used for potentiometer work it is necessary to modify the circuit of Fig. 88 so as to ensure that too large a current is not taken from the cell. This may be effected by including a high resistance in series with the standard cell, thus limiting the current in this branch of the circuit. When the sliding contact P is very near the point of balance, this resistance may be short-circuited. Such a precaution is advisable even when the standard cell is not in use, as it prevents damage to the sensitive galvanometer.

Although a potentiometer is primarily a device for measuring e.m.f.'s and p.d.'s, it may be used for the accurate measurement of current and resistance, providing a standard resistance is available. The use of the potentiometer for various purposes is explained in Chapter IX of Vol. II, in which chapter electrical measuring instruments are dealt with in some detail.

Experiment 13. *To compare the e.m.f's. of a Leclanché cell and a nickel-alkaline accumulator.*

Connect up the circuit shown in Fig. 88, A being a lead-acid accumulator. Obtain a balance first with the alkaline accumulator in the galvanometer circuit and then with the Leclanché cell in the circuit. Record the lengths of CP_1 and CP_2 and determine their ratio. This is the ratio of the e.m.f.'s of the two cells.

47. Electrolysis

Electrolysis is the term given to the phenomena which occur when a d.c. source of potential difference is applied to a liquid electrolyte. The electrolyte takes current from the supply, certain chemical changes occurring in the process. The conductors leading the current into and out of the electrolyte are known as *electrodes*, that connected to the positive terminal of the supply being known as the *anode* and the other as the *cathode.* The whole arrangement is termed an *electrolytic cell.* When a current is passed through an electrolytic cell there is frequently a net change in the composition of the electrolyte; *in all cases* some new substance is produced at one or both of the electrodes. *In a voltaic cell* chemical energy is converted into electrical energy; in an *electrolytic cell* the reverse usually occurs, electrical energy supplied from some external source being converted into chemical energy.

In order to convey to the reader some idea of what is likely to happen in any particular case, it is desirable at this stage that certain specific examples of electrolysis should be discussed.

We shall therefore consider what happens when a current is passed through two cells as illustrated in Fig. 89 (a) and (b). The only difference between the two cells is that in one case the anode is a copper plate and in the other case a carbon plate. In each case the cathode is of copper and the electrolyte a strong copper sulphate solution. In each case on closing the switch copper is deposited at the cathode in a very pure form, but here the similarity in the behaviour of the two cells ends. *In the case of the cell with the copper anode* copper goes into solution from the anode to replace the copper lost by the electrolyte to the cathode. So the electrolyte undergoes no net change. *In the case of the cell with the carbon anode* no reaction occurs between anode and electrolyte. The copper sulphate reacts with the water, oxygen being liberated at the carbon anode and the electrolyte becoming acidified. Further, since this cell consists of two dissimilar electrodes, it tends to act as a voltaic cell; an e.m.f. therefore exists which opposes the externally applied e.m.f. It should be noted that the formation of oxygen at the anode considerably affects the actual magnitude of this *back e.m.f.* Unless the externally applied e.m.f. has a value in excess of this back e.m.f. no current will flow, and so no chemical change will occur. Where the electrodes are of the same metallic material and the electrolyte is a salt of the same metal, no gas is formed, and there is no appreciable back e.m.f. if the electrolyte is stirred. Since with two copper electrodes there is no net chemical change, no electrical energy is converted into chemical energy, but in the case of the second cell electrical energy is undoubtedly converted into chemical energy, the net chemical

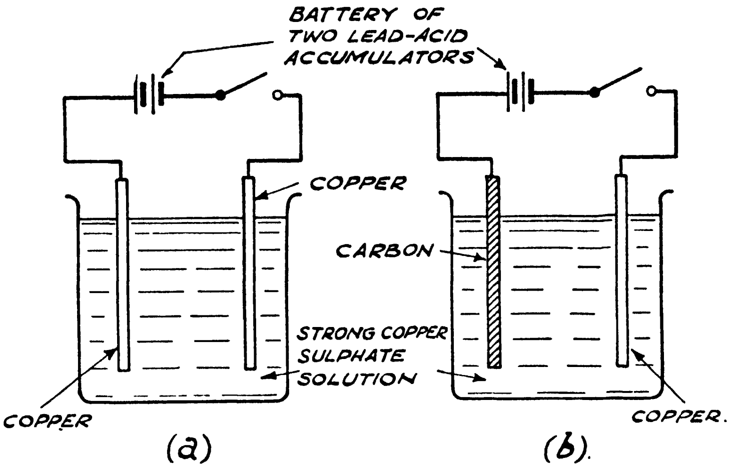

BATTERY OF TWO LEAD-ACID ACCUMULATORS
COPPER
CARBON
STRONG COPPER SULPHATE SOLUTION
COPPER
COPPER.
(a)
(b).

FIG. 89.—THE ELECTROLYSIS OF COPPER SULPHATE SOLUTION.

change being the production of copper, oxygen and sulphuric acid from copper sulphate and water.

Consider now another example of electrolysis—the electrolysis of water acidified with, say, sulphuric acid, using platinum electrodes, these electrodes being inert in a dilute sulphuric acid solution. The sulphuric acid, while taking part in the electrolysis, suffers no net change itself. The net effect of passing a current through such an electrolytic cell is to decompose the water into its constituents, hydrogen and oxygen, hydrogen appearing at the cathode and oxygen at the anode. Owing to the liberation of hydrogen and oxygen there is a *back e.m.f.* of between 1 and 2 volts, the exact value depending on the smoothness of the surfaces of the electrodes. Unless the e.m.f. applied exceeds this back e.m.f. no appreciable current can pass through the cell. *The product of this back e.m.f. and the current gives the power used in the production of hydrogen and oxygen from the water.*

If the e.m.f. of the external battery be E volts, the back e.m.f. of the cell E_b volts, the total resistance of the circuit R ohms and the current I amperes, then:—

$$I = \frac{E - E_b}{R}$$

i.e.,
$$IR = E - E_b$$

or,
$$E = IR + E_b$$

Multiplying throughout by I amp. this equation becomes:—

$$EI = I^2R + E_bI$$

The term EI watts represents the total power supplied by the battery, and I^2R watts the power which is used in overcoming the resistance of the circuit, this latter power being converted into heat. E_bI watts represents the power used in overcoming the back e.m.f. of the cell, this power being the cause of chemical and physical changes in the cell.

To generalise, with an inert anode a gas is usually liberated at the anode, there is a net chemical change in the constitution of the electrolyte, a back e.m.f. is obtained and electrical energy is converted to chemical energy. If the electrolyte is a salt of the metal forming the anode, the metal goes into solution from the anode and is deposited at the cathode, no appreciable back e.m.f. being obtained. If the electrolyte is an acid, hydrogen is liberated at the cathode. The number of possible electrolytic cells is obviously extremely great, and only a very wide knowledge of chemistry could enable one to forecast what would be the result of passing a current through any one of these possible cells.

48. Faraday's Laws of Electrolysis

In 1834 Michael Faraday, by careful experiment, discovered what factors determine the exact amounts of the elements deposited at the cathode or liberated as gases at either electrode during electrolysis. The results of his work are embodied in his two laws of electrolysis. Faraday's second law refers to the *chemical equivalent* (or *equivalent weight*) of an element. The *chemical equivalent* of an element may be regarded as a number which represents how many parts by weight of the given element can enter into combination with 8 parts by weight of oxygen or 1 part by weight of hydrogen (a more exact definition of chemical equivalent is given in the Appendix at the end of this chapter—Section 54).

Faraday's two laws may be stated as follows:—

(1) **The mass of an element deposited or liberated in an electrolytic cell is proportional to the current and to the time for which it passes.**

(2) **The masses of different elements deposited or liberated by the same quantity of electricity during electrolytic action are proportional to the various chemical equivalents of the elements.**

If a mass, m gm., of an element be liberated in a time t sec. by a current I amp., then from Faraday's first law:—

$$m \propto It$$

i.e., $m = zIt$, where z is a constant.

From the second law it follows that this constant will be proportional to the chemical equivalent of the element concerned. For this reason z is known as the **electro-chemical equivalent** (E.C.E.) of the element. Let Q coulombs represent the quantity of electricity involved during the liberation of a mass m gm. of the element, then since

$$Q = It$$

$$m = zQ$$

or,

$$z = m/Q$$

Thus z is equal to the mass of the element liberated divided by the quantity of electricity necessary to liberate it. In other words, the *E.C.E. of an element may be defined as the mass of the element liberated by unit quantity of electricity.* E.C.E.'s are usually expressed in *gm. per coulomb* or in *lb. per ampere-hour.* Table VII (at the end of Vol. I) gives a list of the E.C.E.'s of important elements.

Experiment 14. *To measure the E.C.E. of copper.*

Connect up the apparatus shown in Fig. 90, a battery B of 2 lead-acid accumulators being connected in series with a switch, S, a rheostat, Rh, an ammeter and a copper *voltameter*, consisting of two copper plates X and Y immersed in a strong copper sulphate solution. (A *voltameter* is an electrolytic cell used for the measurement of E.C.E., or for the measurement of current, assuming a value for the E.C.E. of some element deposited or liberated in the cell.) Adjust the rheostat to give a suitable current, i.e., a value such that the *current density* is less than ⅛th amp. per sq. inch of cathode immersed in electrolyte. Remove the cathode, Y, wash and dry it and clean it with emery cloth. Weigh the cathode carefully and then replace it in the voltameter. Close the switch, S, and pass current through the cell for at least half-an-hour, carefully recording the time and ensuring that the current remains at a constant value throughout. Remove the cathode, wash it, dry it in the air and reweigh it. The mass of copper deposited on the cathode may be obtained from the difference of the two weighings. From the mass of copper deposited, the current strength and the time for which it was passed through the voltameter, calculate the E.C.E. of copper.

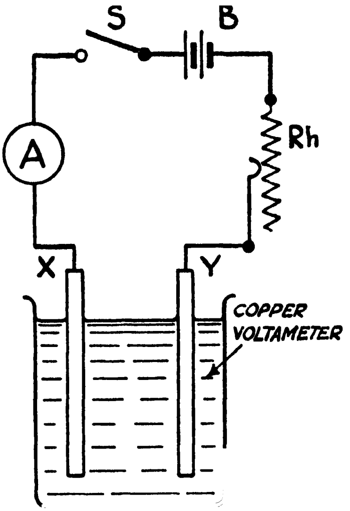


FIG. 90.—CIRCUIT FOR DETERMINING THE E.C.E. OF COPPER.

49. Commercial Applications of Electrolysis

The chief practical applications of electrolysis are the extraction of metals from their ores, the refining of metals, the manufacture of various chemicals and electro-plating, although there are other miscellaneous applications.

Electrolytic Extraction and Refining of Metals.

Electrolytic processes of extraction and purification produce much purer products than most metallurgical processes. In the case of some metals, e.g., aluminium, the electrolytic method is the only economical process available. Aluminium, copper, magnesium, sodium and zinc are examples of metals that can be extracted by electrolytic means, and copper, nickel, gold, silver, iron and lead are examples of those that can be thus refined. The total world production of electrolytically refined copper was of the order of a million tons annually prior to the recent world war, all copper used for electrical purposes being refined by this means. Many thousands of millions of kilowatt-hours of electrical energy are used annually in the electrolytic production of metals.

In the extraction of aluminium the electrolyte consists of a mixture of fused cryolite and alumina, the former being a double fluoride (of aluminium and sodium) and the latter aluminium oxide. To keep the mixture fused, a temperature of something like 1000° C. must be maintained. This high temperature is achieved by the heat developed in the cell due to its resistance; in this connection it must be realised that currents of many thousands of amperes are employed at a p.d. of several volts. Thus this cell is really a furnace which may take a power of several hundred kW. The electrodes are of carbon, the cell itself being lined with carbon which acts as the cathode, heavy carbon rods forming the anode. The liquid aluminium collects on the carbon floor of the cell and is tapped off from time to time.

Magnesium and sodium are also extracted by electrolysis of fused compounds, but the electrolytic extraction of copper and zinc consists of electrolysis of solutions obtained by treating the ores with sulphuric acid. Full details may be obtained from chemistry textbooks. As regards the electrolytic refining of metals, the anode is always made of the " commercially pure " metal, the electrolyte is a solution of a suitable salt of the metal and the pure metal is deposited at the cathode. The impurities go into solution or sink to the bottom of the cell.

Electrolytic Manufacture of Chemicals

A very important chemical compound, caustic soda, can be manufactured from a solution of common salt by electrolysis. The sodium deposited at the cathode is allowed to react with steam to produce caustic soda and hydrogen, the former being run off, the hydrogen, and also the chlorine liberated at the anode, being collected as by-products. Other valuable compounds which can be manufactured by electrolytic processes are potassium permanganate, sodium chlorate, perchlorate and hypochlorite and, as we have seen earlier, hydrogen and oxygen from water.

Electro-plating

In all cases the article to be plated is well cleaned and, if non-conducting, given a coating of powdered graphite. The article is then made the cathode in a cell usually containing for the electrolyte a salt of the metal to be deposited; in the case of chromium-plating the electrolyte is chromic acid. The anode is always a block of the pure metal. In many cases the temperature of the electrolyte must be maintained between very narrow limits, and heat has frequently to be supplied. Metals for which electro-deposition is possible are copper, cadmium, chromium, rhodium, gold, iron, lead, nickel, silver, tin and zinc, and even alloys such as brass and bronze are deposited electrolytically. With some metals, e.g., chromium, a preliminary coating of nickel is desirable.

Acids are frequently added to the electrolyte to reduce its resistance. Potassium cyanide is also added occasionally, since it assists in the production of a deposit of uniform thickness. Successful electro-plating is not easy. There are many difficulties, one of the chief being that if the article to be plated is of irregular shape, the regions most removed from the anode will tend to receive less deposition than other parts.

The quantity of a metal deposited is usually rather less than that which would be expected from Faraday's laws, due to impurities which cause secondary reactions. The ratio of the quantity deposited to the theoretical quantity (known as the *current efficiency*) is usually between 90% and 100%, although in chromium plating less than 15% of the current is used in depositing chromium.

Miscellaneous Applications

Electrolysis is used in the printing industry in reproducing type and in engraving blocks for reproducing pictures. In the case of electro-type a wax mould is made from the type set up and its surface dusted with graphite. Copper or chromium is then deposited upon it, the wax mould then removed from the thin metallic shell, into which molten metal is poured to strengthen it. This metallic plate, which is an exact copy of the original type, is then used for the actual printing. A similar process is employed in the manufacture of gramophone records. The engraving of blocks for printing purposes may also be carried out electrolytically. A copper or chromium block, on which a photograph of the picture to be reproduced is printed, is made the anode of an electrolytic cell, appropriate portions of the plate having been covered over with protective paint. The remainder is etched as the metal from those portions goes into solution. Further applications of electrolysis are the production of thin metallic sheet or foil and the building up of worn machine parts.

APPENDIX

50. Atoms, Ions and Electrolytes

In Chapter II (Section 6) it was pointed out that atoms of metals very easily lose electrons. It should now be noted that atoms of many non-metallic elements very readily tend to attract and gain electrons when they are available. An atom may be likened to an onion in so far as the orbital electrons exist in successive layers. Each layer is incapable of holding more than a certain number of electrons, this number varying with the different layers. When a layer contains its maximum number of electrons the group is known as a *closed* group. Most of the layers in a multi-layer atom consist of closed groups of electrons, but there are usually one or two layers which consist of incomplete groups. It is these incomplete groups which are responsible for chemical changes. They tend to gain electrons in order to make the number up to the maximum, or, alternatively, if there are only one or two electrons in an incomplete layer the atom may readily give them up. Certain gases such as helium, neon and argon are known as inert gases, since they will not combine with any other substance, the reason being that the atoms of which these gases are composed contain only closed groups of electrons. The hydrogen atom, on the other hand, contains only 1 electron. Since the maximum number of electrons the innermost layer of an atom can hold is 2, the normal hydrogen atom readily loses its electron, or, alternatively, gains an extra one. Hydrogen therefore readily combines with other substances.

Let us now consider sodium chloride as an example of a compound which is an electrolyte. Although a molecule of sodium chloride is usually considered to consist of one atom of sodium and one of chlorine, the matter is not quite so simple. A normal sodium atom (Na) possesses 11 orbital electrons, 10 of which form closed groups. The sodium atom therefore readily loses its eleventh electron, the atom becoming a *positively-charged sodium ion*, Na^+. This electron is readily accepted by a chlorine atom (Cl), since the chlorine atom contains 17 electrons, 10 of which form closed groups, the remaining 7 electrons requiring another one to complete a group. When the chlorine atom gains an extra electron it becomes a *negatively-charged chlorine ion*, Cl^-. Now, when an electron is transferred from a sodium atom to a chlorine atom resulting in a sodium ion and a chlorine ion these two ions attract each other because they possess charges of opposite sign. A minute crystal of common salt consists of millions of these sodium and chlorine ions held together in regular formation, as shown in Fig. 91, by the forces of attraction due to the positive and negative charges. (See Chapter IX for an account of the phenomenon of electrostatic attraction.)

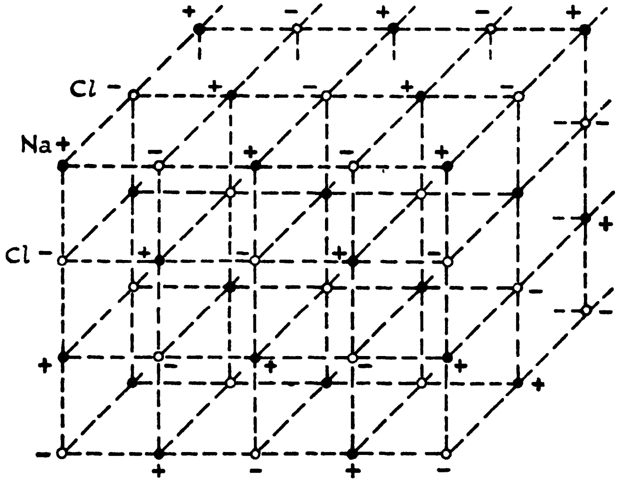


FIG. 91.—THE STRUCTURE OF A SODIUM CHLORIDE CRYSTAL.

In general, an ion can be formed from either a single atom or a group of atoms of various kinds by the addition or subtraction of one or more electrons. An ion is said to be *mono-*, *bi-* or *tri-valent*, according as to whether it is formed by the addition or subtraction of 1, 2 or 3 electrons. A negative ion is formed by the *addition* of electrons; a positive ion by the *subtraction* of electrons. Thus, an aluminium atom possesses 13 electrons, 10 of which form closed groups, the remaining 3 belonging to an incomplete group. It readily loses these 3 electrons, becoming a trivalent positive ion, Al^{+++}. An example of a bivalent negative ion composed of a group of atoms is the sulphate ion, SO_4^{--}. It is composed of a sulphur atom and 4 atoms of oxygen together with 2 electrons. Sulphur and oxygen atoms each possess, in addition to various closed groups of electrons, an incomplete group containing 6 electrons. Fig. 92 shows how, in the sulphate ion, the sulphur atom shares its 6 outer electrons and the 2 extra ones with the 4 oxygen atoms, so that each oxygen atom and also the sulphur atom can be regarded as possessing an outer closed group of 8 electrons. In the case of the positive monovalent ammonium ion, NH_4^+, we have a nitrogen atom, which normally possesses 5 electrons, and 4 hydrogen atoms normally possessing 1 electron each. So between them these 5 atoms normally

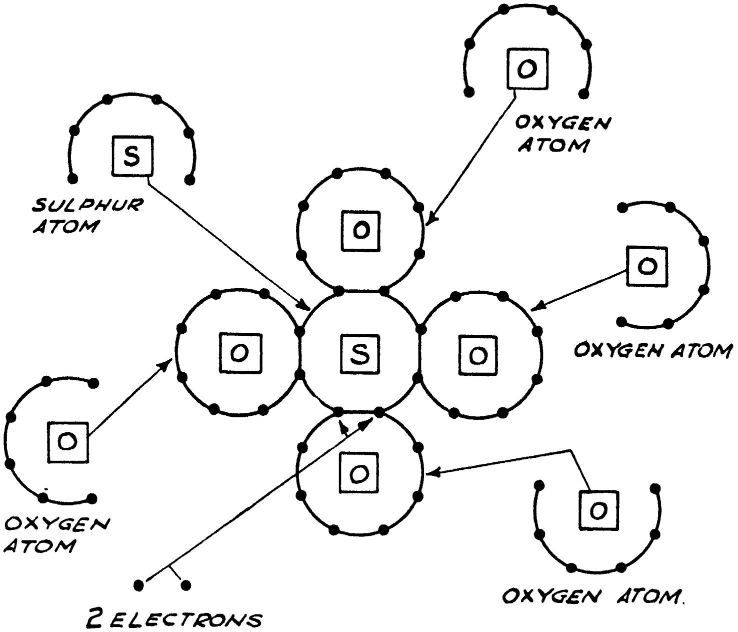


FIG. 92.—A NEGATIVE BIVALENT SULPHATE ION (SO_4^{--}).

possess 9 electrons; one of these electrons is given up, so that the ion possesses a single positive charge, and the remaining 8 electrons are shared as shown in Fig. 93. Thus the nitrogen atom in the ammonium ion can be regarded as possessing a closed group of 8 electrons and each hydrogen atom a closed group of 2 electrons. Thus ammonium chloride is a stable compound because a nitrogen atom and 4 hydrogen atoms possess between them 1 electron too many for stability and because a chlorine atom requires an extra electron for stability. So the spare electron is transferred to the chlorine atom, to the mutual satisfaction of both parties. Ammonium chloride therefore consists of singly-charged positive ammonium ions and singly-charged negative chlorine ions firmly held together by electrostatic attraction.

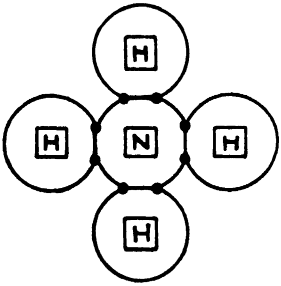
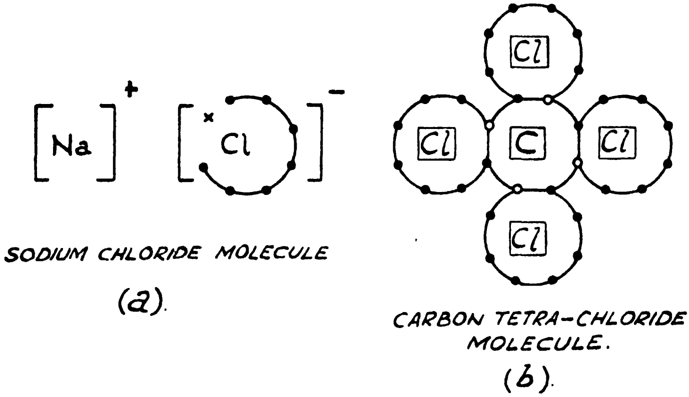


FIG. 93.—A POSITIVE MONOVALENT AMMONIUM ION (NH_4^+).

The student should not be misled into believing that all compounds consist of ions. *Compounds may be divided into electrolytes and non-electrolytes, only the former consisting of ions.* Fig. 94 illustrates the difference between the two types of compound. Fig. 94a represents a sodium chloride molecule which consists of 2 ions formed by the *transfer* of 1 electron, represented in the diagram by a small cross, the other electrons being represented by dots.



SODIUM CHLORIDE MOLECULE

(a).

CARBON TETRA-CHLORIDE MOLECULE.

(b).

FIG. 94.—MOLECULES OF ELECTROLYTIC AND NON-ELECTROLYTIC COMPOUNDS.

Fig. 94b represents a carbon tetrachloride molecule; carbon tetrachloride is a substance renowned for its cleaning properties, since it is an excellent solvent of grease. In this case no electrons are *transferred.* Instead, electrons are *shared.* Ions exist only when electrons have been transferred from 1 atom or group of atoms to another atom or group. In Fig. 94b the 4 electrons originally belonging to the outer incomplete layer of the carbon atom are represented by small circles, and the electrons originally belonging to the chlorine atoms by dots. It will be noticed that owing to this sharing process the carbon atom and each chlorine atom may be regarded as possessing 8 electrons in their outer layers. It might be noted that there are thousands of carbon compounds which are non-electrolytes.

As was stated in Section 39, electrolytes consist of either acids, salts or bases. An *acid* is a substance which yields hydrogen ions in the presence of water. A *base which dissolves in water* is known as an **alkali,** this being defined as a substance which produces *hydroxyl ions* in solution, the hydroxyl ion, OH^-, being composed of an oxygen and a hydrogen atom plus an electron as illustrated in Fig. 95. Various acids differ from one another merely as regards their negative ions; they all possess hydrogen ions. One alkali differs from another only as regards the positive ion; they all possess hydroxyl ions. Thus, a solution of hydrochloric acid in water contains positive hydrogen ions, negative chlorine ions, water molecules and a small proportion of whole hydrogen chloride molecules. A solution of sodium hydroxide in water consists of positive sodium ions, negative hydroxyl ions, water molecules and a small proportion of whole sodium hydroxide molecules. The reason that, at any instant, there exists a number of whole molecules is that the water molecules and the positive and negative ions move about in the liquid, and occasionally a positive and negative ion closely approach each other and electrostatic attraction results in their cohesion. The molecule thus formed soon splits up again as a result of collisions with other ions and molecules, but at any given instant there does exist a number of whole molecules. This number will be comparatively small if the solution is very dilute, since in that case ions of opposite polarity will approach each other less frequently than when there is less water present.

We have seen that a crystal of sodium chloride consists of sodium and chlorine ions in a lattice formation as depicted in Fig. 91.

When dissolved in water, sodium chloride consists of positive sodium ions, negative chlorine ions, a small proportion of whole sodium chloride molecules and water molecules, the various ions and molecules moving about at random. Solution of the crystal in water causes a separation of the sodium and chlorine ions because the force of attraction between the ions is weakened due to the high *permittivity* of water (see Chapter IX, Section 105).

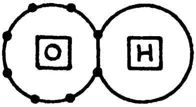


FIG. 95.—A NEGATIVE MONOVALENT HYDROXYL ION (OH^-).

51. The Theory of the Simple Voltaic Cell

It has been stated previously that a simple voltaic cell is a system in which chemical energy is converted into electrical energy. Consider the voltaic cell discussed in Section 41, in which zinc and copper plates are immersed in dilute sulphuric acid. When a metal is placed in an electrolyte there is a tendency for positive metallic ions to go into solution and, simultaneously, there is a tendency for positive ions in the solution to be deposited on the metal. In the case of zinc and copper plates in dilute sulphuric acid there is a greater tendency for zinc ions to go into solution than for hydrogen ions to come out of solution on to the zinc, but at the copper the tendency for hydrogen ions to come out of solution on to the copper is greater than the tendency for copper ions to go into solution. The result is that positive zinc ions go into solution from the zinc plate leaving negative electrons behind, whereas positive hydrogen ions go to the copper plate leaving a surplus of negative sulphate ions in the solution. Thus at each plate there is established a double layer of electric charge, one layer being positive and the other negative (see Fig. 96), this double layer causing a p.d. between each metal plate and the solution, the copper being at a higher potential than the solution and the zinc at a lower potential than the solution. It is found that there exists an e.m.f. of 1·08 volts between the

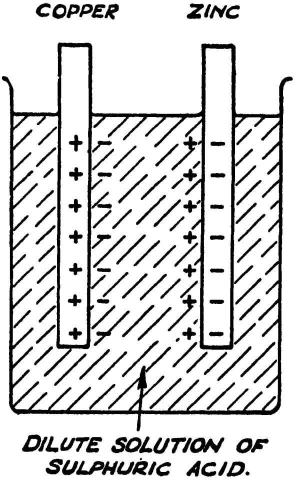


FIG. 96.—THE PRODUCTION OF THE E.M.F. IN A SIMPLE VOLTAIC CELL.

copper and zinc plates, this figure of 1·08 volts being equal to the sum of the two *contact p.d's.* at the zinc–acid and copper–acid boundaries. The double layers of charge at each plate are established almost instantaneously on dipping the plates into the liquid, as can be seen from the fact that a voltmeter connected across the plates will immediately record an e.m.f. of 1·08 volts. The fact that this reading remains constant indicates that these layers do not continue to grow. Positive charges cease to cross the metal–acid boundaries owing to the forces exerted by those charges already existing in the double layer.

Why zinc and copper behave so differently in this respect cannot be explained; it is a fact which must be taken for granted. But it is possible to estimate from the *electro-chemical series of elements* to be found in chemistry text-books what will happen when any two dissimilar metals are used in a voltaic cell. The various elements are arranged in this series in a special order, since this order gives much useful information regarding chemical changes. For our purpose it is useful, since, in general, if two different elements are used as the plates in a voltaic cell, the one higher in the series will be the negative plate and the one lower in the list the positive. That is, the one higher in the list will be at the lower potential. An abbreviated electro-chemical series is given in Table VIII at the end of this book. The position of hydrogen in this series is interesting. Because zinc is higher up in the table than hydrogen, and hydrogen higher than copper, one might expect zinc ions to go from a zinc plate into a solution of hydrogen ions and at the same time expect hydrogen ions to go out of the solution on to the copper. One would expect magnesium, aluminium and iron to behave like zinc and silver, platinum or carbon to behave like copper. In general, the further apart two metals are in the series the bigger will be the e.m.f. of a voltaic cell employing these metals. Thus whereas, in the voltaic cell being discussed, the e.m.f. with zinc and copper plates is 1·08 volts, in the case of a Leclanché cell with zinc and carbon plates the e.m.f. is 1·5 volts. Care must be taken, however, in arriving at conclusions from the use of this table, since the nature of the excitant affects the e.m.f. to some extent. From the relative positions of hydrogen and copper it is possible to see why the e.m.f. of a simple voltaic cell decreases when the cell becomes polarised, hydrogen replacing the copper as the effective positive plate. Copper and zinc are further apart in the table than hydrogen and zinc, and so, from what has been said above, we should expect a smaller e.m.f. when hydrogen usurps the function of the copper plate.

Referring again to the cell of Fig. 96, we have seen that no further reaction occurs, once the contact p.d's. are established, until the plates are connected externally. When this is done electrons immediately flow from the zinc to the copper plate along the connecting wire, since the zinc plate possesses a surplus of electrons and the copper plate a deficit of electrons, positive ions having gone into solution from the zinc plate, leaving electrons behind, and positive ions having been deposited on the copper plate, so making it positively charged. When electrons commence to pass away from the zinc via the external circuit the negative charge on the zinc starts to diminish, and hence the contact p.d. between the zinc and solution tends to fall; so further positive zinc ions are able to pass from the zinc into solution, and they do so, thus maintaining the contact p.d. The process is a continuous one. For every zinc ion going into solution two electrons pass round the external circuit to the copper plate, zinc ions being positive and bivalent. At the copper plate, as each pair of electrons arrive they are seized by two hydrogen ions, a hydrogen molecule resulting. More hydrogen ions come to the copper plate to replace those changed to hydrogen gas. (It will be appreciated, of course, that hydrogen gas consists of hydrogen molecules and has chemical and physical properties quite different from a solution of hydrogen ions.)

Thus, immediately the plates are joined externally a current flows through the wire (i.e., electrons pass from zinc to copper), hydrogen gas is formed on the surface of the copper plate, the zinc goes into solution in ionic form and the electrolyte slowly changes from sulphuric acid to zinc sulphate. The current *within* the cell consists of the movement of positive ions from zinc into solution and from solution to copper. There is a steady drift of positive ions from the negative to the positive plate. Simultaneously there is a steady drift of electrons in the external path from the negative to the positive plate, although it is conventional to say that the current flows from the copper to the zinc. (See Fig. 97.) Fig. 98 illustrates in symbolic and diagrammatic form the fundamental change from a chemical point of view. Although this diagram merely indicates what happens to a single zinc atom and a pair of hydrogen ions, the same changes occur with millions of zinc atoms and hydrogen ions.

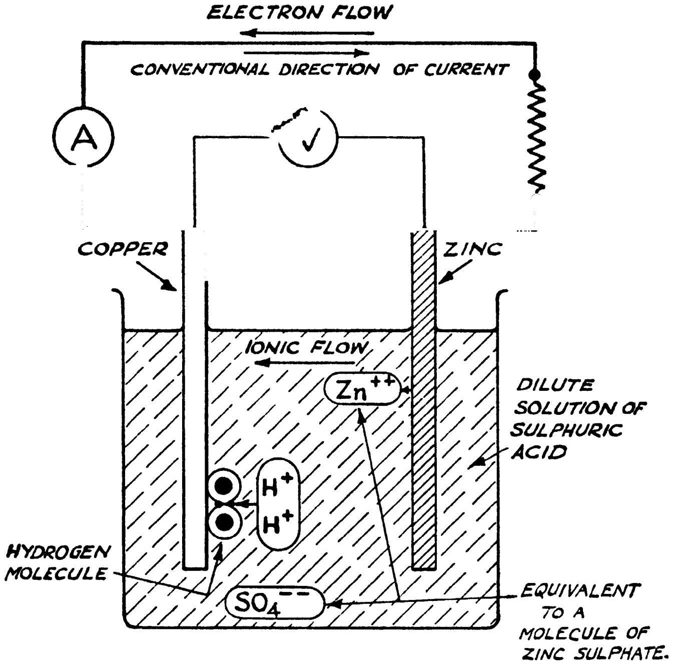

ELECTRON FLOW
CONVENTIONAL DIRECTION OF CURRENT
A
V
COPPER
ZINC
IONIC FLOW
Zn⁺⁺
DILUTE SOLUTION OF SULPHURIC ACID
H⁺
H⁺
HYDROGEN MOLECULE
SO₄⁻⁻
EQUIVALENT TO A MOLECULE OF ZINC SULPHATE.

FIG. 97.—THE CHANGES OCCURRING DURING DISCHARGE IN A SIMPLE VOLTAIC CELL.

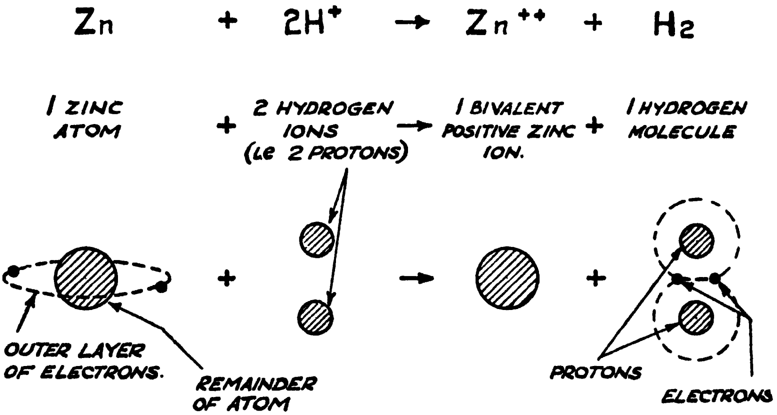

Zn + 2H⁺ → Zn⁺⁺ + H2
1 ZINC ATOM
+
2 HYDROGEN IONS (I.E. 2 PROTONS)
→
1 BIVALENT POSITIVE ZINC ION.
+
1 HYDROGEN MOLECULE
OUTER LAYER OF ELECTRONS.
REMAINDER OF ATOM
PROTONS
ELECTRONS

FIG. 98.—THE NET CHEMICAL CHANGE OCCURRING DURING DISCHARGE IN A SIMPLE VOLTAIC CELL.

52. The Theory of the Leclanché Cell

A sal-ammoniac solution contains positive monovalent ammonium ions and negative monovalent chlorine ions. When a current is taken from a Leclanché cell, positive bivalent zinc ions go into solution, causing ammonium ions to split up into ammonia gas and positive monovalent hydrogen ions. When 1 zinc atom splits up into a zinc ion, which goes into solution, and 2 electrons, which leave the zinc rod and pass round to the carbon rod through the external circuit, 2 ammonium ions are attacked, giving rise to 2 molecules of ammonia and 2 hydrogen atoms. The ammonia is dissolved in the solution and may ultimately escape; the hydrogen ions move to the carbon rod, collect 2 electrons and become a hydrogen molecule. The net chemical reaction can be expressed very concisely by the following equation:—

$$\begin{array}{ccccccccc} Zn & + & 2(NH_4^+) & \longrightarrow & Zn^{++} & + & 2NH_3 & + & H_2 \\ \text{1 zinc atom} & + & \begin{array}{c}\text{2 ammonium}\\ \text{ions}\end{array} & \longrightarrow & \text{1 zinc ion} & + & \begin{array}{c}\text{2 ammonia}\\ \text{molecules}\end{array} & + & \begin{array}{c}\text{1 hydrogen}\\ \text{molecule}\end{array} \end{array}$$

The hydrogen molecules form bubbles of gas which are attacked by manganese dioxide, another oxide of manganese and water being formed, as follows:—

$$\begin{array}{ccccccc} H_2 & + & 2MnO_2 & \longrightarrow & Mn_2O_3 & + & H_2O \\ \begin{array}{c}\text{1 hydrogen}\\ \text{molecule}\end{array} & + & \begin{array}{c}\text{2 molecules of}\\ \text{black oxide}\end{array} & \longrightarrow & \begin{array}{c}\text{1 molecule of}\\ \text{brown oxide}\end{array} & + & \begin{array}{c}\text{1 water}\\ \text{molecule}\end{array} \end{array}$$

Each positive bivalent zinc ion, going into solution, together with two negative monovalent chlorine ions (left spare by the splitting-up of two ammonium ions) is equivalent to a molecule of zinc chloride.

53. The Explanation of Electrolysis

Only when it is realised that an electrolyte is a compound containing ions is it possible to understand how and why the various electrolytic phenomena occur. When a p.d. is applied to an electrolyte the current that flows consists of (a) a drift of electrons in the external circuit from anode to cathode and (b) a drift of positive and negative ions within the electrolyte, the positive ions moving towards the cathode and the negative ions towards the anode. (See Fig. 99.) What happens when the ions reach the electrodes depends on the nature of the electrodes. In the case of the electrolytic cells shown in Fig. 89, on closing the switch positive copper ions immediately move towards the copper cathode and negative sulphate ions towards the anode. *At the cathode*, each copper ion, Cu^{++}, as it arrives, receives 2 electrons, making it into a copper atom, Cu, which is deposited on the copper cathode. *At the anode, in the case of the cell with the copper anode,* the concentration of negative sulphate ions causes more positive copper ions to enter the solution from the anode in an endeavour to neutralise the negatively charged solution. These copper ions were originally copper atoms, a copper atom, Cu, being equivalent to a copper ion, Cu^{++}, plus 2 electrons. So as each copper ion enters the solution it leaves behind on the anode 2 free electrons, which commence to drift round the external circuit through the battery towards the cathode. So, as fast as copper ions are deposited on the cathode, copper ions enter the solution from the anode, the electrolyte undergoing no net change. For every copper ion arriving at the cathode 2 electrons also arrive to neutralise it, and for every copper ion leaving the anode 2 electrons also leave the anode. The function of the external battery is to supply electrons to the cathode and remove them from the anode. Slowly the anode becomes thinner and the cathode gets thicker. Apart from this, the only resultant change in the cell, as a result of applying the external e.m.f., is a tendency for positive copper ions to concentrate round the cathode and negative sulphate ions round

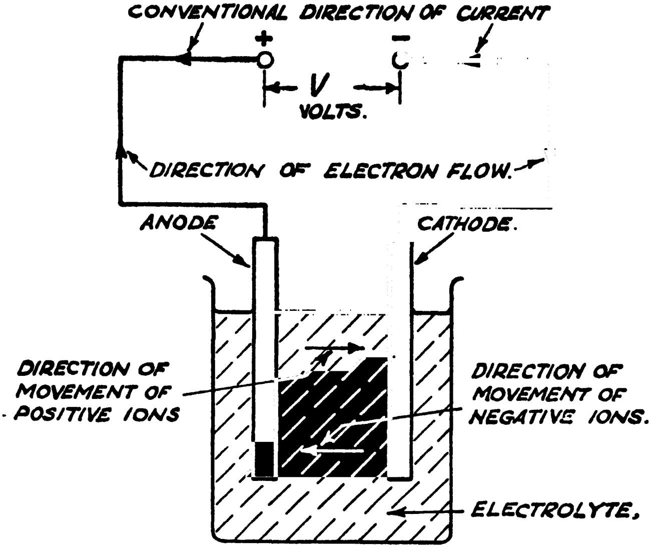

CONVENTIONAL DIRECTION OF CURRENT
+
−
V
VOLTS.
DIRECTION OF ELECTRON FLOW.
ANODE
CATHODE.
DIRECTION OF MOVEMENT OF POSITIVE IONS
DIRECTION OF MOVEMENT OF NEGATIVE IONS.
ELECTROLYTE.

FIG. 99.—THE PHENOMENON OF ELECTROLYSIS.

the anode; before applying the e.m.f. the positive and negative ions would be uniformly distributed throughout the electrolyte.

In the case of the cell with the carbon anode, since carbon is inert in copper sulphate solution, the negative sulphate ions are unable to induce positive ions to come into solution from the anode. In all cases such as this, where no positive ions are forthcoming from the anode to balance the concentration of negative ions, the negative ions produce the requisite number of positive ions by attacking the water, causing it to split up into hydrogen ions, electrons and oxygen molecules. Two sulphate ions cause 2 molecules of water to split up producing 4 hydrogen ions (which balance the 2 sulphate ions since sulphate ions are bivalent and hydrogen ions monovalent), 4 electrons (which proceed to drift round the external circuit from anode to cathode) and 1 molecule of oxygen, which collects on the anode and, with other oxygen molecules, forms a bubble of gas which eventually escapes. Expressed in symbolic form this may be written:—

$$2H_2O \longrightarrow 4H^+ + 4 \text{ electrons} + O_2$$

The substitution of a carbon for the copper anode of Fig. 89a makes the results of electrolysis in the two cells very different. Whereas, with a copper anode, copper ions enter the solution from the anode, with a carbon anode no ions leave the anode, but oxygen is liberated there. In the first cell there is no net change in the constitution of the electrolyte, whereas with a carbon anode hydrogen ions slowly replace copper ions, or, in other words, the electrolyte becomes acidified. As we have seen previously, a further difference is that with a carbon anode a back e.m.f. exists, whereas there is no appreciable back e.m.f. with a copper anode. Any very small back e.m.f. that might be found in the cell with the copper anode would be due to the positive and negative ions tending to concentrate round the cathode and anode respectively. A further point of difference previously mentioned is that whereas in the first cell there is no net chemical change, there is a chemical change in the cell with the carbon anode, the net chemical change being the production of hydrogen ions, oxygen molecules and copper atoms from copper ions and water molecules, thus:—

$$2\,Cu^{++} + 2H_2O \longrightarrow 4H^+ + O_2 + 2Cu.$$

Regarding the electrolysis of water acidified with sulphuric acid using platinum electrodes, the effect of applying a p.d. is to cause hydrogen ions to move to the cathode and sulphate ions to the anode. (It should be realised that water molecules do not ionise to any appreciable extent, but that the water is responsible for the independent existence of the hydrogen ions and sulphate ions of the sulphuric acid.) At the cathode 2 hydrogen ions receive an electron each to become a hydrogen molecule. At the anode the sulphate ions attack the water in the manner described in connection with the electrolysis of copper sulphate solution with a carbon anode; oxygen is liberated as a gas, hydrogen ions are formed, thus balancing the sulphate ions, and electrons are supplied to the anode, from which they drift round the external circuit. As quickly as hydrogen ions go out of solution at the cathode others are formed at the anode. The amount of acid in the solution therefore remains unchanged. The net change is simply that water molecules are decomposed into hydrogen and oxygen, thus:—

$$2H_2O \longrightarrow 2H_2 + O_2.$$

In all cases of electrolysis the fundamental and primary effect of applying a p.d. to the electrodes of a cell is to cause a drift of the positive ions of the electrolyte to the cathode and of the negative ions to the anode. But the final result depends upon whether or not secondary chemical reactions occur at the electrodes, and this depends upon the nature of the electrodes.

54. The Quantitative Laws of Electrolysis

It is obvious from the preceding section that electrolytic decomposition goes hand in hand with the flow of electrons in the external circuit. In other words, the amount of decomposition in an electrolytic cell in a given time is directly proportional to the quantity of electricity which flows into and out of the cell in that time. The mass of any element deposited at the cathode or liberated as a gas at either electrode is therefore proportional to the quantity of electricity which passes through the cell.

The mass of any element liberated at an electrode will also depend upon *the atomic weight* and *the valency* of the element. Atoms of different elements obviously have different atomic weights. The *atomic weight* of an element was originally defined as the ratio of the weight of an atom of the element to the weight of an atom of hydrogen, the atomic weight of hydrogen being taken as 1. This is now slightly modified, the atomic weight of hydrogen being taken not as 1 but as 1·008 (in order to make the atomic weight of oxygen exactly 16), but this modification makes very little difference to the actual values of the atomic weights. Suppose several elements having different valencies are liberated during electrolysis in a number of cells connected in series. Then for every 6 electrons flowing through the external circuit the number of monovalent ions of any one type discharged at an electrode will be 6, the number of bivalent ions 3 and the number of trivalent ions 2. So the weights of the various elements liberated will be proportional to the ratio $\frac{\text{atomic weight}}{\text{valency}}$. This ratio is known as the *chemical equivalent* of the element.

Thus it is obvious from the ionic explanation of electrolysis that the mass of any element liberated during electrolysis will be proportional to the quantity of electricity which passes and to the chemical equivalent of the element. In other words, the ionic explanation is consistent with Faraday's laws of electrolysis.

Exercises on Chapter V

Any additional data required for the solution of these exercises may be obtained from the tables at the end of this book.

1. Calculate (a) the number of grams of copper deposited on the cathode of a copper voltameter containing copper sulphate solution by a current of 3 amp. flowing for an hour, and (b) the number of pounds of copper deposited by a current of 200 amp. flowing for 10 hours.

[(a) 3·55; (b) 5·22.]

2. A silver voltameter and an ammeter are arranged in series with a battery. If the ammeter reading remains constant at 1·90 amp. for half-an-hour, during which time 3·925 gm. of silver are deposited, what is the percentage error in the ammeter reading?

[−2·65.]

3. A metal plate having a surface of 115 sq. cm. is to be silver-plated. If a current of 1·5 amp. be passed for 1 hour 30 minutes, what thickness of silver will be deposited?

[0·075 mm.]

4. (a) For how long could a Leclanché cell give a current of 0·1 amp. before using up 10 gm. of zinc? (b) If such a cell gives a

zero deflection on the galvanometer when the sliding contact of a simple potentiometer is at a point 101·2 cm. along the wire, and a Weston standard cadmium cell gives zero reading at a point 68·0 cm. along the wire, what is the e.m.f. of the Leclanché cell?

[(a) 82·43 hr.; (b) 1·52 V.]

5. How long will it take to deposit, from a copper sulphate solution, a coating of copper 0·05 mm. thick on an area of 118 sq. cm. if the supply p.d. is at 4·5 volts and the total resistance of the circuit is 2·3 ohms?

[2·269 hr.]

6. It is required to deposit $\frac{3}{64}$ inch of nickel on the cylindrical surface of a worn circular shaft 8 inches in diameter and 15 inches long, the current to be maintained at constant strength and adjusted so that the current density is 18 amp. per sq. foot at the beginning of the process. Assuming a current efficiency of 96%, estimate how long the process will take and the number of ampere-hours required.

[52·2 hr.; 2460.]

7. What percentage of the power supplied is utilised in chemical action in an electrolytic cell in which the p.d. across the electrodes is 6 volts, the resistance between electrodes 0·2 ohm and the current flowing 11·2 amp.? What will be the back e.m.f. of the cell?

[62·7; 3·76 V.]

8. What will be the cost of the energy used in the production of 1000 cu. ft. of hydrogen by electrolysis of water if the cost of energy is 2d. per kWh, the voltage drop per cell 3·5 and the current efficiency 95%? A litre (1000 c.c.) of hydrogen weighs 0·089 gm.

[£2 1s. 7d.]

9. Calculate the number of pounds of copper which can be purified per hour by electrolysis in a vat taking a current of 1000 amp. at a terminal p.d. of 4·5 volts. Estimate the energy consumption per ton of copper purified.

[2·61; 3862 kWh.]

10. 392 lb. of aluminium are produced per week of 120 working hours in a cell taking an average current of 6000 amp. What is the average current efficiency? If the average p.d. across the cell is 6·15 volts and energy costs 0·35d. per unit, what is the energy cost per ton of aluminium produced?

[72·6%; £36·9.]

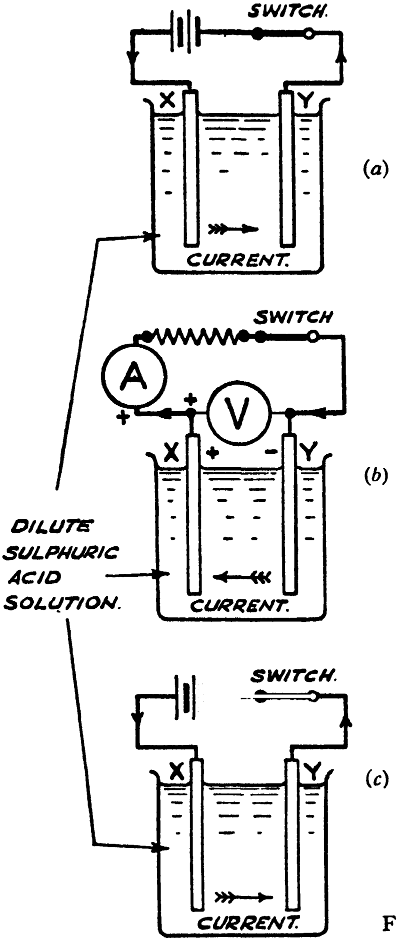
CHAPTER VI

ACCUMULATORS

55. The Acid Accumulator—Introduction

In section 47 of the previous chapter the electrolysis of water acidified with sulphuric acid was discussed. If, instead of using platinum for the electrodes, lead be used, as illustrated in Fig. 100a, hydrogen is liberated at the cathode, Y, but instead of oxygen being liberated at the anode, X, the lead of the plate X is attacked and the lead on the surface converted to lead dioxide (formerly known as lead peroxide), a chocolate-brown powder. In a few minutes the whole of the surface of the plate, X, immersed in the electrolyte becomes coated with an exceedingly thin layer of lead dioxide. This thin layer protects the remainder of the lead plate, X, from further oxidation, so bubbles of oxygen gas are then produced, as is the case when platinum electrodes are used.

If a voltmeter be now connected across the two plates, an e.m.f. of about 2 volts will be registered, the oxidised plate, X, being positive with respect to the plate, Y. In other words, the cell has become a voltaic cell and will supply a current if the two plates be connected by conducting materials as shown in Fig. 100b. As the cell delivers this current both lead plates become covered with a coating of lead sulphate, a greyish-white powder, the electrolyte becoming simultaneously weaker. Naturally, when both plates are covered with the same material there can be no p.d. between the plates. When the e.m.f. and current have fallen to zero, the cell is said to be *discharged.* To *re-charge* the cell, that is to restore it to its former condition in which the plate X is covered with a layer of lead dioxide at a potential of approximately 2 volts higher than that of plate Y, it is necessary to connect the plates X and Y to the positive and negative terminals respectively of a source of e.m.f. considerably in excess of 2 volts—see Fig. 100c. If the processes illustrated in Figs. 100b and 100c be repeated alternately the chemical changes occurring on the surface of the positive plate penetrate further and further into the plate, so that more of the lead becomes converted to *active material,* the cell eventually being capable of supplying a current for a very considerable period and requiring a much longer period than previously to become fully charged. The **capacity** of the cell increases with each repetition of the charging and discharging processes, the capacity of a cell being



On X—lead becoming converted to lead dioxide.
On Y—lead remaining as lead.

On X—lead dioxide becoming converted to lead sulphate.
On Y—lead also becoming converted to lead sulphate.

On X—lead sulphate becoming converted to lead dioxide.
On Y—lead sulphate becoming converted to spongy lead.

Fig. 100.—A Simple Lead–Acid Cell.

the quantity of electricity, measured in *coulombs* or *ampere-hours*, which it delivers in passing from the charged to the discharged state.

It should be observed that, whereas in the initial charging process illustrated in Fig. 100a hydrogen gas is evolved at the plate Y, in the succeeding charging processes no gas is liberated at either electrode until the lead sulphate at the two plates has been converted to lead dioxide in the one case and spongy lead in the other, spongy lead being lead in a somewhat porous condition.

The cell, once it has received its preliminary charge, is in a *reversible* condition. Electrical energy is then supplied to the cell when it is in a discharged condition (Fig. 100c), the electrical energy being transformed into and stored as chemical energy. This chemical energy may then be utilised whenever required by re-converting it to electrical energy, the two plates being simply connected through a conducting circuit as in Fig. 100b. Such a *reversible* cell is known as a *secondary cell*, an *accumulator* or a *storage cell.* There are two types of accumulator in frequent use, the lead-acid cell, the principle of which has been described above, and the nickel-alkaline cell, described later.

56. The Two Types of Lead Plate—Planté and Faure

Planté in 1859 devised the lead accumulator by the process described in the previous section, repeatedly charging and discharging the cell for 12 months, after which a thickness of about 1 mm. of active material was obtained. Plates produced by this method are known as **formed plates.** Formed plates, being very heavy, are not normally used nowadays in portable batteries, although the positive plates of stationary batteries, where weight is not important, are almost invariably of the formed type. The process of forming has been reduced to a matter of hours by the use of catalysts in the electrolyte, a catalyst being a substance which speeds up a chemical reaction. It is believed that the action of the catalysts in the forming of accumulator plates is to give the active material first formed a very porous condition, so that further action may proceed below the outer surface. In order to increase the amount of active material on a Planté-type positive plate, the surface area is increased, the usual method being to cast or shape the plate with deep corrugations as in Fig. 101. In another type of formed plate spirals of corrugated lead ribbon are inserted into a large number of circular holes in a framework of lead with which antimony is alloyed (see Fig. 102). The active material is formed on the surface of the ribbon, which projects a few millimetres to each side of the plate. During the process of formation the spirals expand assuming a rivet shape, since the holes of the grid are countersunk on both sides. The advantage of this type of plate is that the framework, being of a lead-antimony alloy, is not affected by the chemical action and is much stronger than the normal type of plate.

In 1881 Faure pasted the lead plates of a cell with compounds of lead. On passing a current through the cell the pastes were converted to lead dioxide and spongy lead respectively. It was found necessary to make the lead framework of the plates in the form of a grid, since charging causes an increase in volume of the active material; consequently the active material pasted on the surfaces of plain lead plates does not adhere to the plates with sufficient tenacity. Negative plates are almost always **pasted plates**, since the finely-divided spongy lead on a negative formed plate tends after a time to form into a non-porous mass, with consequent reduction in the capacity of the cell.

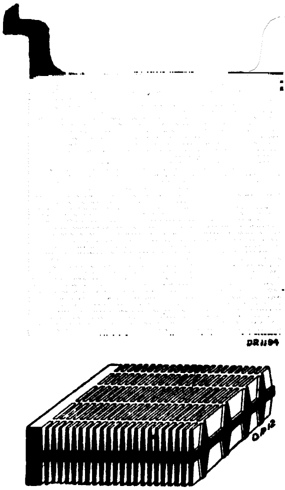


FIG. 101.—PLANTÉ POSITIVE PLATE AND ENLARGED SECTION.

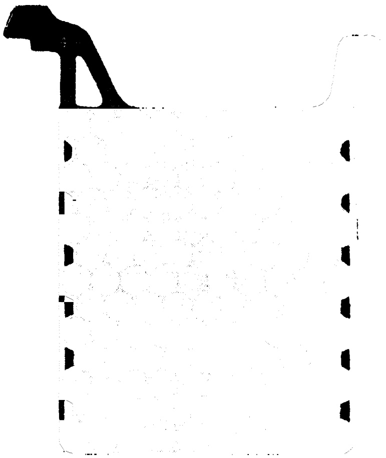


FIG. 102.—PLANTÉ POSITIVE PLATE OF THE ROSETTE TYPE.

The essential difference between formed and pasted plates is that in the former case the active material is produced by electrolysis out of the lead of the plate itself, whereas in the case of pasted plates the active material is applied mechanically by being pressed into the pockets between the intersecting ribs of a thin lead grid (see Fig. 103). Negative Faure plates are also frequently of the *box type*, consisting of a lattice with outer faces of finely perforated lead sheet (Fig. 104). Fig. 105 illustrates a type of positive pasted plate designed for electric vehicles where lightness, high capacity, large discharge currents and mechanical strength are particularly desired.

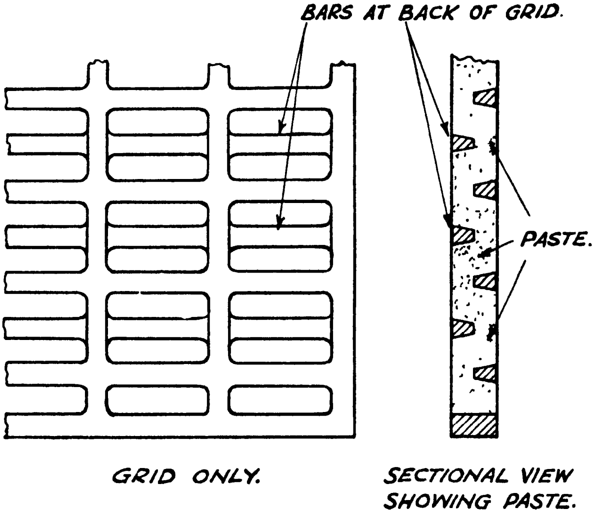


FIG. 103.—FAURE-TYPE GRID AND SECTION WITH PASTE.

Vertical lead rods, connected at top and bottom to horizontal lead bars are each surrounded by an ebonite tube, the intervening space being filled with paste to which the electrolyte penetrates through very fine horizontal slits in the ebonite tubes.

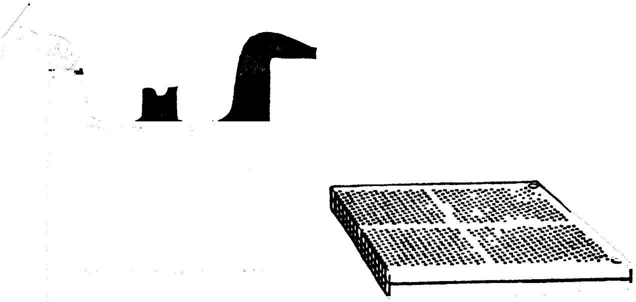
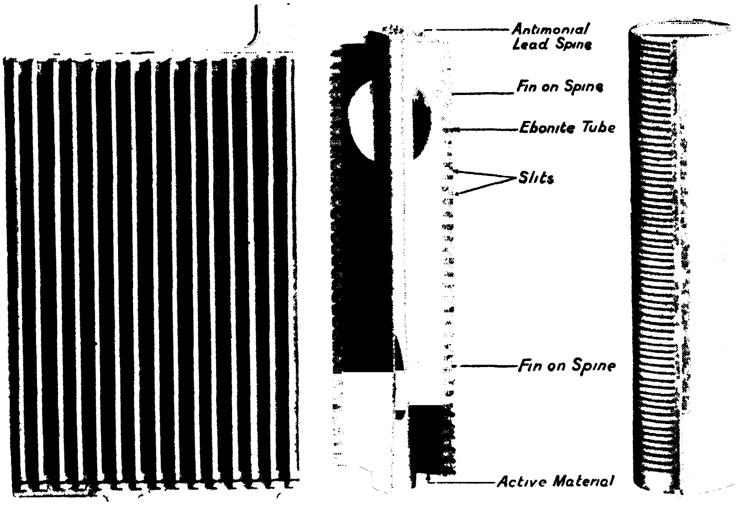


FIG. 104.—CASTING FOR BOX-TYPE NEGATIVE PASTED PLATE AND ENLARGED SECTION.

The pastes used are complicated, but consist chiefly of an oxide of lead. Either litharge, red lead or a mixture of the two is used plus a liquid which causes the paste to set into a sort of cement. Plates intended to be positives are connected up as anodes and those intended to be negatives as cathodes in a cell containing sulphuric acid. On passing a current through such a cell the oxygen evolved at the anode converts the lead oxide into lead dioxide and the hydrogen liberated at the cathode reduces the lead oxide to spongy lead. The lead framework of pasted plates contains from 5% to 10% of antimony. This alloy, in addition to its hardness, strength and acid-resisting properties, expands on solidification, a property very useful during casting. By using such an alloy it is possible to construct the framework of very thin, light plates.



Complete positive plate. Cross-section of assembled tube. Side view of ebonite tube.

FIG. 105.—" IRONCLAD " POSITIVE PLATE.

57. The Construction of Acid Accumulators

The positive active material of an acid cell undergoes large changes in volume during charge and discharge as compared with the negative active material. Moreover, the spongy lead is tough as compared with lead dioxide, which is comparatively brittle. So the tendency for the positive active material to disintegrate and fall to the bottom of the cell is much greater than for the negative material. The positive plate is therefore the weaker of the two, a negative pasted plate having roughly twice the life of a positive. For this reason the two outside plates in acid cells are usually negative plates, an odd number of plates being used; an outside positive plate would tend to become deformed due to chemical action occurring on the inside to a much greater extent than on the outside.

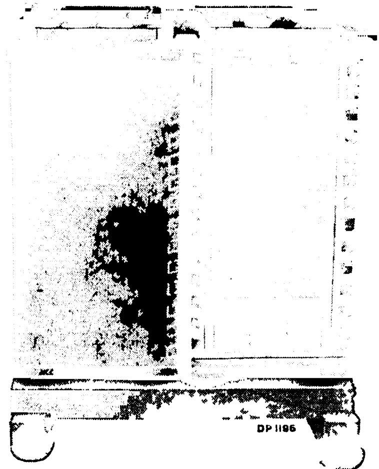


FIG. 106.—LARGE LEAD-ACID CELL IN LEAD-LINED WOOD BOXES

Since the construction of accumulators depends largely upon the use for which they are designed, it is desirable to treat stationary and portable cells separately.

(*a*) *Stationary Batteries*

The chief consideration in the design of cells for use in stationary batteries is durability, since weight and size are not of vital importance. The usual practice is to employ Planté positive plates and box-type negatives. Where the containers are of glass, the plates, positive and

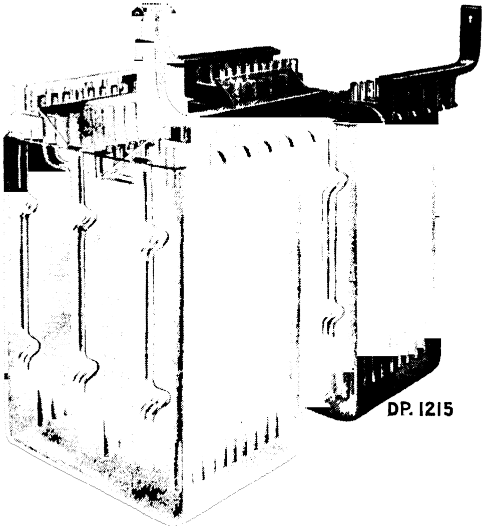


FIG. 107.—LEAD-ACID CELLS WITH GLASS TUBE SEPARATORS AND BOLTED CONNECTIONS.

negative alternately, are suspended by lugs usually from the edges of the container. For large cells, lead-lined wooden containers are usually employed (Fig. 106), the plates in such cells being suspended from the edges of sheets of glass placed inside and resting on the bottom of the cell. Vertical glass tubes are often employed for separating adjacent plates (see Fig. 107). The plates are frequently cast with *guides* into which the separator tubes are slotted; alternatively, the tubes are retained in position by means of horizontal glass tubes. *Spray arresters*, i.e., sheets of glass which prevent the escape of acid spray during gassing, also help to retain the separator tubes in position.

Owing to the fact that, with glass separators, an internal short-circuit can easily occur due to a piece of spongy lead growing on a negative plate, and so bridging the space between the negative plate and an adjacent positive plate, there is an increasing tendency for wood separators to be used in preference to glass separators. Wood separators consist of thin sheets of wood rendered porous by treatment with an alkaline solution which removes all resinous material.

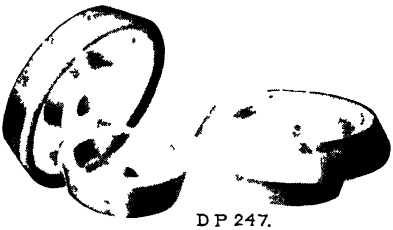


FIG. 108.—OIL-FILLED GLASS INSULATORS.

Internal short-circuits may also occur, due to active material which has fallen away from the positive plates being deposited at the bottom of a cell. To minimise the danger of the sediment, which accumulates over a long period, short-circuiting positive and negative plates, it is essential that considerable space be allowed underneath the plates (see Fig. 107). This illustration also shows the end springs fitted so as to hold the plates firmly together.

Glass insulators are usually employed to support each cell, and in the case of high-voltage batteries the insulators are of the oil-filled type, as illustrated in Fig. 108. Large cells in glass containers are often placed on insulated wooden trays filled with sawdust so as to secure an even distribution of the weight. The reason for the precautions regarding the insulation of the cells is that, owing to a certain amount of unpreventable acid spray, the surfaces of the containers, on which there is always a large amount of moisture due to condensation, acquire a good conducting layer. Unless the cells

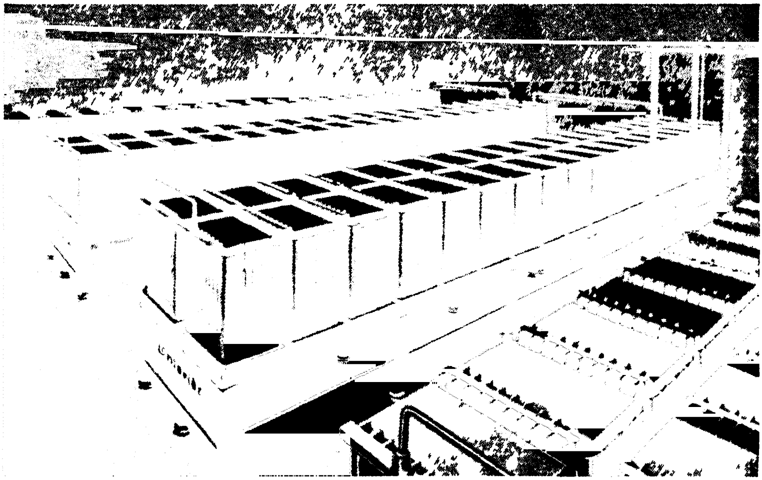
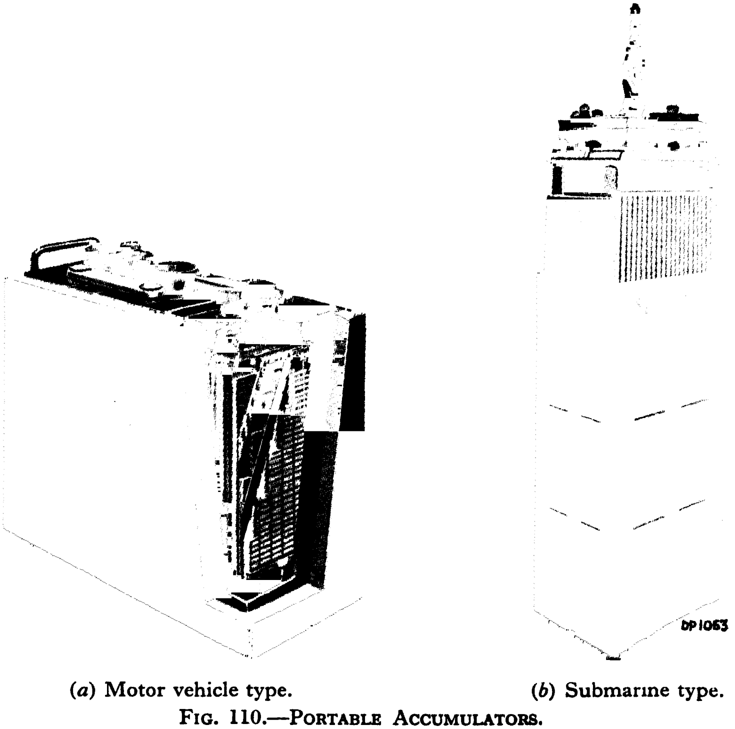


Fig. 109.—Emergency Lighting Battery at the Leeds Store of Lewis's, Ltd.



(*a*) Motor vehicle type. (*b*) Submarine type.

Fig. 110.—Portable Accumulators.

are well-insulated from earth there might be a considerable leakage current from the positive to the negative terminal of a high-voltage battery. Cells are usually carried, at a convenient height for inspection, on wooden stands, which should also rest on porcelain insulators if the e.m.f. of the battery be 200 volts or more (Figs. 106 and 109).

The lugs on the tops of all the positive plates of a cell are welded to a stout terminal bar, the negative plates being joined in a similar manner to a second bar. The positive terminal bar of one cell is then joined to the negative of an adjacent cell, and so on; the bars may be either welded (i.e., lead-burnt) or bolted together. The latter method is illustrated in Fig. 107; the reader will recognise the former method as being that adopted in the case of a motor-car battery.

(*b*) *Portable Cells*

Portable cells include those used on vehicles, and also cells which may be carried by hand. The fundamental consideration in the design of such portable cells is a high ratio of capacity to weight. Usually it is also desirable to keep down the size of the cell. Pasted plates are used for lightness, and the thickness of the separators, usually thin sheets of grooved wood, is reduced to a minimum. Most of the space available within the container is occupied by the very large number of plates employed, as illustrated in Fig. 110. To compensate for the small volume of acid, more concentrated acid is used than with stationary cells.

The pasted positive plate is the weakest feature of portable cells, owing to the readiness with which such a plate loses its active material when subject to vibration. Portable cells, it should be remembered, are widely used on vehicles, and in such cases are particularly subjected to vibration. Much attention has been given to the problem of devising a method of retaining the active material in the grid. The "ironclad" positive plate of Fig. 105 represents one such method. Another method employs a separator consisting of a special kind of felt made from spun glass-wool. The material when placed in contact with the positive plate prevents the escape of active material, but is porous as regards gas and liquid. In use a thin sheet of porous wood or perforated ebonite is placed between the negative plate and the glass wool adjacent to the positive plate.

The containers of small cells used for the low-tension supply of battery-operated radio sets are nowadays usually of glass, but cells used on electric vehicles or as motor-car starting and lighting batteries possess containers of ebonite or a hard rubber compound. The fact that these opaque containers hide the plates and electrolyte level from view is, of course, a disadvantage. To prevent spilling of the electrolyte portable cells are either sealed at the top, or are provided with lids. The plates are either suspended from the lid or are supported on blocks from the bottom of the cell.

58. General Characteristics of Acid Cells

(1) *Specific Gravity.* The internal resistance of a charged acid accumulator is a minimum when the specific gravity of the electrolyte is between 1·20 and 1·25. Since the working efficiency of a cell is very much dependent on the internal resistance, it is customary to use a specific gravity of about 1·22 in charged stationary batteries. As indicated in the previous section, a larger specific gravity (up to about 1·28) may be used in certain types of portable cells. An increase in specific gravity gives an increase in e.m.f., and less likelihood of freezing in cold climates; on the other hand, there is the danger of increased local action and a shortening of the life of the wooden separators where these are used. With a typical stationary cell the specific gravity falls to about 1·17 when the terminal p.d. falls to 1·8 volts, the point at which discharging should be stopped. With portable cells, owing to the small volume of electrolyte, the specific gravity is allowed to vary over a larger range, from, say, 1·28 to 1·16, before discharging should cease. Fig. 111a shows how the specific gravity of a typical accumulator increases at a uniform rate while the cell is being charged at an 8-hour rate, the current

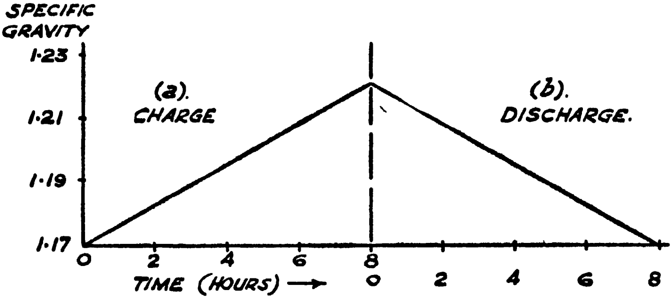


FIG. 111.—CHANGES IN S.G. OF ELECTROLYTE DURING CHARGE AND DISCHARGE OF A LEAD-ACID CELL.

remaining constant throughout; Fig. 111b illustrates the uniform rate of fall in the specific gravity when a constant discharge current is taken from the cell. This steady rate of change in the value of the specific gravity is explained in the Appendix. The specific gravity of the electrolyte is measured by means of a hydrometer. With an open-type stationary battery it is usually possible to place the hydrometer into the liquid between the plates of the individual cells. Portable cells are, however, usually sealed and the volume of electrolyte small, so in such cases it is necessary to use a syringe type of hydrometer, as illustrated in Fig. 112. To use this type of hydrometer the rubber tube fitted to the bottom of the glass tube is immersed in the acid by way of the filler vent. On squeezing the rubber bulb at the top of the syringe and releasing it, acid is sucked up into the glass tube, thus enabling an S.G. reading to be taken on the small hydrometer float within it. Care should be taken to ensure that the whole of the acid withdrawn from a cell is replaced after reading the specific gravity.

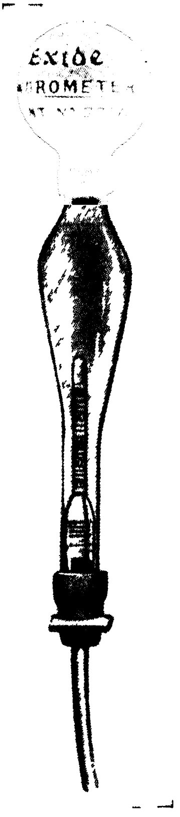


FIG. 112. — THE SYRINGE TYPE OF HYDROMETER.

(2) *E.M.F.* If an acid accumulator be charged, the *terminal voltage* will rise to 2·6 or 2·7 volts. Immediately on opening the circuit the e.m.f. will have the abnormally high value of 2·4 or 2·5 volts, but if the cell be allowed to stand for an hour or so the e.m.f. will fall to about 2·1 volts if the specific gravity of the acid is about 1·22. If the cell be now discharged until the *terminal voltage* is 1·8 volts and the circuit then opened, the e.m.f. observed immediately on opening the switch will have a rather low value of about 1·9 volts. On standing for an hour or so, however, the e.m.f. will slowly rise until it reaches a steady value of about 2·0 volts. Thus a fully-charged acid cell has a normal e.m.f. of about 2·1 volts and a fully-discharged cell a normal e.m.f. of about 2·0 volts, but several hours may elapse after charging or discharging before these steady e.m.f's. are reached.

During the charging process oxygen begins to evolve from the anode at a *p.d.* of about 2·3 volts and hydrogen from the cathode at about 2·4 volts, " gassing " occurring fairly freely above 2·45 volts. Towards the end of charging the *e.m.f.* is abnormally high, due to the presence of gas and due to the fact that, owing to the slow diffusion of the electrolyte, the concentration of the acid in the pores of the active material becomes abnormally high. The reason for the still higher terminal voltage is that, *on charge, the terminal p.d. is equal to the e.m.f. plus the voltage drop in the cell due to its internal resistance.*

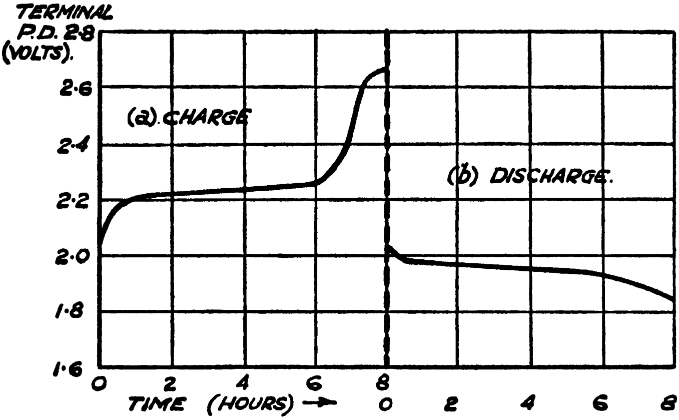


FIG. 113.—CHANGES IN T.P.D. DURING CHARGE AND DISCHARGE OF A LEAD-ACID CELL.

Fig. 113 illustrates how the *terminal p.d.* of a typical cell varies, the current being kept constant, while the cell is charged and discharged at an 8-hour rate. In interpreting these curves the following relationships should be kept in mind:—

During charge:

Terminal p.d. = E.M.F. + (Current × Internal Resistance)

During discharge:

Terminal p.d. = E.M.F. – (Current × Internal Resistance)

(3) *Capacity.* In practice the capacity of an accumulator is always measured in ampere-hours, one ampere-hour (1 Ah) being the quantity of electricity conveyed by a current of 1 ampere in 1 hour. A cell of capacity 120 Ah at the 10-hour rate will deliver a current of 12 amperes for 10 hours before the terminal p.d. falls to the lowest permissible value. The capacity of a cell varies, however, with the rate of discharge. The higher the rate of discharge, the less

the capacity. Thus a cell having a capacity of 120 Ah at a 10-hour rate would have a capacity of approximately 60 Ah at a 1-hour rate. Curve A in Fig. 114 illustrates the relationship between the capacity of a lead-acid cell and the rate of discharge, the curve being roughly applicable to all types and sizes of lead-acid cell. (Curves B and C refer to alkaline accumulators—see Section 61.) The curve illustrates why a motor-car battery becomes discharged when it is used for only a few minutes to drive the starter-motor during cold weather, under which conditions a current of hundreds of amperes may be taken from the battery.

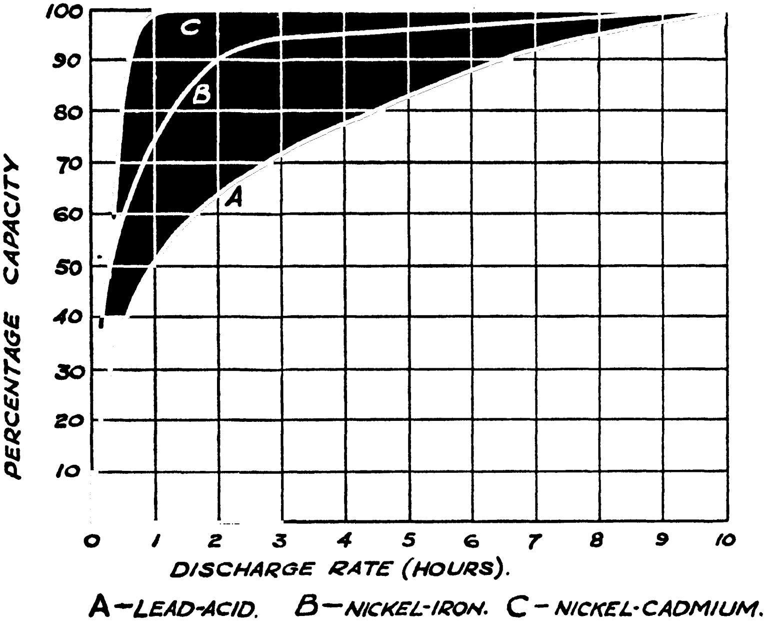


FIG. 114.—RELATIONSHIPS BETWEEN CAPACITY AND DISCHARGE RATE FOR VARIOUS TYPES OF CELL.

The chief cause of the variation in capacity at different discharge rates is to be found in the rate of diffusion of the electrolyte. The specific gravity of the acid in the pores of the active material is rapidly reduced at high rates of discharge. Diffusion takes place relatively slowly. The rate at which the terminal voltage falls is decided by the rate at which the specific gravity of the acid in contact with the active material falls, as compared with the rate at which it is replaced by stronger acid as a result of diffusion. It should be noted, however, that, if a cell discharged at a high rate to the permissible final voltage be allowed to stand for a time, a further discharge can be obtained from it.

The capacity of a cell for a given discharge rate is determined by the amount of active material on the plates. Thus the larger the plates, the greater the capacity. The capacity of a given cell is also very much dependent upon temperature, chiefly because of the manner in which temperature affects the rate of diffusion of the electrolyte. Near the freezing point of water there is a considerable reduction in the capacity, and near the freezing point of the electrolyte the capacity may be almost reduced to zero. At 40° F., at the 1-hour rate, the capacity of a normal lead-acid cell is only about 80% of the capacity at 60° F. Thus we have another factor contributing to the difficulties experienced with motor-car batteries during cold weather. In this connection it may be noted that lead-acid cells with special low-temperature characteristics were, during the late world war, developed for use in arctic regions; unfortunately, however, detailed information regarding these cells was not available at the time of publication.

(4) *Internal Resistance.* The internal resistance of an accumulator is made up of the resistances of the plates and connections, the active material and the electrolyte. A cell on charge has a slightly lower resistance than when being discharged. Much depends upon the concentration of the acid in the pores of the active material, and this is high while the cell is on charge. The higher the concentration of the acid, the less its resistivity.

The internal resistance of an accumulator is always a very small fraction of an ohm, being as small as 0·0001 ohm in the case of very large cells, the plate area and the distance between the plates being the deciding factors; the larger the plates and the less the distance between the plates, the smaller the internal resistance.

(5) *Efficiency.* In discussing the efficiency of an accumulator one has to distinguish between the *ampere-hour efficiency* and the *watt-hour efficiency*. The former may be defined as the ratio of the *quantity of electricity* which can be taken from a cell to the quantity which must be put back into the cell in order to restore it to its original condition. Expressed as a percentage the ampere-hour efficiency may be defined as follows:—

Ampere-hour efficiency

$$= \frac{\text{No. of ampere-hours available during discharge}}{\text{No. of ampere-hours to be put in during charge}} \times 100\%$$

The Ah efficiency is always stated for a given set of discharge and charge conditions, usually based on constant values of current. The values of Ah efficiency for most lead storage cells at the 10-hour discharge rate lie between 90% and 95%, i.e., 100 Ah must be put back for every 90–95 Ah taken out.

The watt-hour efficiency is more important, since it comes within the normal meaning of efficiency as used in engineering, i.e., it is the energy efficiency. It may be defined as follows:—

$$\text{Watt-hour efficiency} = \frac{\text{No. of watt-hours available during discharge}}{\text{No. of watt-hours to be put in during charge}} \times 100\%$$

For a constant current value, the number of watt-hours is equal to the number of ampere-hours multiplied by the average terminal voltage. Thus we see that, for the usual conditions in which the currents during discharge and charge are kept constant:—

$$\text{Watt-hour efficiency} = \text{Ah efficiency} \times \frac{\text{Average terminal p.d. during discharge}}{\text{Average terminal p.d. during charge}}$$

The watt-hour efficiencies of most lead-acid cells lie between 70% and 80%.

If an efficiency value for a storage cell is to convey any definite meaning, it must be obtained, not only for a stated set of discharge and charge conditions, *but also with the cell in a cyclic state*, i.e., in a state such that a given cycle of discharging and charging can be repeated continually. Unless this cyclic state is attained, the performance of the accumulator will depend upon its previous history, i.e., upon the precise condition of the active materials on the plates, and of the electrolyte.

The Ah efficiency of a given cell when the latter has reached this cyclic state will depend upon the various factors previously noted as affecting the capacity of a cell, namely, discharge rate and temperature.

The Wh efficiency is also dependent upon the current value and upon the internal resistance of the cell for another reason. High values of current and internal resistance raise the terminal p.d. during charge and lower it on discharge. Thus, for a high Wh efficiency a low rate of discharge and a low internal resistance are essential.

59. Charging, Care and Treatment of Acid Cells

Accumulators to be charged are connected in series to a direct-current supply having a p.d. of at least 2·7 volts for every cell. Thus if 50 cells are to be charged, the terminal voltage of the supply should be at least 135 volts. *The positive terminal of the supply must be connected to the positive terminal of the battery.* It is essential also to have the necessary means of varying the voltage applied to the battery. This can be done by means of a variable resistance as shown in Fig. 115. If, however, the source of supply is a shunt generator provided with fine control of field current (see Vol. II, Chapter IV), the variable resistance may be dispensed with, since varying the field current of a shunt generator gives a wide voltage range. This is an undoubted advantage, since any unnecessary resistance in the circuit involves a waste of power. Where only a fixed voltage supply is available the variable resistance frequently takes the form of a number of lamps or resistors in parallel, any number of which may be switched into the circuit. If an alternating-current supply only is available, a rectifier may be used to convert the a.c. to a d.c. supply.

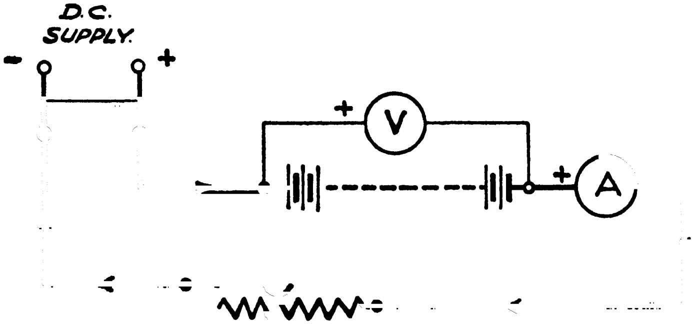


FIG. 115.—A SIMPLE CHARGING CIRCUIT.

Two factors—the temperature and the amount of gassing—limit the rate at which a cell may be charged with safety. Cells with formed positive plates can withstand the highest rates of charge, but the temperature should not be allowed to rise unduly nor should copious gassing be allowed. The upper temperature limit for the electrolyte is usually specified by the manufacturer of the cell, and is in the region of 100–110° F., depending upon the type of cell. The normal rate of charge for cells with Planté type positive plates is between one-quarter and one-third of the 1-hour discharge rate. When the terminal voltage reaches 2·45, or the temperature 90° F., the rate should be reduced to a value not exceeding the lower of the above limits. The rates given above may be arrived at for any given cell by dividing the Ah capacity of the cell *at the 10-hour rate* by 6 or 8. (The Ah capacity at the 10-hour rate is twice that at the 1-hour rate—see curve A, Fig. 114.) Portable cells should be charged at rather lower rates.

A lead-acid cell can be considered fully charged when the terminal voltage ceases to rise, when the specific gravity has reached the normal value corresponding to the fully-charged state and when the cell is gassing freely from *both* plates. It is usual to allow the cell to gas freely from both plates for 10 minutes or so, to ensure that the cell is really fully charged. If a cell is regularly allowed to gas freely for a longer period, an excessive amount of active material is loosened from the plates, giving rise to an unusually large amount of deposit at the bottom of the cell. *Overcharging should certainly be avoided, but it is even more important that the cell should not be frequently undercharged.* If a cell be regularly under-charged some of the lead sulphate never gets removed and the small crystals grow in size, the lead sulphate hardening so much that it becomes impossible to remove it, with consequent reduction of the cell's capacity. This phenomenon is known as *sulphation* and very frequently leads to buckling of the plates. Sulphation also results from over-discharging, i.e., discharging below the final permissible voltage. There is no danger of over-discharging if the cell is re-charged whenever the *e.m.f.* falls to 1·85 volts.

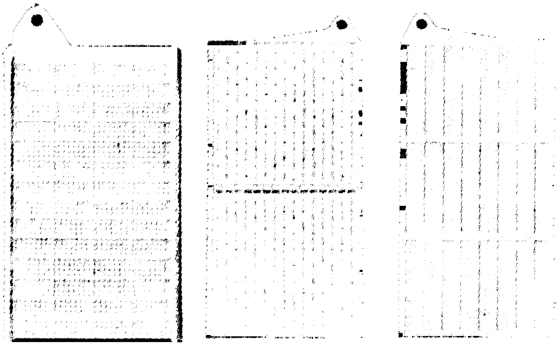
Thus we see that *acid cells should not be consistently under-charged, over-charged or over-discharged.* If, in addition, the cells are not charged or discharged above the normal rate and as far as possible are kept fully charged, if short-circuits, both internally and externally, are avoided, if corroded terminals are also avoided by keeping them covered with grease, if the electrolyte is kept well above the plates by " topping up " with distilled water, a long life for the cells will be ensured.

The chief causes of deterioration of acid cells are:—

(1) local action at the positive plates between the lead and lead dioxide resulting in disintegration of the positive plates.

(2) sulphation which occurs chiefly at the negative plates and

(3) local action at the negative plates between the lead and metallic impurities, chiefly manganese and platinum. It is in order to avoid the deposition of metallic impurities on the negative plate that distilled water is used for topping-up, tap water frequently containing minute traces of harmful impurities.



(*a*) " NIFE " positive or negative plate. (*b*) Edison positive plate. (*c*) Edison negative plate.

FIG. 116.—POSITIVE AND NEGATIVE PLATES OF ALKALINE CELLS.

60. Alkaline Cells

(1) *Types and Construction of Alkaline Cells*

Several types of accumulators employing an alkaline electrolyte have been produced and developed, two of them, the Jungner and Edison types, on a wide commercial scale. Edison and Jungner cells differ essentially only in the negative active material, which consists, in the charged Edison cell, of finely divided iron, and in the case of the charged Jungner cell of a mixture of cadmium and iron. In each case the positive active material consists of nickel hydroxide and the electrolyte of a 20–25% solution of pure potassium hydroxide having a specific gravity of from 1·16 to 1·19. In a more recently developed type of alkaline cell, the Drumm cell, whereas the positive active material is nickel hydroxide as with the other cells, the negative material is zinc and the electrolyte a solution of zinc oxide and caustic potash.

The active material of Jungner and Edison cells is contained in perforated nickel-plated steel tubes or pockets which are assembled into nickel-plated steel frames to form the positive and negative plates. Fig. 116 illustrates the construction of the various types of plate employed, and Figs. 117 and 118 show complete Edison and " Nife " (Jungner type) cells cut away so as to reveal the construction. The separators consist of thin ebonite or hard-rubber rods, and the containers are of sheet steel with welded joints, these containers being usually nickel-plated on the outside to prevent corrosion.

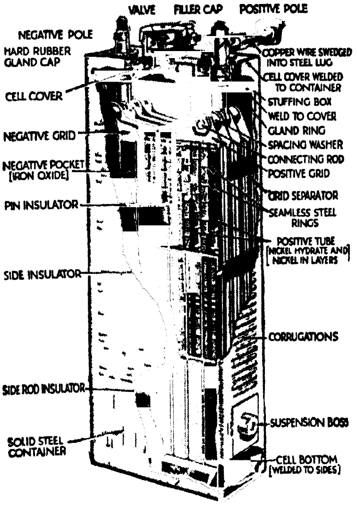


FIG. 117.—EDISON NICKEL-IRON CELL.

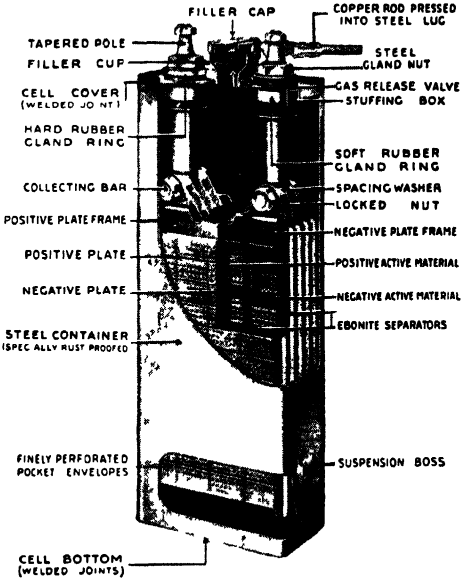


FIG. 118.—" NIFE " NICKEL-CADMIUM CELL.

The plates are comparatively thin, and as they are placed very close together, an alkaline cell occupies less volume than an equivalent acid cell. Owing to the fact that potassium hydroxide absorbs carbon dioxide from the atmosphere, the containers are made as airtight as possible with the aid of a combined valve and filler cap. The individual cells in a battery are insulated from each other by means of suspension bosses carried in ebonite insulators supported in wooden crates.

To improve the conductivity of the active material and so reduce the internal resistance of the cells, the nickel hydroxide of the Edison cell is intimately mixed with flakes of pure nickel and the iron with mercuric oxide; in the Jungner cell graphite is mixed with the nickel hydroxide. Cadmium, the chief negative active material of the Jungner cell, possesses good conductivity, but it is mixed with iron, since this improves the porosity; moreover, the iron takes a part in the reactions of the cell along with the cadmium.

(2) *Chemical Changes during Charge and Discharge*

Unlike the electrolyte in acid accumulators, the strength of the potassium hydroxide does not vary, there being no net change in its concentration during charge and discharge. Although there is no *net* change in the specific gravity of the potassium hydroxide solution, the electrolyte does of course play a part in the changes which take place in the active materials, transferring certain constituents from one plate to the other.

During discharge the iron of the Edison cell is converted into ferrous hydroxide, the positive active material being reduced from nickelic hydroxide to nickelous hydroxide. In the Jungner cell the positive active material is changed in the same manner, whereas at the negative plates the iron and cadmium are changed to ferrous and cadmium hydroxide respectively. During charge the reverse process takes place, but, in addition, gas is evolved, hydrogen being given off from the cathode when the terminal p.d. exceeds a value between 1·45 and 1·55 volts (depending on the rate of charge) and a little oxygen from the anode just before the cell becomes fully charged.

(3) *Charge and Discharge Curves*

The e.m.f. of a charged nickel-cadmium cell lies between 1·3 and 1·4 volts and that of an Edison cell slightly higher, the average e.m.f. during discharge being, in each case, slightly more than 1·2 volts. Fig. 119 shows charge and discharge curves for a 5-hour rate for typical Jungner and Edison cells. During discharge the voltage variation in the case of the nickel-iron cell is considerably greater than in the case of the nickel-cadmium cell. Moreover, the Edison cell requires a charging p.d. rising from 1·55 to just over 1·8 volts. In the Jungner cell the p.d. during charge rises from just over 1·3 to rather more than 1·6 volts, two-thirds of the charge being at a p.d. less than 1·47 volts. The comparatively low charging p.d. in the latter case is an advantage, since it involves considerably less evolution of gas, and thus gives rise to a higher efficiency. It also makes "trickle-charging" possible, a process which is impossible with the Edison cell owing to the fact that at low current densities most of the energy is dissipated in gas evolution if the terminal p.d.

is more than 1·45 volts. For higher charging rates, and in the case of other types of nickel-cadmium cell, the p.d. at the end of charge may rise to about 1·75 volts. The reason that when a nickel-cadmium cell is two-thirds charged there is a comparatively steep rise in the voltage is that up to this point it is the cadmium portion of the active material which is involved in chemical change, whereas the higher

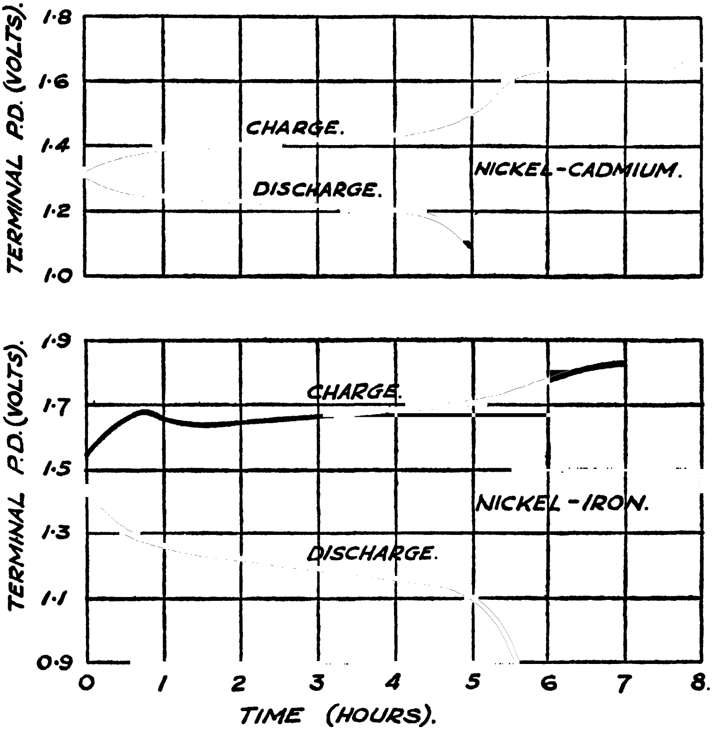


FIG. 119.—CHARGE AND DISCHARGE CURVES FOR ALKALINE CELLS.

voltage is due to the iron portion, although even when the iron becomes involved the p.d. does not rise so high as in the nickel-iron cell, owing to the low resistance of the cadmium-iron mixture.

(4) *Charging and Maintenance of Alkaline Cells*

Alkaline cells are charged by the same methods as acid cells, a d.c. supply of 1·7 to 1·8 volts per cell being required. With the nickel-iron type gassing occurs from the beginning of charge, and so cannot be used as an indication that the cell is charged. Neither can the S.G. be used as an indication, since, as we have seen, it remains constant throughout charge and discharge. It is therefore necessary to rely upon the voltage. An Edison cell may be regarded as charged when the terminal p.d. *at the normal charging rate* reaches 1·8 volts and a Jungner cell when the p.d. reaches 1·6/1·75 volts, depending on the type. Over-charging cannot harm alkaline cells, and it is desirable to over-charge rather than under-charge. The charging rate should never be so high that the temperature exceeds 115° F., or, for some types, 130° F., and the cells should not be continuously operated above these temperatures. Alkaline cells should always be charged when the terminal p.d. at the 5-hour rate becomes as low as 1·1 volts. Discharge beyond this point results in reduced capacity and a shortened life.

Actually alkaline cells can stand up to careless treatment far better than acid cells, being able to withstand, without undue depreciation, high rates of charge and discharge and long periods of idleness. The chief precaution to be taken is to ensure that the electrolyte level does not fall below the tops of the plates, this reduction in level being due to evaporation and gassing. Topping up with distilled water is essential, as with acid cells. Owing to evaporation, gassing and absorption of carbon dioxide due to leaky filler plugs, the specific gravity of the electrolyte is liable to vary with time. It should not be allowed to fall below 1·16 or rise above 1·19. Moreover, the electrolyte should be renewed periodically according to the makers' instructions, since, owing to absorption of carbon dioxide, the potassium hydroxide solution becomes changed to potassium carbonate and the cell loses capacity. The capacity is, however, restored on renewing the electrolyte.

61. Comparison of Lead-acid and Nickel-alkaline Cells

Many factors have to be taken into account in comparing accumulators, the chief ones being initial cost, life, mechanical strength, ability to withstand misuse, weight, bulk, e.m.f., internal resistance, capacity and efficiency. Moreover, any estimate as to which type of cell is most suitable will very largely depend upon the purpose for which the cells are required.

In the first place, in two equivalent batteries, one of acid cells and the other of alkaline cells, the number of acid cells will be only three-fifths of the number of nickel-cadmium cells, since the average p.d.'s on discharge are roughly in the ratio of 5 : 3. If the batteries are to be really equivalent, the acid and alkaline cells must have equal capacities. The initial cost of five nickel-cadmium cells will be two or three times the cost of three acid cells of the same capacity. The higher initial cost is, however, offset by the much longer life of alkaline cells, this longer life being due to the fact that the latter do not suffer from the defects of acid cells. There is, of course, no sulphation with alkaline cells, and the active material is held much more tenaciously than in acid cells, the amount of material falling away from the plates of alkaline cells being quite negligible. An important merit of alkaline cells, to which reference has already been made, is their ability to withstand careless treatment and undue demands such as over-charging, over-discharging, long periods of idleness and high rates of charge and discharge. Not only do these cells benefit from the use of an alkaline rather than an acid electrolyte, but the use of steel instead of lead plates is a very considerable advantage in so far as it is claimed to be impossible to buckle the plates of alkaline cells by any electrical means. The mechanical strength of alkaline cells is possibly their chief merit. This feature, combined with their low weight and bulk, makes them eminently suitable as portable cells, particularly for use on vehicles of all types. Even allowing for a 5 : 3 ratio in the number of cells in batteries of equal p.d., alkaline batteries are much lighter and less bulky than batteries of acid cells. It is also claimed that the nickel-cadmium cell does not suffer from "self-discharge", even when standing idle for long periods. This characteristic is, of course, of outstanding importance for certain applications.

At one time the internal resistance of an alkaline battery was considerably greater than that of an equivalent acid battery, but special low-resistance nickel-cadmium cells are now available, so that there is now no reason why alkaline cells should not be used for any purpose. Acid cells certainly possess higher efficiencies than alkaline cells, typical ampere-hour and watt-hour efficiencies for nickel-cadmium cells being 80% and 65% respectively as against figures of, say, 90% and 75% for acid cells. The efficiencies of nickel-iron cells are lower than those of the nickel-cadmium type, chiefly due to the excessive gassing during charge.

On the other hand, the capacity of an alkaline cell does not fall away so rapidly with high rates of discharge as does the capacity of an acid cell, due to the fact that the alkaline electrolyte does not change in strength during discharge. Curves B and C in Fig. 114 illustrate how the percentage capacity of typical alkaline cells remains fairly constant until the rates of discharge become excessively *high.* Thus, for a discharge rate of 1 hour, whereas the capacity of an acid cell is only 50% of the capacity at the 10-hour rate, the figures for nickel-iron and nickel-cadmium cells are respectively 76% and 99%. At one time the capacity of alkaline cells fell away at low temperatures very much more rapidly than in the case of acid cells, but the makers of " Nife " accumulators claim that their cells are actually superior in this respect to acid cells and draw attention to the fact that " Nife " batteries operate on mining locomotives within the Arctic circle.

In view of the lower initial cost and higher efficiencies of acid cells, they are generally preferred to alkaline accumulators for use in stationary batteries, where they are likely to receive due care and attention, but it would appear that nickel-cadmium cells, in view of their mechanical strength, lightness and ability to withstand arduous conditions, are more satisfactory for traction purposes, and particularly in cases where exceptional vibration is encountered, as on motor-cycles.

62. Applications of Accumulators

(*a*) *Stationary batteries*

The chief applications of this type of cell are indicated below. In the first two applications the lead-acid cell is the type commonly employed, but in (3) and (4) the nickel-alkaline cell is being used to an increasing extent.

(1) In Post Office telephone exchanges, the Post Office being, in 1939, the largest user of storage batteries in the country.

(2) In d.c. power-stations and sub-stations, to provide a reserve of energy during peak-load periods. Formerly, this system was frequently employed, particularly in stations supplying a traction load, but the expansion of the national Grid system, coupled with the change-over from d.c. to a.c. for distribution networks, has resulted in a marked diminution in the importance of this application of storage cells.

(3) For emergency lighting in theatres, cinemas, hospitals, banks and large stores. The battery is usually arranged so that it automatically supplies the essential lighting and other services in the event of a failure of the main power supply.

(4) For the operation of control gear in modern power-stations and sub-stations.

(b) *Portable and Semi-portable Cells*

The purposes for which these cells are employed are even more numerous than those for which stationary cells are used. In almost every case both the lead-acid and nickel-alkaline types of cell are equally suitable on the score of performance, but, as indicated previously, the initial cost of the latter type is considerably higher, although the life is very much longer. The most important applications of portable and semi-portable cells are as follows:—

(5) On motor-cars, buses and heavy road vehicles for starting, lighting, L.T. supply for the ignition system and other minor purposes.
(6) In aircraft for lighting, radio and various control circuits.
(7) On trains, for lighting and for the kitchens.
(8) As the source of motive power on battery electric vehicles of all types, e.g., delivery vans, colliery and shunting locomotives and trucks.
(9) As the source of motive power for under-water propulsion in submarines.
(10) For the L.T. supply to radio equipment and, in certain cases, for the H.T. supply as well.
(11) In miners' hand- and cap-lamps.

In the case of the battery electric vehicle the saving in weight occasioned by the use of a nickel-alkaline battery instead of a lead-acid battery is very important, as it enables additional pay-load to be carried.

APPENDIX

63. Theory of the Lead-acid Cell

(a) On Charge

When an acid cell in the discharged state is subjected to a charging p.d., the hydrogen ions of the sulphuric acid drift towards the cathode (which becomes the negative plate of the accumulator when charged) and sulphate ions drift towards the anode. Lead sulphate is slightly soluble in sulphuric acid, existing in solution as lead ions and sulphate ions. The hydrogen ions arriving at the cathode react with the lead sulphate existing there in ionic form, and the sulphate ions arriving at the anode react with the lead sulphate which has entered into solution from the anode, thus:—

At the cathode:—

$$(\mathrm{Pb}^{++} + \mathrm{SO}_4^{--}) + \quad 2\mathrm{H}^+ \quad + \begin{array}{c}\text{2 electrons}\\ \text{arriving from}\\ \text{the external}\\ \text{circuit}\end{array} \rightarrow (2\mathrm{H}^+ + \mathrm{SO}_4^{--}) + \quad \mathrm{Pb}$$

$$\text{i.e.,}\ \begin{array}{c}\text{1 lead sulphate}\\ \text{molecule}\end{array} + \begin{array}{c}\text{2 hydrogen}\\ \text{ions}\end{array} + \text{2 electrons} \rightarrow \begin{array}{c}\text{1 sulphuric acid}\\ \text{molecule}\end{array} + \begin{array}{c}\text{1 lead}\\ \text{atom}\end{array}$$

As the lead ions of the lead sulphate become lead atoms which are deposited on the negative plate, more lead sulphate dissolves, the process continuing until the whole of the lead sulphate is converted to spongy lead.

At the anode:—

$$(\mathrm{Pb}^{++} + \mathrm{SO}_4^{--}) + \quad \mathrm{SO}_4^{--} \quad + \quad 2\mathrm{H}_2\mathrm{O} \quad \rightarrow$$

$$\begin{array}{c}\text{1 lead sulphate}\\ \text{molecule}\end{array} + \begin{array}{c}\text{1 sulphate}\\ \text{ion}\end{array} + \begin{array}{c}\text{2 molecules}\\ \text{of water}\end{array} \rightarrow$$

$$\mathrm{PbO}_2 \quad + (4\mathrm{H}^+ + 2\mathrm{SO}_4^{--}) + \text{2 electrons which move from the anode round the external circuit}$$

$$\begin{array}{c}\text{1 lead}\\ \text{dioxide}\\ \text{molecule}\end{array} + \begin{array}{c}\text{2 sulphuric acid}\\ \text{molecules}\end{array} + \text{2 electrons}$$

The total change at the two electrodes may be represented as follows:—

$$2\mathrm{PbSO}_4 + \mathrm{H}_2\mathrm{SO}_4 + 2\mathrm{H}_2\mathrm{O} + \text{2 electrons} \longrightarrow \mathrm{Pb} + \mathrm{PbO}_2 + 3\mathrm{H}_2\mathrm{SO}_4 + \text{2 electrons}$$

and the *net* change as follows:—

$$2\mathrm{PbSO}_4 + 2\mathrm{H}_2\mathrm{O} \longrightarrow \mathrm{Pb} + \mathrm{PbO}_2 + 2\mathrm{H}_2\mathrm{SO}_4 \quad . \quad (\mathrm{I})$$

Thus for every 2 electrons passing round the circuit, 1 molecule of lead sulphate on the anode is converted to lead dioxide, 1 molecule of lead sulphate on the cathode is converted to spongy lead and 2 molecules of sulphuric acid replace 2 molecules of water in the solution. It is this arrival of sulphate ions into the solution which is responsible for the increase of specific gravity. The number of sulphate ions (i.e., the number of sulphuric acid molecules) entering the solution is directly proportional to the quantity of electricity passing through the cell; hence the straight-line graph of Fig. 111.

(b) On Discharge

There exists at the surfaces of the positive and negative active materials of a charged acid accumulator a double-layer of electric charge, as explained, in the case of a simple voltaic cell, in Section 51 of the appendix to the previous chapter. This is because the tendency of positive lead ions on the negative plate to enter the solution is greater than the tendency of positive hydrogen ions in the solution to be deposited on the lead plate, and because there is a strong tendency for positive hydrogen ions to collect on the surface of the positive active material. So at the negative plate, the lead atoms split up into positive lead ions and electrons; for every doubly-charged lead ion entering the solution, two electrons are left behind on the plate. Similarly, at the positive plate for every two singly-charged positive hydrogen ions deposited on the plate one doubly-charged negative sulphate ion is left behind in the solution. Thus the state of affairs is as shown in Fig. 120. There is thus a contact p.d. between each plate and the solution, the e.m.f. of the cell being equal to the sum of the two contact p.d's.

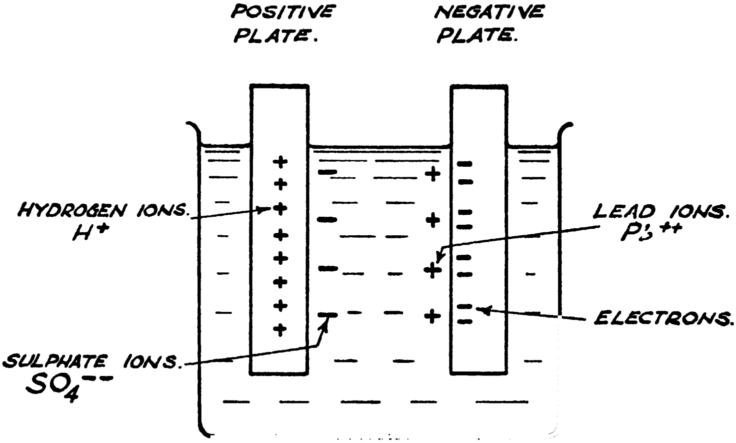


FIG. 120.—THE PRODUCTION OF THE E.M.F. IN A LEAD-ACID CELL.

On connecting the terminals through a conducting circuit, electrons flow round from the negative to the positive plate; more positively charged lead ions can therefore go into solution from the negative plate, since they are no longer held to the plate by the attraction of the electrons on the plate. Similarly, more positive hydrogen ions are attracted to the positive plate by the negative electrons which arrive there.

At the negative electrode sulphuric acid molecules attack the lead ions as quickly as they are liberated, lead sulphate being formed and hydrogen ions liberated. At the positive electrode, as fast as hydrogen ions are deposited the surplus sulphate ions attack the lead dioxide, this substance being converted to lead sulphate, and the oxygen in the lead dioxide combining with the hydrogen ions to form water molecules. Thus, *at the negative plate*:—

$$\begin{aligned} Pb + (2H^+ + SO_4^{--}) &\longrightarrow (Pb^{++} + 2\text{ electrons}) + (2H^+ + SO_4^{--}) \\ &\longrightarrow (Pb^{++} + SO_4^{--}) + 2H^+ + \begin{array}{c}\text{2 electrons which pass}\\ \text{round the circuit to the}\\ \text{positive plate}\end{array} \end{aligned}$$

At the positive plate:—

$$\begin{aligned} PbO_2 + (4H^+ + 2SO_4^{--}) + \begin{array}{c}\text{2 electrons which}\\ \text{arrive from the}\\ \text{negative plate}\end{array} &\longrightarrow \\ (Pb^{++} + SO_4^{--}) + SO_4^{--} &+ 2H_2O \end{aligned}$$

The net total change at the two plates is as follows:—

$$Pb + PbO_2 + 2H_2SO_4 \longrightarrow 2PbSO_4 + 2H_2O \quad . \quad (II)$$

It will be seen that the change indicated by equation II is precisely the reverse of the change indicated by equation I. These equations represent the molecular changes which take place each time two electrons pass round the circuit, in equation I the two electrons being part of a charging current and in equation II part of a discharging current.

In the above theory no mention has been made of gassing. Towards the end of charge two processes occur simultaneously. In addition to the changes indicated by equation I, hydrogen and oxygen are produced at the cathode and anode respectively, as described at the end of Section 53 in the appendix to the previous chapter. This gassing cannot be avoided if the cells are to be fully charged. But the energy necessary to produce the gas during charge cannot be recovered during discharge. The production of the hydrogen and oxygen is an irreversible process, whereas the changes discussed in the present section are reversible.

64. Theory of the Alkaline Cell

In this section the theory of the Edison-type cell will be discussed, since the negative active material is iron, whereas in the Jungner-type cell a mixture of iron and cadmium are used. The iron and cadmium behave in a similar manner, but it is more convenient to refer to a single element than to two elements.

The chemical reactions occurring in alkaline cells are rather complicated, but the following reversible equation is usually accepted as a useful summary of the *net* changes which take place during the charge and discharge of Edison-type cells:—

$$2\,Ni(OH)_3 + Fe \rightleftharpoons 2\,Ni(OH)_2 + Fe(OH)_2$$

$$\text{i.e.,}\quad \begin{array}{c}\text{2 molecules}\\ \text{of nickelic}\\ \text{hydroxide}\end{array} + \begin{array}{c}\text{1}\\ \text{atom}\\ \text{of iron}\end{array} \rightleftharpoons \begin{array}{c}\text{2 molecules}\\ \text{of nickelous}\\ \text{hydroxide}\end{array} + \begin{array}{c}\text{1 molecule}\\ \text{of ferrous}\\ \text{hydroxide}\end{array}$$

Potassium hydroxide, KOH, in solution exists in the form of positive, singly-charged potassium ions, K^+, and negative, singly-charged, hydroxyl ions, OH^-.

On charge, potassium ions drift towards the cathode and hydroxyl ions towards the anode. *At the cathode* the potassium ions abstract the hydroxyl ions from ferrous hydroxide molecules, which are slightly soluble in potassium hydroxide solution, existing in ionic form as ferrous ions, Fe^{++}, and hydroxyl ions, OH^-. The reaction is as follows:—

$$(Fe^{++} + 2OH^-) + 2K^+ + \begin{array}{c}\text{2 electrons}\\ \text{arriving from}\\ \text{the external}\\ \text{circuit}\end{array} \longrightarrow (2K^+ + 2OH^-) + Fe$$

$$\text{i.e.,}\quad \begin{array}{c}\text{1 ferrous}\\ \text{hydroxide}\\ \text{molecule}\end{array} + \begin{array}{c}\text{2 potassium}\\ \text{ions}\end{array} + \text{2 electrons} \longrightarrow \begin{array}{c}\text{2 potassium}\\ \text{hydroxide}\\ \text{molecules}\end{array} + \begin{array}{c}\text{1}\\ \text{iron}\\ \text{atom}\end{array}$$

At the anode, the hydroxyl ions from the electrolyte react with the nickelous hydroxide (also slightly soluble), converting it into nickelic hydroxide, 2 electrons being given up to the anode thus:—

$$(2Ni^{++} + 4OH^-) + 2OH^- \longrightarrow (2Ni^{+++} + 6OH^-) + \begin{array}{c}\text{2 electrons which}\\ \text{move from the}\\ \text{anode round the}\\ \text{external circuit}\end{array}$$

$$\text{i.e.,}\quad \begin{array}{c}\text{2 molecules of}\\ \text{nickelous}\\ \text{hydroxide}\end{array} + \begin{array}{c}\text{2 hydroxyl}\\ \text{ions from}\\ \text{the electrolyte}\end{array} \longrightarrow \begin{array}{c}\text{2 molecules of}\\ \text{nickelic}\\ \text{hydroxide}\end{array} + \text{2 electrons}$$

Thus for every 2 electrons passing round the circuit the ions of 2 potassium hydroxide molecules separate and react as above, but it will be noticed that 2 other potassium hydroxide molecules are produced at the cathode, so that there is no net change in the total concentration of the electrolyte. For every 2 electrons passing, the net reaction is simply:—

$$Fe(OH)_2 + 2Ni(OH)_2 \longrightarrow Fe + 2Ni(OH)_3$$

On discharge, electrons flow round the external circuit from the negative to the positive plate for the same reason as with the cells previously discussed; namely, that a surplus of electrons exists on the negative plate, and positive ions on the positive plate. A double layer of electric charge exists at each plate, giving rise to an e.m.f. The double layer is produced at the negative plate by ferrous ions, Fe^{++}, entering the solution, leaving electrons behind. At the positive plate it is produced by potassium ions, K^+, collecting on the surface of the positive active material, leaving a surplus of hydroxyl ions, OH^-, in the solution.

As electrons flow round the external circuit from the negative to the positive plate, more ferrous ions are able to enter the solution, where they can react with the potassium hydroxide as follows:—

$$\begin{aligned}\mathrm{Fe} + (2\mathrm{K}^+ + 2\mathrm{OH}^-) &\longrightarrow (\mathrm{Fe}^{++} + 2\text{ electrons}) + (2\mathrm{K}^+ + 2\mathrm{OH}^-)\\ &\longrightarrow (\mathrm{Fe}^{++} + 2\mathrm{OH}^-) + 2\mathrm{K}^+ + \begin{matrix}\text{2 electrons which pass}\\ \text{round the circuit}\end{matrix}\end{aligned}$$

At the positive plate the electrons arriving there allow more potassium ions to come from the solution to the plate, where they attack the nickelic hydroxide, removing 2 hydroxyl ions from each molecule, these hydroxyl ions entering the solution and neutralising the potassium ions. The two electrons are taken up by the nickel ions, which are thus changed from triply-charged nickelic ions to doubly-charged nickelous ions, as follows:—

$$(2\mathrm{Ni}^{+++} + 6\mathrm{OH}^-) + \begin{matrix}\text{2 electrons which arrive}\\ \text{from the negative plates}\end{matrix} \longrightarrow (2\mathrm{Ni}^{++} + 4\mathrm{OH}^-) + 2\mathrm{OH}^-$$

Once again there is no net change in the total concentration of the electrolyte since, although the ferrous ion attacks 2 molecules of potassium hydroxide at the negative plate, yet 2 potassium ions are liberated, and these, together with the 2 hydroxyl ions liberated at the positive plate, are actually equivalent to 2 potassium hydroxide molecules. The net total change on discharge for the passage of 2 electrons is simply:—

$$\mathrm{Fe} + 2\mathrm{Ni(OH)}_3 \longrightarrow \mathrm{Fe(OH)}_2 + 2\mathrm{Ni(OH)}_2$$

Exercises on Chapter VI

1. If you wished to charge a 12-volt motor-car battery at approximately 6 amp. from a 230-volt d.c. supply by connecting the battery and supply in series with a group of 60-watt, 220-volt carbon filament lamps in parallel, how many lamps would you employ?
[22.]

2. If a battery of 65 storage cells is being charged by a d.c. generator with a terminal p.d. of 160 volts, what will be the charging current, assuming the e.m.f. and internal resistance of each cell

while on charge to be 2·25 volts and 0·005 ohm respectively, and the resistance of the leads from generator to battery 0·075 ohm?

[About 34·4 A.]

3. Calculate the supply voltage necessary for charging a battery of 110 cells at 30 amp. (a) at the beginning of the charge and (b) at the end of the charge, if each cell possesses a terminal p.d. of 2·1 volts at the beginning and 2·7 volts at the end of the charge. Allow 0·06 ohm for the resistance of the connecting leads.

[(a) 232·8 V; (b) 298·8 V.]

4. An alkaline battery of 40 cells is to be charged at 15 amp. The e.m.f. of each cell is 1·40 volts at the commencement of the charge and 1·75 volts at the end of the charge, the internal resistance remaining constant at 0·007 ohm per cell. Find the supply p.d. required to charge this battery (a) at the commencement of the charge and (b) at the end of the charge. The resistance of the connecting leads is 0·10 ohm.

[(a) 61·7 V; (b) 75·7 V.]

5. A lead-acid battery of 50 cells is to be charged at 25 amp. from a 200-volt d.c. supply, using a series rheostat. If the e.m.f. of each cell may be taken as 1·95 volts at the commencement of the charge and 2·55 volts at the end of the charge, find the range of series resistance values required. The internal resistance of each cell may be assumed to remain constant at 0·002 ohm and the resistance of the connecting leads may be neglected.

[4·0 Ω to 2·8 Ω.]

6. If the e.m.f. of a lead-storage cell varies between 2·10 and 1·99 volts when allowed to discharge, and that of a nickel-cadmium alkaline cell between 1·35 and 1·11 volts, the internal resistances in the two cases being 0·001 and 0·003 ohm respectively, calculate how many cells will be required to deliver a current of 40 amp. at a terminal p.d. of 110 volts (1) at the commencement of discharge, (2) at the end of discharge, if the battery is composed of (a) acid cells, (b) alkaline cells.

[(1a) 54; (1b) 90; (2a) 57; (2b) 112.]

7. Calculate the ampere-hour and watt-hour efficiencies of a lead-acid cell which when charged for 10 hours using a current of 18 amp. at an average p.d. of 2·26 volts is discharged in the same time by a current of 17·2 amp., the average p.d. during discharge being

1·98 volts. Use Fig. 114 to estimate what current would be necessary to discharge this cell at a 1-hour rate.

[95·5%; 83·7%; 86 A.]

8. A lead-acid cell gave the following results during a discharge test:—

Time (hr.) .	0 (switched on)	0·25	0·5	1·0	2·0	3·0	4·0	5·0
Current (amp.)	20·1	19·6	19·55	19·5	19·4	19·2	18·85	18·2

During the subsequent charge the current was kept constant at 12·5 amp. for 9 hours and reduced to 7·5 amp. for the remaining hour. Determine the ampere-hour efficiency of the cell under these conditions.

[79·9%.]

9. A lead-acid cell is discharged at a constant current of 12 amp. for 10 hours and subsequently charged at the 8-hour rate by a constant current of 16·5 amp. The terminal p.d. of the cell varies during discharge and charge in the following manner.

Time (hr.) . .	0	0·25	0·5	1	2	3	4	5	6	7	7·5	8	9	10
Discharge p.d. (V)	2·05	2·01	2·005	2·0	2·0	2·0	1·995	1·985	1·975	1·965	--	1·95	1·92	1·85
Charge p.d. (V) .	2·05	2·14	2·18	2·20	2·21	2·22	2·23	2·24	2·26	2·38	2·60	2·65	—	—

Determine (a) the ampere-hour efficiency and (b) the watt-hour efficiency of the cell under these conditions.

[(a) 90·9%; (b) 79·1%.]

10. A battery is to have a capacity of 150 Ah at the 10-hour rate. Using Fig. 114, determine the approximate discharge current available from the battery (1) at the 2-hour rate and (2) at the 1-hour rate, if the battery is composed of (a) lead-acid cells, (b) nickel-iron alkaline cells and (c) nickel-cadmium alkaline cells.

[(1a) 48 A; (1b) 67·5 A; (1c) 75 A; (2a) 75 A; (2b) 112 A; (2c) 148·5 A.]

CHAPTER VII

MAGNETIC FIELDS

65. The Importance of Magnetic Fields

Electro-technology is concerned with the *production* and *use* of electrical energy, which is normally *produced* with the aid of a *magnetic field*, in a machine known as a generator (or dynamo). Electrical energy owes much of its importance to its *use* in connection with the electric motor, but only by employing magnetic fields are we able to use electrical energy to drive electric motors. It will be seen, therefore, that the magnetic field occupies a position in electro-technology almost as important as the electric current itself. As a matter of fact, whenever a current is caused to flow in a conductor a magnetic field is simultaneously brought into existence. So it is essential at this stage that the student should acquire a clear conception of what is meant by the term *magnetic field.* It is difficult, however, to form a conception of what the term implies without first considering the characteristics of a simple magnet.

66. Magnets

A magnet is a piece of iron, steel, cobalt or nickel possessing the properties outlined below.

(a) A magnet attracts small pieces of iron, steel, cobalt or nickel, and these metals, together with a number of alloys containing iron, are the only substances capable of being strongly magnetised. They are known as **ferro-magnetic** materials.

(b) The centres of attraction are near the ends of the magnet. This may be verified by placing a magnet in a heap of iron filings; on removal of the magnet the iron filings will be seen to be clinging, in two clusters, to the ends of the magnet. The actual centres of attraction, known as the *poles* of the magnet, may be exactly located by placing a small compass needle in various positions near each end of the magnet; the compass needle will always point towards the pole situated at the end at which the compass needle is placed (see Fig. 121). The line joining the poles is called the *magnetic axis* of the magnet.

(c) When suspended so as to be free to rotate about a vertical axis, the magnet sets itself in a specific direction, which varies at different parts of the earth's surface, but which is always roughly North and South. In other words, a magnet possesses the compass property. A compass needle is merely a magnet suspended so as to be free to rotate in a horizontal plane. The pole of a magnet which points towards the north when the magnet is free to rotate in a horizontal plane is known as the *north-seeking pole*, or, more briefly, the **north pole,** and the other as the **south pole.**

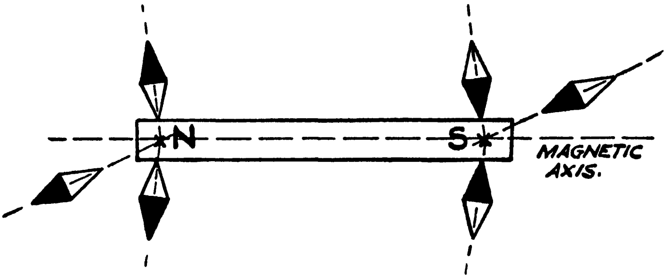


FIG. 121.—LOCATION OF THE POLES IN A BAR MAGNET.

(d) When a magnet, A, is freely suspended and the north and south poles of another magnet, B, are brought in turn up to each of the two poles of the suspended magnet, it is found that similar poles repel each other, whereas dissimilar poles attract each other. The apparatus used for demonstrating this effect is illustrated in Fig. 122—it is fairly obvious that the stand used for supporting the magnet A in this and similar demonstrations should not be made of iron or steel, otherwise forces of attraction will exist between the poles of the magnet and adjacent parts of the stand.

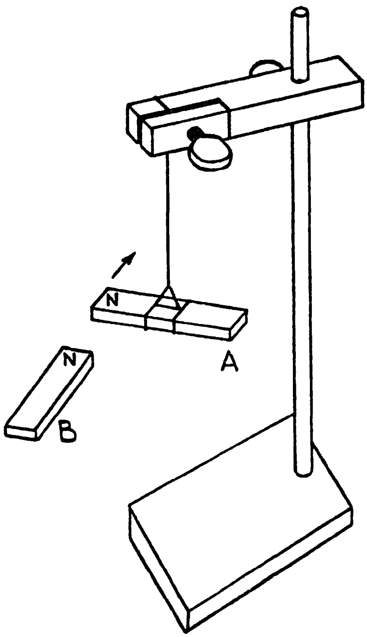


FIG. 122.—APPARATUS FOR DEMONSTRATING REPULSION AND ATTRACTION BETWEEN MAGNETIC POLES.

(e) A magnet may be used to confer its properties on other ferro-magnetic materials.

(f) A single pole cannot be isolated. One cannot have a N. pole without a S. pole. Poles exist only in pairs, and the two poles of a magnet are equal in strength.

A *pole of unit strength*, termed a **unit pole,** is defined as that pole which, when placed in air 1 cm. from an identical pole, repels it with a force of 1 dyne. The **pole strength** of any other pole is measured by the force in dynes with which it would repel a unit pole if placed 1 cm. away from it. No name is given to the unit of pole strength, it is simply termed a c.g.s. unit; we might, for example, refer to a pole strength of 25 c.g.s. units.

67. The Nature of Magnetic Fields

The reason that a magnet can attract small pieces of ferro-magnetic materials, that two similar poles repel each other and that a magnet may be used to confer its properties on other materials capable of being magnetised, is that the space round a magnet is not in its normal state. That this is so will be seen if a magnet be laid out flat on a bench, a sheet of cardboard placed over the magnet and iron filings sprinkled over the cardboard. On tapping the cardboard the filings will arrange themselves in curved lines as shown in Fig. 123. These curved lines are called **lines of magnetic force.** Compass needles placed in various positions around the magnet will also rotate until they lie along the curved lines taken up by the iron filings; if the magnet be removed the compass needles do not stay in these positions. The fact that the magnet causes iron filings and compass needles to move into special directions shows that the magnet exerts forces on the filings and compass needles. These forces are very strange ones, since it is impossible to see by what mechanism the forces act. Forces in machinery are usually conveyed along rods, by way of belts or through gear-wheels; a horse tows a barge by pulling on a rope; a boy propels a bicycle because the force he exerts on the pedals is conveyed to the rear wheel by means of the cranks, gear-wheel and chain. But it was seen that the compass needles and iron filings in the vicinity of the magnet moved without any visible mechanical means. It is very mysterious that when a magnet is held over a nail, the nail should jump up towards the magnet; there is no obvious mechanism acting upon the nail.

We are compelled to assume that empty space possesses uncanny properties. It is as though a magnet held over a nail causes the space round the magnet, and therefore in the vicinity of the nail, to be " strained " in some manner; it is as though there is a vortex in the space which sucks up the nail. The compass needles and iron filings placed in the vicinity of the magnet are sucked round into special directions. The lines of magnetic force give some indication of the pattern of the " strain ". It is this unusual state of space which is known as a **magnetic field.** The *direction of a magnetic field* at any point may be defined as the direction indicated by the N. pole of a compass needle if placed at the point.

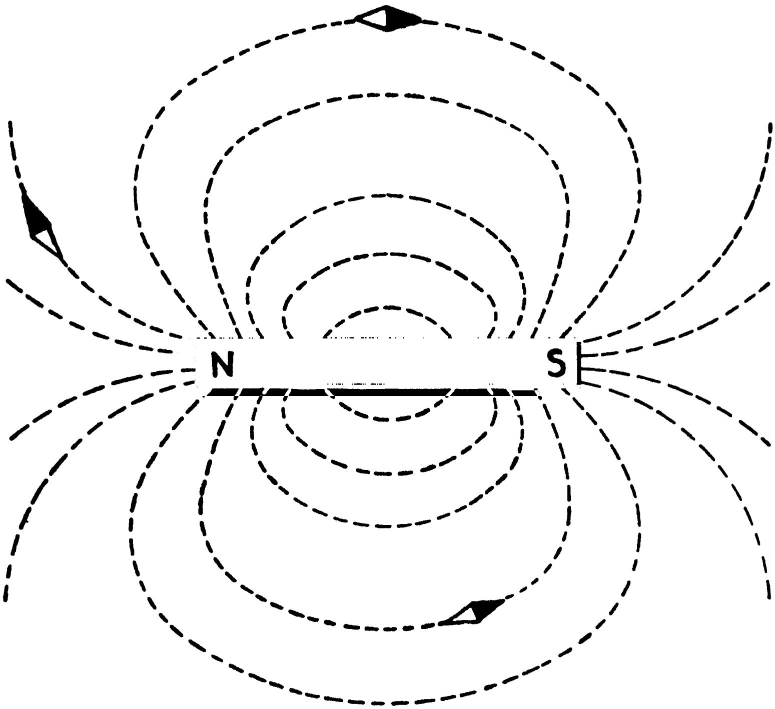


Fig. 123.—Lines of Magnetic Force Round a Bar Magnet.

The reason a compass needle swings round into the direction of a line of magnetic force is that there is a force, F dynes, acting in one direction on the N. pole and an equal force acting in the opposite direction on the S. pole. Thus a *couple* acts upon the compass

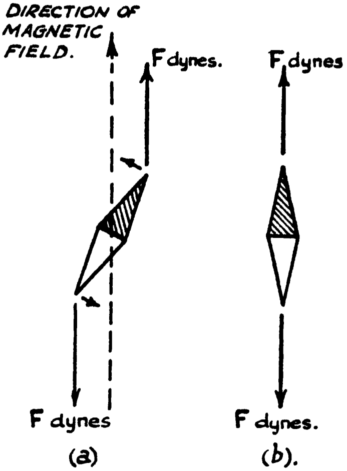


Fig. 124.—The Behaviour of a Compass Needle in a Magnetic Field.

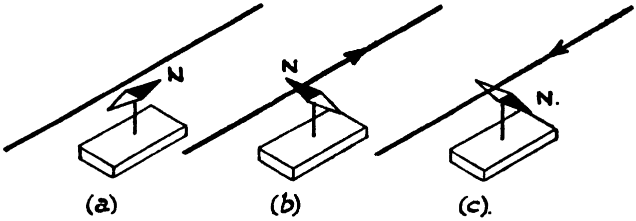


Fig. 125.—The Behaviour of a Compass Needle Near a Current-carrying Conductor.

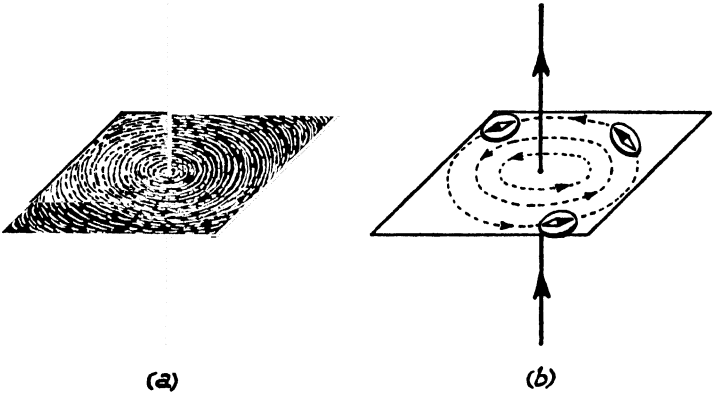


Fig. 126.—The Magnetic Field Round a Straight Current-carrying Conductor.

needle, causing it to turn from the state shown in Fig. 124a to that shown in Fig. 124b.

The force in dynes exerted on a unit N. pole placed at a point in a magnetic field is defined as the **magnetic force** (symbol: H). The unit of magnetic force is a *dyne per unit pole.* This is commonly referred to simply as a **c.g.s. unit** or alternatively as an **Oersted.** The term **magnetising force** is synonymous with magnetic force, see p. 229. A *line of magnetic force* may now be defined as a line drawn in a magnetic field such that its directions at various points along its length are the directions of the magnetic force at those points. The magnetic force, in general, varies in strength and direction from point to point along a line of magnetic force.

The reason a compass needle, when placed at a point on the earth's surface well removed from ferro-magnetic materials, takes up a position roughly N. and S. is that, owing to a number of causes, the earth behaves as a huge magnet, and so possesses a magnetic field of its own. The lines of magnetic force in the earth's field run roughly N. and S., and although the earth's magnetic force is comparatively weak, it is sufficient to rotate the magnetic axis of the compass needle into the direction known as the *magnetic meridian.*

68. Magnetic Fields Round Current-carrying Conductors

It is impossible to produce an electric current without also producing a magnetic field. Thus when a current flows through a wire, not only do things happen within the wire itself, but changes occur in the surrounding space. That this is the case may be readily tested by arranging a length of wire above and parallel to a pivoted compass needle, as in Fig. 125a. On passing a current through the wire the magnetic needle swings round until it lies roughly at right angles to the length of the wire. If the direction of the current is reversed the compass needle reverses, as illustrated in Fig. 125 (b) and (c).

By arranging a straight conductor in a vertical position then with the aid of a piece of cardboard and iron filings or a compass needle, the magnetic field pattern may be exhibited as shown in Fig. 126 (a) and (b). The advantage of using a compass needle is that the direction in which the N. pole points gives the direction of the lines of magnetic force. If such an experiment be carried out it will be found that the lines of magnetic force are all circular. Fig. 127 illustrates the direction of the lines of magnetic force round a straight current-carrying conductor; in Fig. 127a the current is supposed to be flowing down the wire or into the paper, this being represented in the conventional manner by a cross, whereas in Fig. 127b the dot represents current flowing upwards. Maxwell's *corkscrew rule*

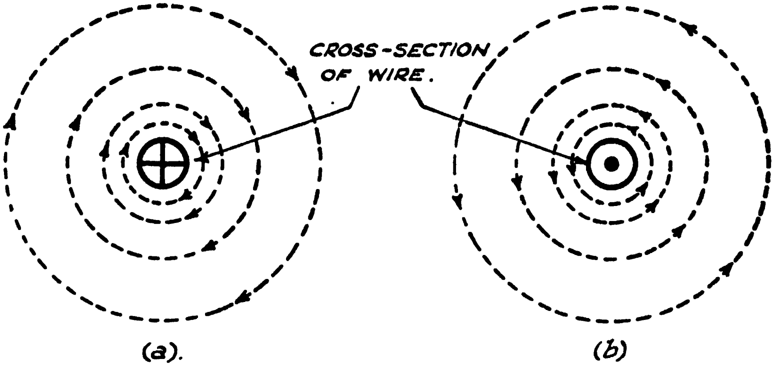


FIG. 127.—MAXWELL'S CORKSCREW RULE.

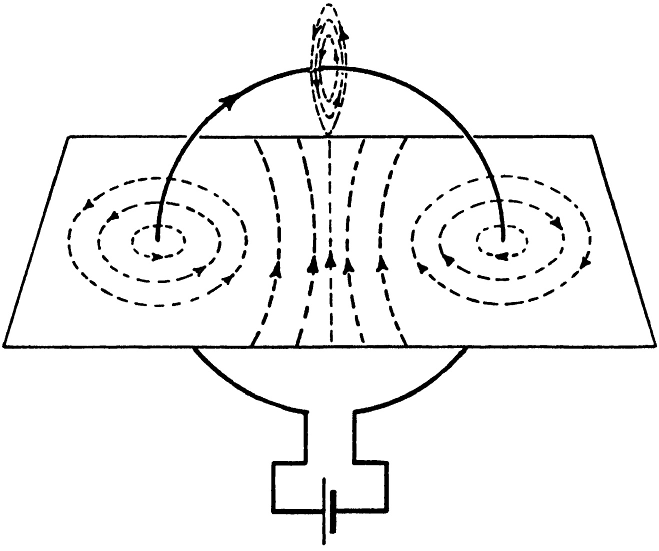


FIG. 128.—THE MAGNETIC FIELD ROUND A CIRCULAR CURRENT-CARRYING CONDUCTOR.

enables one to remember the direction of the lines of magnetic force. It states that if one imagines a right-handed screw to be screwed along the wire in the direction of the current, then the direction of the magnetic field is given by the direction of rotation of the screw.

It should be noted that in the case of a straight conductor, unless the current is exceptionally large, the magnetic force at any point is very weak. As one might expect, the magnetic force is strongest at points near to the conductor.

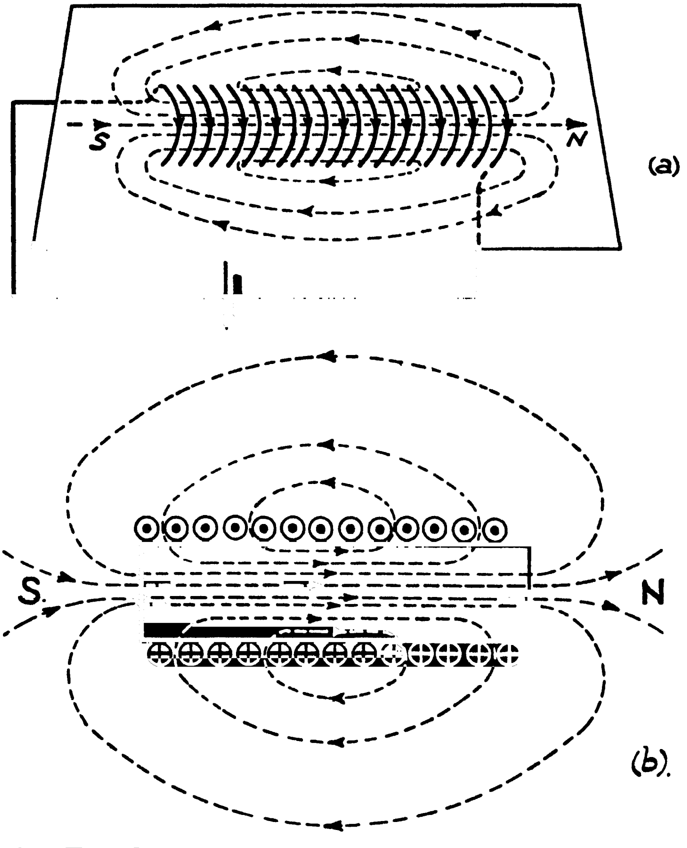


FIG. 129.—THE MAGNETIC FIELD IN THE VICINITY OF A CLOSELY-WOUND CURRENT-CARRYING SOLENOID.

Fig. 128 shows the magnetic field pattern in the space surrounding a conductor bent in the form of a circle. The lines of magnetic force at the very centre of the circular loop lie along the axis of the loop, i.e., their direction is at right angles to the plane containing the conductor. Very close to the conductor the lines take the form of small rings which are almost circles, but the larger rings do not even approximate to a circular form.

The magnetic field in the space within and around a current-carrying solenoid is of considerable importance, as will be seen later. A *solenoid* is a long coil of closely-wound circular turns as shown in Fig. 129. Both Fig. 129a and Fig. 129b, which represents a sectional view, depict the arrangement of the lines of magnetic force caused by a current flowing through a solenoid. The lines of magnetic force are distributed very much as they are round a bar magnet, and the solenoid may be regarded as possessing N. and S. poles as shown in Fig. 129b. When the turns on a solenoid are not closely wound, some of the lines of magnetic force enclose one turn only, as illustrated in Fig. 130. When the current viewed from one end of a solenoid rotates in a clockwise direction, that end is the S. pole. Viewed from the other end the current rotates in a counter-clockwise direction, and that end is a N. pole. This rule may be remembered by writing the letters S. and N. and putting arrows at the ends of the letters as shown in Fig. 131.

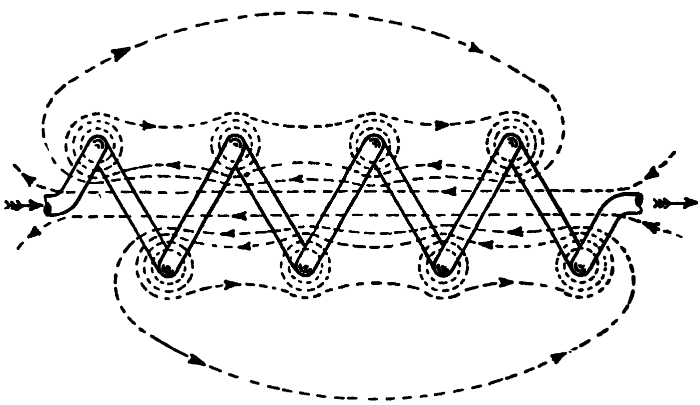


FIG. 130.—THE MAGNETIC FIELD IN THE VICINITY OF AN OPEN CURRENT-CARRYING SPIRAL.

Alternatively, the corkscrew rule may be applied to the solenoid. In this case the R.H. screw is assumed to rotate in the direction of current flow, and advances in the direction of the lines of magnetic force existing *inside* the solenoid.

A solenoid wound to form a closed ring (known as a *toroidal* coil) as in Fig. 132 exhibits no polarity, and the lines of magnetic force, which are circular, are more or less confined to the space within the coil. In the space outside the coil the magnetic force is quite negligible.

Experiment 15. *To show that the magnetic fields round a bar magnet and a straight current-carrying solenoid are similar.*

Divide a sheet of cardboard of dimensions 12 in. × 9 in. into two equal parts by the line XY. Cut a slot in the centre of each of the two halves so as to take the magnet and solenoid to be used in the experiment, as illustrated in Fig. 133. These should each be 3 or 4 ins. in length, the solenoid possessing 100 turns or more. A sheet of paper should be attached to the cardboard by means of a drawing-pin in each corner, the paper having the slots and line XY to correspond with the cardboard. With the aid of a compass needle, arrange the cardboard so that the line XY lies along the magnetic meridian.

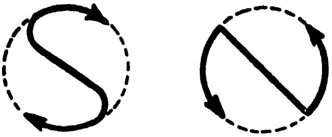


Fig. 131.—Rule for the Polarity of a Solenoid.

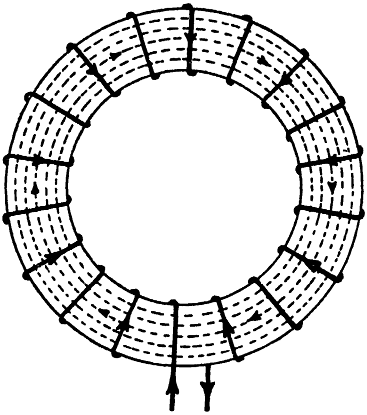


Fig. 132.—The Magnetic Field within a Toroidal Coil.

Insert the bar magnet, with its N. pole pointing south, in one of the slots, raising the cardboard above the bench slightly so that half the magnet lies below and half above the plane of the paper attached to the cardboard. Place a small compass needle near the N. pole of the magnet, and make a pencil mark on the paper vertically below the N. pole of the needle. Then move the needle forward until its S. pole is just over this mark. Make another mark below the N. pole of the needle in its new position. Again move it forward until its S. pole is over this mark. Repeat the process until the line reaches the S. pole of the magnet or until the line runs off the half of the paper being used for the magnet. Afterwards join the points together to produce a line of magnetic force. Draw other lines by the same process until a pattern similar to that shown in Fig. 133 is obtained. Remove the magnet to a considerable distance and insert the solenoid in the other slot. Pass a current of at least 5 amp. through the solenoid in such a direction that its S. pole points towards the north. Plot lines of magnetic force using the process described in connection with the magnet. If the above instructions are followed, the earth's field will have a negligible effect on the two field patterns, which will be found to be almost identical. Finally, the sheet of paper may be removed from the cardboard and filed with the student's notes on the experiment.

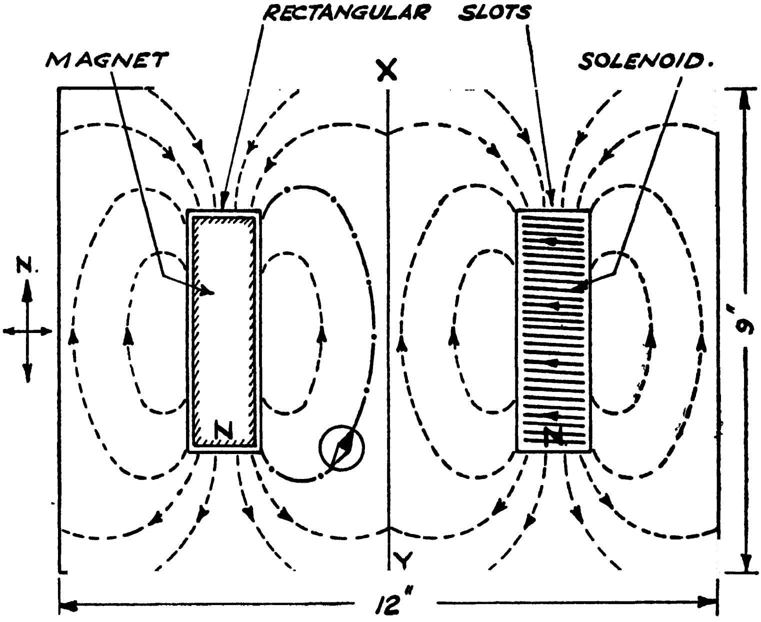


FIG. 133.—APPARATUS USED IN EXPT. 15.

69. Magnetic Flux

A number of lines of magnetic force is referred to as a *magnetic flux* (symbol Φ), and so the c.g.s. unit of magnetic flux is 1 *line*, sometimes called 1 *Maxwell.* Thus we might say that emerging from a given solenoid is a magnetic flux of 1500 lines or 1500 Maxwells. Magnetic flux can be measured with a *fluxmeter*—an instrument which has the appearance of a special galvanometer. In using a fluxmeter, some form of *search coil* must be employed. Thus to measure the flux emerging from the N. pole of a magnet, a coil of 50 to 100 turns should be wound to fit fairly closely over the magnet, but not so closely that it cannot slide freely along the magnet. This coil should then be placed over the magnet at its mid-point, the free ends being connected to a fluxmeter (see Fig. 134). If the coil be then slipped off the magnet and removed to a point some considerable distance from it, the pointer of the fluxmeter will be deflected through a number of divisions, representing a large number of *Maxwell-turns.* By dividing this number of Maxwell-turns by the number of turns on the search coil, the amount of magnetic flux emerging from the magnet can be determined.

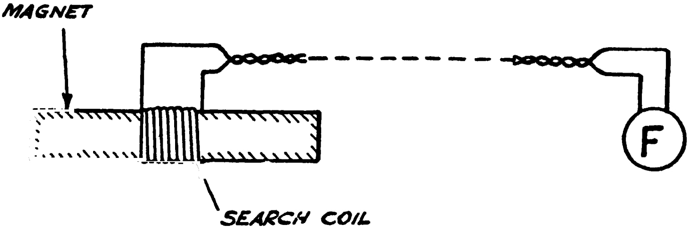


FIG. 134.—USE OF FLUXMETER.

70. Magnetic Flux Density

It will be observed by referring to Fig. 129b that at a point within a current-carrying solenoid the *crowding of the lines* is much greater than at a point outside the solenoid. But the *force* that would be exerted on a unit magnetic pole placed at a point within the solenoid is also much greater than the force that would be exerted on a unit pole placed at a point outside the solenoid, as could be observed by placing a compass needle in the two positions, since a compass needle oscillates more rapidly the stronger the magnetic force acting upon it. We have seen that the reason a pole placed in a magnetic field experiences a force is that the space is *strained* in some manner. We might expect that the greater the *strain* the greater the force the magnetic pole will experience. Thus it would appear that space is strained more within the solenoid, where the lines are crowded, than outside the solenoid, where the lines are not so crowded. For this reason we express this strain in terms of the number of lines passing through 1 sq. cm. of area in a plane at right angles to the direction of the lines, i.e., we use the number of lines per sq. cm. as a quantitative measure of the state of *strain.*

For simplicity, we *assume* that where the magnetic force *at a point in air* is 1 dyne per unit pole the *magnetic strain* is 1 *line per sq. cm.* The number of lines per sq. cm. in any region is known as the **magnetic flux density** (symbol: B). The c.g.s. unit of magnetic flux density, **a line per sq. cm.**, is sometimes referred to as a **Gauss.** It should be realised that, in order to aid us in visualising the *amount* of strain in any space, we have *imagined* that the number of lines in any given magnetic field is limited and quite definite. The ratio of the magnetic flux density, B gauss, at a point in a medium, to the magnetic force, H oersteds, at the same point is termed the **permeability** (symbol μ) of the medium under the given conditions. When the medium is *air* (or, more strictly, *a vacuum*), for which the permeability is denoted by μ_0, it follows from the statement above that, using the c.g.s. system of units, *the permeability* (μ_0) *is unity.*

71. The Distinction between Magnetic Force, H, and Magnetic Flux Density, B

Because *at a point in air* where the magnetic force is 4 dynes per unit pole we assume the flux density to be 4 lines per sq. cm., it might be thought that flux density and magnetic force are different aspects of the same thing and that one of them is superfluous. It is therefore perhaps desirable to state at this stage, although it is primarily a matter for Volume II, that at a point within a piece of iron, subject to a magnetic field, the numerical values of B and H are not the same; in such a case, the flux density, measured in lines per sq. cm., may well be many hundreds of times the magnetic force, measured in dynes per unit pole. The magnetic force within a piece of iron is usually defined as the force that would be exerted on a unit N. pole placed in an infinitely thin channel cut in the iron in the direction of the lines of magnetic force, as illustrated in Fig. 135. To measure such a force is, of course, a practical impossibility, but in practical cases, where it is desired to know the magnetic force within a piece of iron, it can usually be calculated (see Vol. II, Chapter II).

An analogy perhaps affords the best way of showing the inherent difference between magnetic force, *H*, and flux density, *B*. When any *stress* is applied to an object it suffers from *strain*. An object subjected to a force becomes distorted. Some materials show a large amount of strain when subjected to quite a small stress. Consider, for example, a rubber cord suspended from a spring balance as in Fig. 136. Suppose that when a weight of 4 lb. is hung from the bottom of the cord, the latter is stretched 4 in. Through the medium of the rubber cord the force of 4 lb. wt. is applied to the spring balance, which immediately registers an increase of 4 lb. wt. The stretch of 4 in. is caused by attaching the 4 lb. weight, but only by the stretching of the rubber is the force of 4 lb. wt. conveyed and applied to the spring balance. The rubber cord exerts a downward force of 4 lb. wt. on the spring balance, which in turn exerts an upward force of 4 lb. wt. on the cord. So immediately the 4 lb. wt. is suspended from the rubber cord the latter is subjected to two equal and opposite forces of 4 lb. wt. as shown in Fig. 136(c), and is simultaneously stretched. The stress and strain are interdependent—they go hand in hand. But no one is likely to confuse the force of 4 lb. wt. on the spring balance with the 4 in. stretch of the rubber cord.

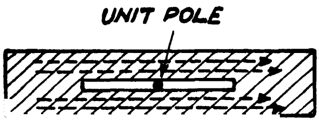


FIG. 135.—ILLUSTRATING THE CONCEPTION OF MAGNETIC FORCE IN IRON.

Consider now the case of a straight conductor carrying no current and a unit N. pole placed a short distance from it as in Fig. 137. Immediately a current is passed through the conductor the unit pole experiences a force. This force is only conveyed to the pole through the straining of the intervening space. Yet it is equally correct to regard the strain, i.e., the magnetic field, as having been set up as a result of the magnetic forces caused by the current flow. An important point of similarity with the analogy is that equal and opposite forces exist at opposite ends of the intervening medium; for, if the unit pole experiences a force of, say, 4 dynes, so does the conductor carrying the current, but in practice the conductor is usually rigidly fixed, and so does not move and thus demonstrate the existence of this force. (The forces experienced by current-carrying conductors are dealt with in Chapter VIII, Section 80.) It is obvious that the magnetic force on the unit pole corresponds to the force on the spring balance, and the magnetic field corresponds to the stretching of the cord. In other words, magnetic force corresponds to the stress and flux density to the strain in the analogy.

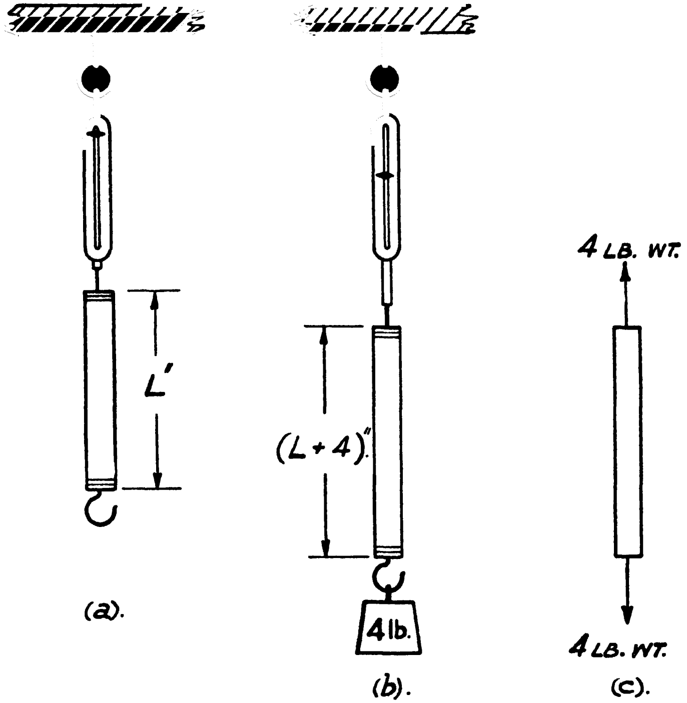
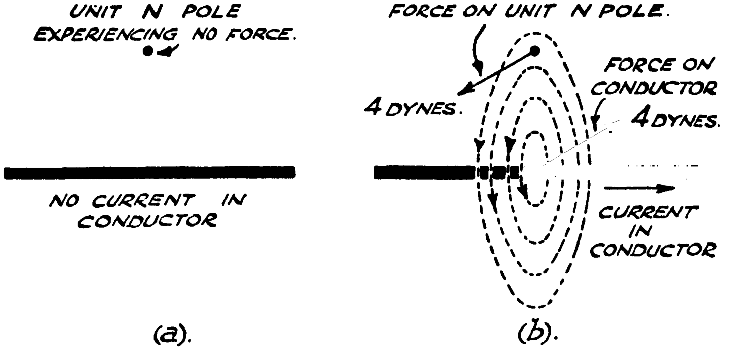


FIG. 136.—ANALOGY ILLUSTRATING THE DIFFERENCE BETWEEN B AND H.

72. The " Ether "

In discussing magnetic fields we have regarded a magnetic field as a strained condition of empty space. Since it is difficult to visualise empty space being *strained*, many scientists and engineers regard empty space as being filled with a medium called the *ether*. The medium has no material properties, and so cannot be perceived by the senses; its existence can only be deduced from phenomena such as those described in this and the next chapter. The *ether*, or empty space, certainly possesses electromagnetic properties, and so it is often referred to as the *electromagnetic medium*. It is merely a matter of personal preference as to whether one speaks of empty space, of the ether or of the electromagnetic medium. The properties of this medium are of great significance not only to electrical engineers, but also to physicists, and should be of interest to everyone, since radiant heat, light rays, wireless waves and X-rays are all waves in this electromagnetic medium. We depend upon the properties of this medium for warmth, vision and entertainment!



(a). (b).

FIG. 137.—THE ACTION AND REACTION BETWEEN A MAGNETIC POLE AND A CURRENT-CARRYING CONDUCTOR.

73. Formulæ for the Magnetic Force near Certain Current-carrying Conductors

Proofs of formulæ in electromagnetism usually involve a knowledge of the calculus. Only in very exceptional cases will a student reading this chapter for the first time possess this knowledge. So the reader must take for granted the formulæ stated in this section. The proofs of these formulæ are actually given in the Addendum at the end of this volume. The reader is advised to study these proofs after he has studied the integral calculus, which will usually not be for some time after first reading this present volume. In the meantime there is no reason why the student should not become acquainted with formulæ, a knowledge of which will definitely be required before reading Chapter II of Vol. II.

At a point a distance r cm. from an infinitely long straight conductor carrying a current of I amp., the magnetic force, H c.g.s. units, is given by the formula

$$H = \frac{2I}{10r}$$

No conductor is, of course, infinitely long, but in practice the formula holds good for a conductor which is very long compared with the distance r cm. Not only do lines of magnetic force exist outside the conductor, but they exist also within the conductor itself. The magnetic force possesses its maximum value at the surface, and is zero at the axis of the conductor, also at points well removed from the conductor as shown in Fig. 138.

At the centre of a circular conductor of n turns (bunched together so that they can be regarded as lying in one plane) *each turn having*

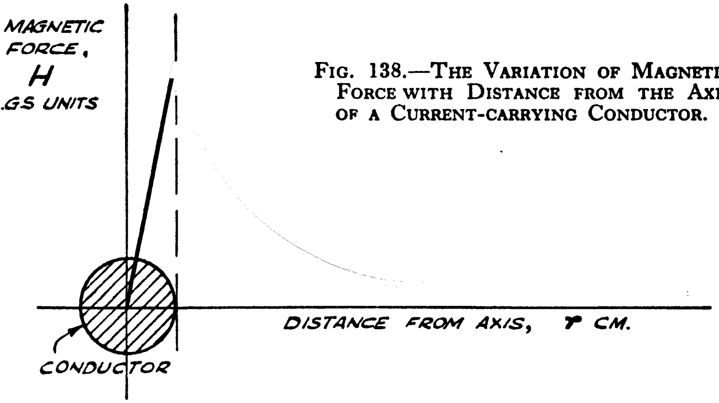


FIG. 138.—THE VARIATION OF MAGNETIC FORCE WITH DISTANCE FROM THE AXIS OF A CURRENT-CARRYING CONDUCTOR.

a radius of r cm., and carrying a current of I amp., the magnetic force H c.g.s. units, is given by the formula

$$H = \frac{2\pi nI}{10r}$$

Within an infinitely long, uniformly-wound solenoid carrying a current of I amp., and having n turns in a length l cm., i.e., one having n/l turns per cm., the magnetic force, H c.g.s. units, is given by the formula $H = \frac{4\pi}{10} \cdot \frac{nI}{l}$. No solenoid is infinitely long, but there is negligible error in using the formula in the case of a solenoid the length of which is very large compared with the diameter.

The above formula is, of course, applicable to the case where the solenoid is bent round until the ends meet, thus forming a toroidal coil—see Section 68 and Fig. 132. Thus, *on the axis* of a toroidal coil carrying a current of I amp. and having n turns uniformly spaced over a mean circumference of l cm., the magnetic force, H c.g.s. units, is given by the formula $H = \frac{4\pi}{10} \cdot \frac{nI}{l}$. If the mean radius of the toroid be r cm. (see Fig. 132), the formula becomes $H = \frac{2nI}{10r}$.

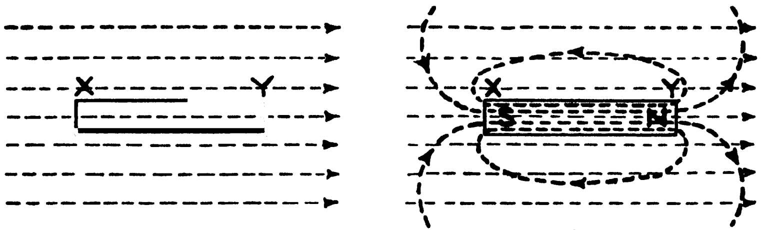
It will be observed that in each of the above formulæ the number 10 occurs in the denominator; the reason is that the ampere was originally defined to be 10 times the size of another earlier unit of current, the electromagnetic unit. In other electromagnetic formulæ various constants consisting of powers of 10 will be observed. These constants are all due to the manner in which the ampere, volt and ohm were originally defined (see Section 77).

In each of the cases above, increasing the number of conductors has exactly the same effect as increasing the current. This is a result which might be expected, since one conductor carrying a current of 15 amp., and 5 conductors, side by side, each carrying 3 amp., are essentially the same thing; in each case 15 coulombs flow past a point each second. In all cases the magnetic force at a given point in the neighbourhood of a current-carrying circuit is proportional to the product of the number of turns and the current in each turn. If we examine the formula giving the magnetic force, H oersteds, on the axis of a toroidal coil, this being a case in which the magnetic force is constant over the entire length of the magnetic path, l cm., we find that $H \propto \frac{nI}{l}$. This result is of some importance and may be expressed in words as follows: *For a magnetic path over which the magnetic force is uniform, the value of this magnetic force is proportional to the number of " ampere-turns " per unit length of path.*

74. Induced Magnetism and Methods of Magnetisation and Demagnetisation

Whenever a piece of ferro-magnetic material is placed in a magnetic field it becomes a magnet while it is in the field, that end which points in the direction of the lines of force becoming a N. pole. Fig. 139a shows a bar of iron, XY, placed in a field consisting of parallel lines of force. The fact that at different parts of this field the flux density is constant means that there is a constant magnetic force of H c.g.s. units throughout the field. The bar of iron will become a magnet with the end Y a N. pole. How strong a magnet it becomes will depend on the magnitude of the magnetic force, H. Since magnetisation is always due to the existence of a magnetic force, the expression *magnetising force* is frequently used instead of *magnetic force*, and in future this expression will usually be employed, since it is the one recommended by the British Standards Institution. Another frequently-used alternative expression is the term *magnetic field strength.* These three expressions mean precisely the same thing.

The result of magnetising the bar of iron shown in Fig. 139a is to cause extra flux to emerge from the end Y, which is now a N. pole, as shown in Fig. 139b. Thus the magnetic flux density in the space occupied by the iron has been increased from its original value in air, B_A lines per sq. cm., to a much greater value, B_I lines per cm., in the iron. B_A was equal to H but B_I may be hundreds of times greater than H. Magnetism is said to be *induced* into the bar of iron by the magnetising force, and so the greatly increased flux density is frequently referred to as *induction density*, or as *magnetic induction.* However, the expression *magnetic flux density* is more frequently used, and is the expression we shall use throughout these volumes. In the above case the fact that B_I may be, say, 500 times greater than H, means that the *permeability* (see Section 70) of the iron has a value of 500. The permeabilities of various ferro-magnetic materials differ considerably; for any given material the value of permeability varies with the flux density (see Chapter II, Vol. II).



(a). (b).

FIG. 139.—THE MAGNETISATION OF A BAR OF IRON PLACED IN A MAGNETIC FIELD.

Magnets can be made only by the above process of subjecting ferro-magnetic materials to a magnetising force. The magnetising force may be that due to another magnet, or it may be due to the flow of electric currents. Thus if a bar magnet is placed adjacent to a bar of iron as in Fig. 140, the end X of the iron will become a S. pole and the end Y a N. pole. Polarity of the opposite kind is induced in the end of a bar of iron placed adjacent to one pole of a bar magnet. A bar such as XY of Fig. 140 on becoming magnetised is capable of magnetising an iron nail, which in turn is capable of magnetising another and so on. As a matter of fact, whenever a magnet attracts a piece of ferro-magnetic material it does so by first converting it into a magnet; the effect is thus due to the attraction between unlike poles.

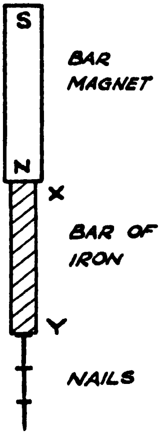


Fig. 140.—Magnetisation by Induction.

Whenever a ferro-magnetic material is subjected to a magnetising force and the magnetising force is subsequently removed, there is a tendency for the material to remain a magnet. On subjecting soft iron to a strong magnetising force quite a strong temporary magnet is produced, on account of the high permeability of this material, but on removal of the magnetising force the soft iron loses almost all its magnetism. Hard steel, on the other hand, possesses a much lower permeability, but on removal of the magnetising force quite a large portion of the acquired magnetism remains permanently. This property is expressed qualitatively by saying that hard steel possesses a higher **retentivity** than soft iron. The magnetic properties of iron are very much affected by the presence of small percentages of other metals, the matter being dealt with in some detail in Chapter II of Volume II.

Permanent magnets are made by subjecting bars of iron alloyed with other metals such as tungsten, aluminium, nickel and cobalt to the intense magnetising force produced by passing a heavy direct current through a selenoid consisting of many turns per centimetre of its length (see Fig. 141). On removal of the magnetising force much of the induced magnetism remains, although some of it is lost later due to rough usage and the minute mechanical vibrations

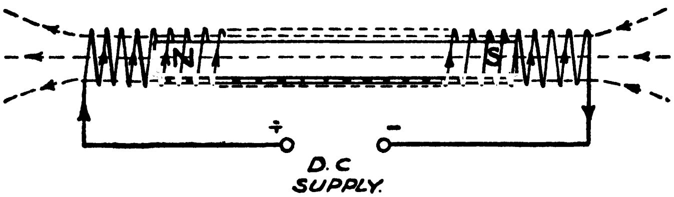
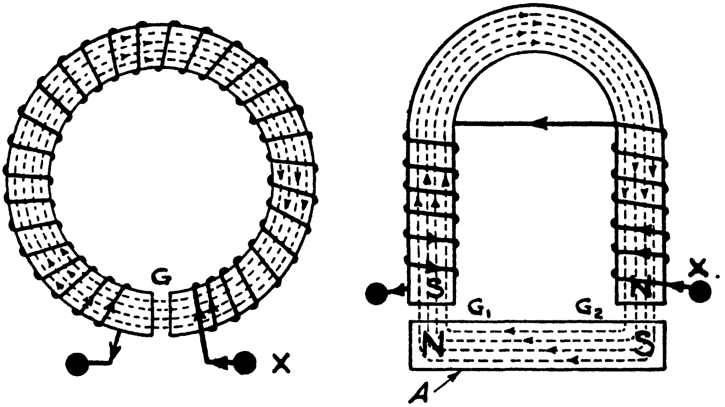


Fig. 141.—Magnetisation by Means of a Solenoid.

which are always present. Magnets used in instruments such as moving-coil ammeters and voltmeters are deliberately subjected to mechanical vibration, so that the magnetism which remains after such treatment is retained permanently.

Two methods are available for demagnetising a bar of ferromagnetic material. In the first of these methods the bar is raised to a red heat of about 800° C. and allowed to cool in the absence of a magnetic field. In order to satisfy this condition it is necessary to allow the bar to cool while lying in a direction at right angles to the earth's magnetic meridian in order to avoid magnetisation by the earth's field. This method is unsatisfactory, since the heating and cooling process is likely to affect the mechanical and magnetic properties of the material. The second method, which is much more satisfactory, is to subject the bar to a rapidly alternating magnetising force of diminishing magnitude. This may be achieved by placing the specimen in a solenoid as in Fig. 141, but with the solenoid connected to an a.c. supply, the voltage of which can be reduced slowly to zero by means of a potential slide (see Section 23; Chapter III) or some alternative arrangement. Actually it is usually unnecessary to reduce the current in the solenoid; it is quite sufficient to withdraw the specimen slowly, since as it leaves the solenoid it is subjected to an alternating magnetising force of gradually diminishing peak value.



(a) (b).

FIG. 142.—FORMS OF ELECTROMAGNET.

75. Electromagnets

The most powerful magnets available are those obtained by winding a solenoid round a ferro-magnetic core. Added to the lines produced by the solenoid are those due to the magnetised ferro-magnetic material. It is possible to produce by such an *electromagnet* more than a thousand times as many lines as would be produced by the solenoid in the absence of the ferro-magnetic core.

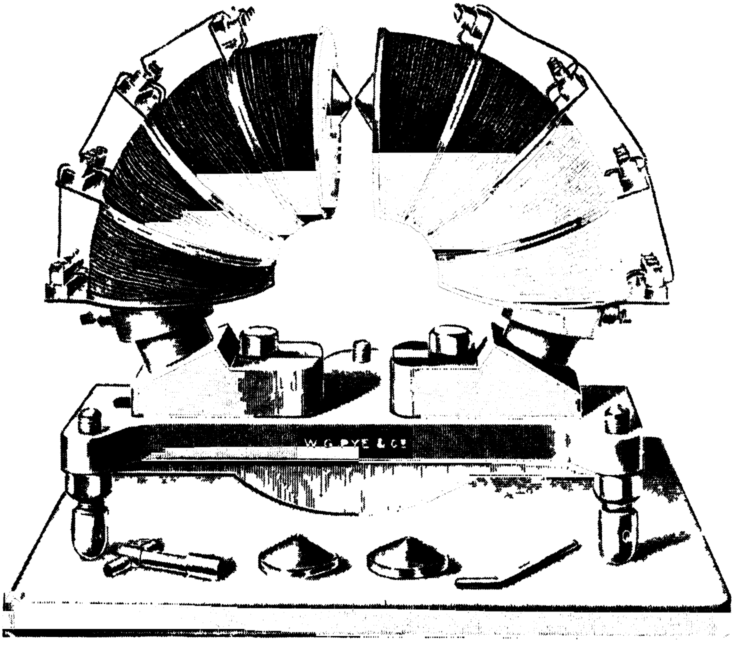
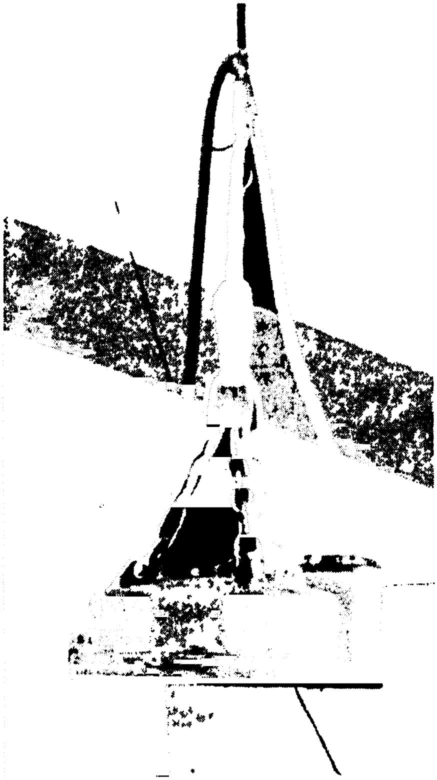


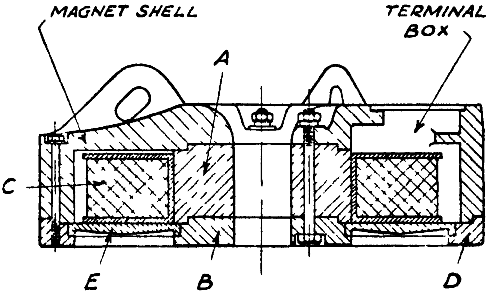
FIG 143 —LABORATORY ELECTROMAGNET

A simple form of electromagnet is illustrated diagrammatically in Fig. 142a. The core is in the form of a ring with a short air-gap G, and it carries a uniformly-distributed winding. To produce the maximum amount of magnetic flux in the core it is essential that the lines of magnetic force shall have the *shortest possible air path* (see Chapter II, Vol. II). Fig. 143 illustrates a practical example of this form of electromagnet, designed for laboratory use and capable of producing a very high magnetic flux density—of the order of 50,000 lines per sq. cm.—in the narrow space between the tips of the conical pole pieces. In this, as in all types of electro magnet, a ferro-magnetic material such as soft iron or annealed cast steel is used for the core, as the *high permeability* of such materials enables the maximum amount of magnetic flux to be produced. Another example of an electromagnet arranged on this principle is the *field magnet* used in all types of electric motor and generator (see Chapter VIII).

For many practical applications, electromagnets with U-shaped cores, as illustrated in Fig. 142b, are employed. In this case the direction of the winding, commencing, say, from terminal X, must be the same throughout, as in the ring electromagnet shown in Fig. 142a. This ensures that the magnetic lines of force pass downwards through, say, the right-hand limb of the U-shaped core and upwards through the other limb. Instead of the single short air-gap G in Fig. 142a, we have in Fig. 142b two short air-gaps G_1 and G_2 between the core and the



(*a*) Exterior view of a 26 inch dia. lifting magnet designed for under-water operation.



(*b*) Sectional view of a circular lifting magnet

FIG. 144.—LIFTING MAGNETS.

armature A, the latter being a bar of ferro-magnetic material of high permeability. It is clear that, as mentioned in the previous section in connection with Fig. 140, polarities of the opposite kind will be induced at those portions of the armature opposite the ends of the core (see Fig. 142b). A force of attraction exists between each of these pairs of opposite poles, the total force exerted between the core and armature being termed the *tractive force*. This force is utilised in each of the following practical examples of the electromagnet.

Lifting magnets are extensively used for handling pig iron, steel ingots and plates, also iron and steel scrap. Fig. 144a illustrates the external appearance of a special type of lifting magnet, designed for under-water operation in the reclamation of tools, bolts and other material inadvertently dropped into the river Thames during the construction of the new Waterloo bridge. The diameter of this magnet is 26 inches, its power consumption being 2 kW at 220 volts. The lifting capacity is about 5 tons with solid blocks of steel, but is obviously reduced when handling loose material such as bolts and tools. Fig. 144b shows the internal construction of a similar type of lifting magnet. The shell is a steel casting, to which is bolted the inner pole A and ring B, also the outer ring D forming the other pole. The winding C is housed in the recess between these poles and is protected at the lower face by a ring E of non-magnetic steel, the latter being a special alloy of iron with about 11% to 13% of manganese. When the lifting magnet is in use the current in the winding produces a magnetic flux which passes, say, downwards through the inner pole A into the iron or steel being lifted, and returns via the ring D and the shell. Very large tractive forces can be developed in magnets of this type. In this, as in most other electromagnets in which the tractive force is utilised, it is desirable to use ferro-magnetic materials possessing *low values of retentivity*. This ensures that, when the current in the electromagnet winding ceases, the tractive force falls to a value as low as possible. Soft iron and certain low-carbon steels possess this property.

Other applications of the electromagnet, similar in principle to the lifting magnet, are: (a) the *magnetic chuck*, used for holding the work on the face-plate or table of a machine tool during the machining operation, (b) the *magnetic clutch*, employed for coupling the " driving " and " driven " shafts in power transmission work, (c) *magnetic brakes*, in which the brake-shoes are held against either (i) a brake drum, or (ii) the track rails in the case of tramcars or electric trains, by the tractive force due to an electromagnet, and (d) the *magnetic separator*, used to remove stray ferro-magnetic materials from non-magnetic bulk passing over a special conveyor belt.

The electric bell. The continuous-ringing or trembler type of electric bell is illustrated diagrammatically in Fig. 145. The U-shaped core C is fixed, and carries the electromagnet winding.

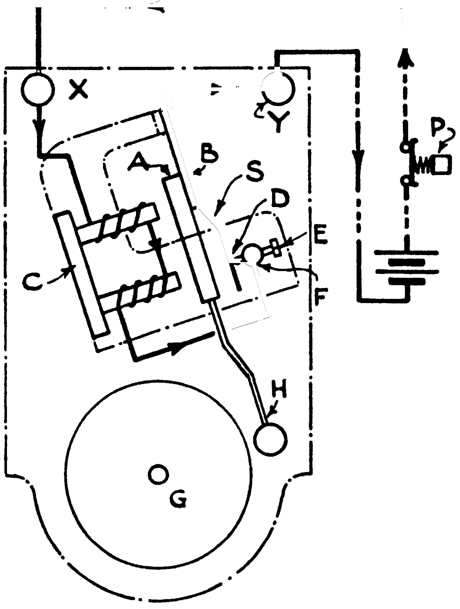


FIG. 145.—THE TREMBLER TYPE OF ELECTRIC BELL.

The armature A is supported from a bracket by means of a flat spring B, and has attached to it a flexible leaf-spring S which makes contact at D with an adjustable contact screw E, carried by the pillar F, the latter being insulated from the frame. The path for the current in the bell is from terminal X through the electromagnet windings to the pillar F and thence via the contact at D and spring S to terminal Y. When the bell-push P is pressed, current flows round the path described, and the armature A is attracted to the core C, thus separating the contacts at D. The current now ceases, and the spring B restores the armature to its original position, thus re-closing the contacts at D and enabling the cycle to be repeated rapidly for as long as the push P is pressed. A hammer H attached to the armature A strikes the gong G.

The bell indicator, used when a single electric bell may be operated by any one of a number of separate bell-pushes, is also an application of the simple electromagnet. Connected in series with each of the bell-pushes there is a solenoid with a soft iron core, as illustrated in Fig. 146, this core being capable of attracting a pivoted armature to which an indicator flag is attached. On pressing the bell-push P_1, for example, an *intermittent* current flows from the battery B through the indicator solenoid S_1 and the bell, thus causing the corresponding armature and indicator flag to " waggle " (see Fig. 147). All the bell-indicator solenoids are mounted side by side in a single box, the separate flags being visible through the glass front of the box.

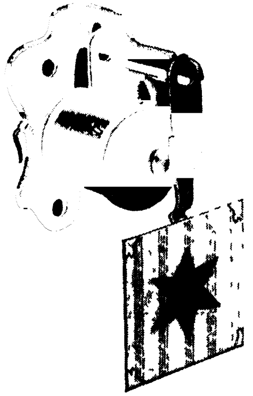


Fig. 146.—A Bell Indicator Movement.

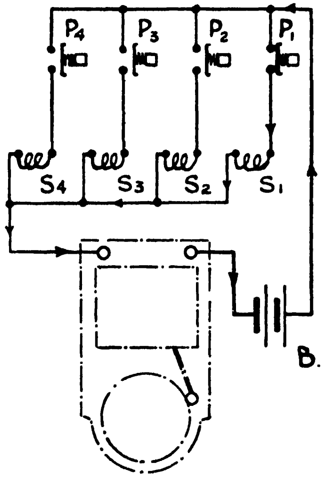


Fig. 147.—A Bell Indicator Circuit.

The electromagnetic relay is another important application of the electromagnet. The tractive force is, in this case, utilised to attract an armature whose movement is employed to " make " or " break " a single pair or a set of contacts. A typical relay, in which the movement of the armature controls a set of contacts, is shown in Fig. 148, this particular type of relay being very frequently used in telephone work. A relay may be described as a magnetically-operated switch, and since the current required to operate a relay is frequently of the order of a few milliamperes only, such devices are extremely useful when it is desired to effect changes in circuit connections from a remote point.

The circuit-breaker, a device acting on a similar principle to the relay, is used for automatically breaking a circuit when the current reaches a dangerously high value, i.e., it performs the same function as a fuse (see Section 37, Chapter IV). In a circuit-breaker the electro-magnet coil carries the main current of the circuit, and when this reaches a pre-determined value the tractive force is sufficient to move the armature, thus " tripping " a toggle device, which allows the main contacts to open under the action of a powerful spring.

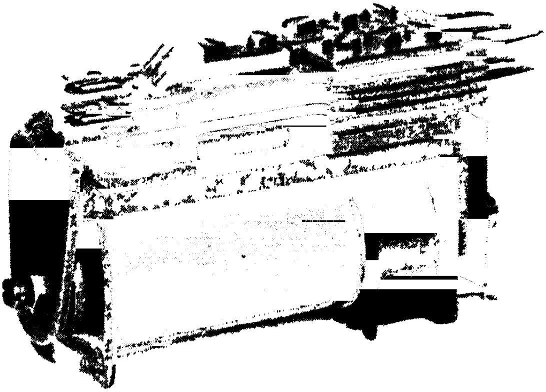


Fig. 148.—A Telephone Relay.

A further application of the electromagnet is the *automatic cut-in and cut-out*, used to control the charging current fed to a motor-car battery from the engine-driven dynamo.

This section should not be concluded without some reference to the *telephone receiver*, the action of which is based on the production of a tractive force by an electromagnet. Fig. 149 illustrates a telephone receiver, of the type used in the modern hand micro-telephone, i.e., microphone and receiver combined in one unit. A permanent magnet in the form of a short bar, composed of one of the newer permanent magnet materials (e.g., an alloy of aluminium, nickel and iron known as " alni "), is clamped to the light alloy casing by means of two nickel-iron alloy brackets, which also act as pole-pieces. Each of these pole-pieces carries a winding, the two coils being joined in series. A thin diaphragm of a special iron-cobalt-vanadium alloy (" Permendur ") or some alternative material such as a silicon steel (" Stalloy ") is clamped across the end of the metal casing by means of the moulded earpiece, a small air gap existing between this diaphragm and the pole-pieces. Thus

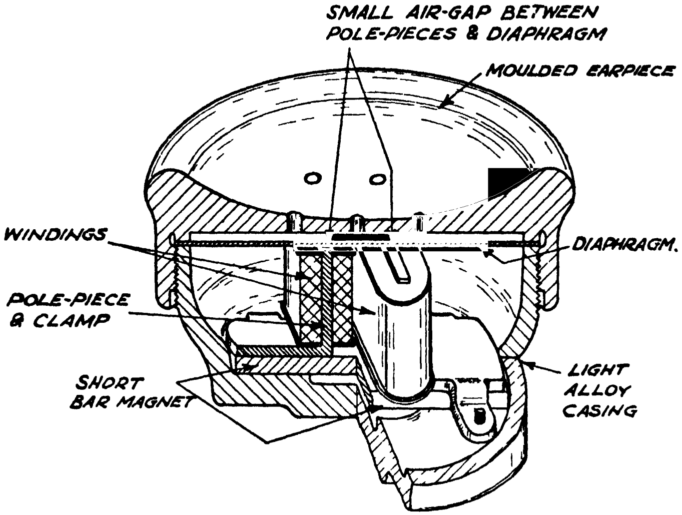


FIG. 149 —A TELEPHONE RECEIVER

the diaphragm is attracted slightly towards these pole-pieces even when no current is flowing in the coils. When the fluctuating speech current from the remote end of the line flows through these coils, variations occur in the tractive force exerted on the diaphragm, and the latter vibrates, causing sound-waves to be emitted. The reason for using a *permanently magnetised* core for the electromagnet cannot be discussed fully at this stage, but it may be stated that such a core increases the range of the variations in the tractive force exerted on the diaphragm.

There is another method by which a tractive force may be produced by electromagnetic means; this is frequently referred to as the *coil and plunger principle.* In Fig. 150, A is a solenoid and B a bar of iron partly introduced into the solenoid. If the current in the solenoid is sufficiently strong and the solenoid is firmly fixed, the bar B will be pulled into the solenoid until it occupies a symmetrical position with respect to the latter. If the direction of the current in the solenoid is as shown, the direction of the magnetic field *inside* the solenoid is from right to left, the left-hand end of the bar B acquiring N polarity, and the right-hand end, S polarity. Thus the N. pole of the bar tends to move in the direction of the lines, i.e., towards the left, and experiences a force P lb. wt. The S. pole of the bar experiences a force Q lb. wt., but towards the right and of smaller magnitude than the force P lb. wt., since the S. pole is situated in a weaker magnetic field than the N. pole. Thus the net force exerted on the bar is (P–Q) lb. wt. and the bar is pulled further into the solenoid until the forces P lb. wt. and Q lb. wt. become equal—this occurring when the bar is symmetrically placed with respect to the solenoid. It should be noted that the effect is identical, i.e., the bar is still pulled into the solenoid, when the direction of the current in the latter is reversed. The simple coil-and-plunger arrangement just described is often modified—in order to obtain a stronger and more uniform tractive force—by the introduction of a fixed iron core into the solenoid at one end. The coil-and-plunger principle is utilised in electromagnets designed to operate large circuit-breakers and magnetic brakes, and in the *attraction type* of *moving-iron measuring instrument* (see Chapter IX, Vol. II).

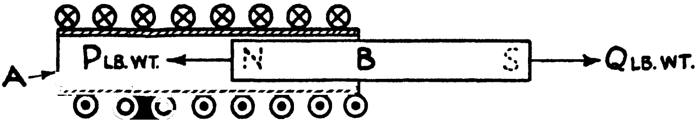


FIG. 150.—A COIL AND PLUNGER.

76. The Electronic Theory of Magnetism

One striking feature of electromagnetism is that introducing a bar of iron into a solenoid has precisely the same effect as increasing the number of ampere-turns on the solenoid; in each case there is an increase in magnetic flux. This fact is not so strange if an atom or molecule can be assumed to contain electrons rotating in orbits—a theory which helps to explain many other physical phenomena. These electronic orbits are, in effect, small circular currents, and a *circular current*, as we have seen, possesses its own magnetic field, flux emerging from one face, which may be regarded as the N. pole, and re-entering at the other face, which is the S. pole.

There are usually many electrons in an atom, and it is feasible to believe that some electrons in any one atom may rotate in one direction and some in another, so that a cancelling-out effect is possible. Thus, whereas some kinds of atoms are, in effect, small magnets, others are not. Materials, the atoms and molecules of which are magnets, are called *paramagnetic* substances, but very few of the paramagnetics are also ferro-magnetics.

There is experimental evidence for believing that magnetised ferro-magnetic materials consist of a very large number of minute particles *with their N. poles all pointing more or less in the same direction*, each particle containing possibly millions of atoms. Thus these microscopic particles, known as *domains*, are small magnets. But when the material is unmagnetised the domains lie in directions at random, and so the magnetic fields due to the various domains also lie in different directions; there is thus no net flux in any one direction. The process of magnetisation consists in subjecting a ferro-magnetic material to a magnetising force which rotates the magnetic axes of the domains into the direction of the magnetising field. So when the material is magnetised not only do the *magnetic atoms* in any one domain have all their N. poles pointing in one direction, but all the domains point in the same direction, providing the magnetising force is large enough.

It was at one time believed that ferro-magnetism was due to the independent rotation of the atoms into the direction of the magnetising force, but it now appears that ferro-magnetism is due to these groups of atoms rotating as a whole. For weak magnetising forces the domains will be only partially rotated into the direction of the field, and so the material will be only partially magnetised.

There is at present no satisfactory explanation for the existence of domains in ferro-magnetic materials, neither can it be explained why ferro-magnetism occurs in so few materials. But the general theory outlined above does show that the large flux produced by ferro-magnetic materials is due to the electronic currents in atoms; in other words, *all* magnetic fields are produced by electric currents. *Magnetic flux is a property of the electric current, and of the electric current alone.*

77. Electromagnetic and Practical Units

In previous chapters it has been stated that the common units of current, p.d., resistance and quantity of electricity were the ampere, volt, ohm and coulomb respectively. No mention was made of the magnitude of these units. As a matter of fact they were derived from another system of units, the *c.g.s. electromagnetic system.* This system of units was introduced in the days when quantitative electrical laws were being discovered and when it became essential to think in terms of formulæ. As the name indicates, the whole system is based on the magnetic effect of an electric current. The electromagnetic unit of current is defined in terms of the force a current exerts on a unit pole, a unit pole having previously been defined in terms of the force it exerts on a similar pole. As already stated in Sections 3 and 4 of Chapter I, the c.g.s. units of force and work are the dyne and erg respectively. The c.g.s. unit pole has already been defined in terms of the dyne and the centimetre.

Actually there are two c.g.s. systems of electrical units, the electromagnetic and electrostatic systems. Just as the former is based on the definition of the unit pole, so the latter is based on the definition of the unit charge (see Chapter IX, Section 108).

The c.g.s. electromagnetic unit of current is defined as that current which, flowing in a conductor in the form of a circular arc of length 1 cm. and 1 cm. in radius, would exert a force of 1 dyne on a unit magnetic pole placed at the centre of the arc. Why the definition takes this peculiar form can only be appreciated by reading the Addendum at the end of the book; this deals with the mathematical theory of electromagnetism, and can only be read with understanding when the student possesses a knowledge of the calculus.

The c.g.s. electromagnetic unit of quantity is that quantity of electricity which in 1 second crosses any section of a conductor through which 1 c.g.s. unit of current is flowing.

The c.g.s. unit of work (**or energy**) is the **erg** and the **c.g.s. unit of power** is a power of **1 erg per sec.**

The c.g.s. electromagnetic unit of p.d. is the p.d. existing between two points between which 1 erg of work has to be done to convey unit quantity of electricity from one point to the other (see Chapter II, Section 10, regarding the definition of the volt).

The c.g.s. electromagnetic unit of resistance is that resistance which allows unit current to flow when unit p.d. is applied to it.

When electricity was put to commercial use, and, instead of being simply a section of pure science, became an important branch of technology, it was realised that the magnitudes of the electromagnetic units were unsatisfactory from the practical point of view, and so *practical units* were derived, in quite an arbitrary manner, from the electromagnetic system as shown in the table below.

	Name of practical unit.	One practical unit is equal to:—
Current . .	Ampere	1/10 of a c.g.s. electromagnetic unit
Quantity . .	Coulomb	1/10 " " " "
Energy . .	Joule	10^7 c.g.s. units
Power . .	Watt	10^7 " "
E.M.F. or P.D. .	Volt	10^8 c.g.s. electromagnetic units
Resistance .	Ohm	10^9 " " "

The student may now appreciate why certain powers of 10 occur in important formulæ, e.g., those given in Sections 73, 80, 86 and 87. They occur because *the electromagnetic units of the various electrical quantities were defined so as to make certain formulæ as simple as possible.* If practical units are used the corresponding formulæ are not quite so simple.

78. The Rationalised M.K.S. System in Electromagnetism

This system of units, which was recommended for general use by the International Electrotechnical Commission in 1950, and is being adopted to an increasing extent, has a number of important differences when compared with the c.g.s. systems.

By choosing the metre, the kilogram and the second as the three fundamental units in mechanical science, the *joule* becomes the m.k.s. unit of work or energy (see Chapter I, Section 4). A new unit of force, the *newton*, is, however, required (see Chapter I, Section 3). In the m.k.s. system of electrical units the inclusion of the *ampere* as the fourth fundamental unit (see Section 94 for definition) results in the adoption of all the familiar *practical units* for the electrical quantities (the coulomb, volt, ohm, etc.).

A further important feature of the m.k.s. system of units is the choice of values for the *permeability and permittivity of a vacuum* (for definitions see Sections 70 and 108), these constants being denoted by μ_0 and ϵ_0 respectively. It was the *arbitrary choice of unity* for both μ_0 and ϵ_0 that was responsible for the *two separate c.g.s. systems of units*, namely the c.g.s. electromagnetic and electrostatic systems. In the m.k.s. system now being generally adopted, **the value of μ_0 is chosen as $4\pi \times 10^{-7}$**, this value being linked with the definition of the ampere (see Chapter VIII, Section 94). In the same system of units, the value chosen for ϵ_0 (see Chapter IX, Section 109) conforms with the relationship connecting μ_0 and ϵ_0 with the velocity of propagation of light and other electromagnetic waves (c metres per sec.).* Conformity with this relationship ensures that *a single system of electrical units* is obtained, including all the quantities met with in both electromagnetism and electrostatics. It should be mentioned here, although the matter is more logically dealt with in detail at a later stage (see Chapters II and V of Vol. II), that both the permeability and permittivity of a vacuum are not pure numbers, but are associated with definite units.

The introduction of the factor 4π into the value for μ_0, which practice may be adopted in any system of units, is the feature termed *rationalisation*. It has a number of useful and important effects, among which is the establishment, in the rationalised m.k.s. system of units, of the **ampere-turn per metre** (abbreviation AT/m) as the unit of magnetising force (see Section 73). The relationship between the c.g.s. unit of magnetising force or *oersted* and the corresponding rationalised m.k.s. unit, the *ampere-turn per metre*, may be derived as follows.

Consider the magnetising force, H oersteds, or H' amp-turn per metre, *on the axis of a toroidal coil carrying a current of I amp. and having n turns uniformly spaced over a mean circumference of l cm., or l′ metres.* As stated in Section 73, $H = \frac{4\pi}{10} \cdot \frac{nI}{l}$, and since the magnetising force is constant over the length of magnetic path, l' metres, $H' = \frac{nI}{l'} = 100 \,.\, \frac{nI}{l}$. Hence, assuming that $\frac{nI}{l}$ has unit value, a magnetising force of $\frac{4\pi}{10}$ oersteds is equivalent to 100 amp-turn per metre, or **1 amp-turn per metre $= \frac{4\pi}{1000}$ oersted** (0·01256 oersted).

---

* Prof. J. Clerk Maxwell discovered in 1865 that $c = (\mu_0\epsilon_0)^{-\frac{1}{2}}$.

Using this relationship, the formulæ stated in Section 73 are modified as follows, in terms of rationalised m.k.s. units.

At a point a distance r′ metres (100*r′ cm*) *from an infinitely long straight conductor carrying a current of I amp.*, the magnetising force, H' amp-turn per metre, is given by—$H' = \frac{I}{2\pi r'}$.

At the centre of a circular conductor of n turns (bunched together so that they can be regarded as lying in one plane) *each turn having a radius of r′ metres and carrying a current of I amp.*, the magnetising force, H' amp-turn per metre, is given by—$H' = \frac{nI}{2r'}$.

Finally, *within an infinitely long, uniformly-wound solenoid carrying a current of I amp., and having n turns in a length l′ metres*, the magnetising force, H' amp-turns per metre, is given by—$H' = \frac{nI}{l'}$.

The rationalised m.k.s. unit of *magnetic flux* is the **Weber** (abbreviation Wb), **a flux of 1 weber being equivalent to 10^8 c.g.s. lines** or maxwells (see Chapter VIII, Section 94). *Magnetic flux density* is expressed in **webers per square metre** (abbreviation Wb/m^2) and the student can quickly verify the fact that **a flux density of 1 Wb/m^2 is equivalent to 10^4 c.g.s. lines/cm^2 or 10^4 gauss.**

APPENDIX

79. Magnetic Poles and Magnetic Flux

(*a*) *C.G.S. electromagnetic system of units*

It is an experimental fact that the force between two magnetic poles is inversely proportional to the square of the distance between the two poles. From this fact, and the definition of the c.g.s. unit pole, it follows that if two poles of strengths m_1 and m_2 c.g.s. units respectively are placed d cm. apart in air (or, more precisely, in a vacuum), the force, F dynes, between the poles will be given by the formula

$$F = m_1 m_2 / d^2$$

Since the field strength, H c.g.s. units, at a point, P, a distance r cm. from a pole of strength m c.g.s. units, is equal to the number of dynes experienced by a unit pole placed at the point, P, it follows that

$$H = \frac{m \times 1}{r^2} = \frac{m}{r^2}$$

Consider a sphere of radius r cm. with a pole of strength m c.g.s. units at the centre. Since every point on the surface of this sphere is at a distance r cm. from the pole, the field strength, H c.g.s. units at all points on the surface of the sphere, is equal to m/r^2 c.g.s. units. Therefore the flux density at all points on the surface of the sphere is m/r^2 lines per sq. cm. Since the surface area of the sphere is $4\pi r^2$ sq. cm., the total flux cutting the surface of the sphere is $m/r^2 \times 4\pi r^2$, i.e., $4\pi m$ lines. All these lines emanate from the pole of strength m c.g.s. units. Thus **from a pole of strength m c.g.s. units there emerge $4\pi m$ lines.** Every c.g.s. unit pole situated in air or any other non-magnetic medium gives rise, therefore, to a magnetic flux of 4π c.g.s. lines.

(*b*) *Rationalised m.k.s. system of units*

The rationalised m.k.s. unit pole is defined, from the outset, as *the pole which, when situated in air, produces unit magnetic flux* (1 *weber*). It follows, therefore, that if we consider a sphere of radius R metres, with a pole of strength p rationalised m.k.s. units at the centre, in air, the total magnetic flux cutting the spherical surface is p webers and the magnetic flux density on this surface is $\frac{p}{4\pi R^2}$ Wb/m². The magnetising force on the spherical surface, H' amp-turns per metre, is therefore given by $H' = \frac{p}{4\pi\mu_0 R^2}$. But the magnetising force at any point may be defined as the force, in newtons, exerted on a rationalised m.k.s. unit N. pole placed at that point. Hence the force, F' newtons, exerted between two poles of strengths p_1 and p_2 rationalised m.k.s. units, placed R metres apart, in air, will be given by the formula $F' = \frac{p_1 p_2}{4\pi\mu_0 R^2}$.

Exercises on Chapter VII

Any additional data required for the solution of these exercises may be obtained from the tables at the end of this book.

1. What will be the magnetising force, expressed in c.g.s. units, in the following cases?

(a) At a point 5 inches from a long straight conductor carrying a current of 40 amp.

(b) At the centre of a circular loop of diameter 7·5 inches consisting of 3 turns and carrying a current of 12 amp.

(c) At the centre of a long thin solenoid uniformly wound with 5 turns per cm. and carrying a current of 10 amp.

[(a) 0·63; (b) 2·38; (c) 62·8.]

2. What current must flow through a long thin air-cored solenoid of diameter 1 inch and having a uniform winding of 20 turns per inch in order to establish a total flux of 500 lines?

[9·97 A.]

3. A circular wooden ring, of mean diameter 15 cm., has a circular cross-section 2·5 cm. in diameter and is uniformly wound with 500 turns of wire. Find the steady current necessary to produce a total magnetic flux of 200 lines in the ring.

[3·06 A.]

4. (a) Convert to c.g.s. electromagnetic units (i) 2·6 amp. (ii) 0·025 joule, (iii) 7·5 millivolts, and (iv) 1·75 microhms.

(b) Convert to practical units (i) 1500 c.g.s. electromagnetic units of quantity of electricity, (ii) $0{\cdot}5 \times 10^5$ c.g.s. electromagnetic units of p.d. and (iii) 30,000 c.g.s. electromagnetic units of resistance.

[(a) (i) 0·26; (ii) 250,000; (iii) $7{\cdot}5 \times 10^5$; (iv) 1750. (b) (i) 15,000 C.; (ii) 0·5 mV; (iii) 30 microhms.]

5. In a radio circuit a direct-current of 2 microamp. is passed through a coil of resistance 3 ohms for 5 seconds. Calculate (a) how many c.g.s. electromagnetic units of p.d. would be necessary to establish this current, (b) how many ergs of energy would flow and (c) what quantity of electricity, expressed in c.g.s. electromagnetic units, would pass in the time stated.

[(a) 600; (b) 0·0006; (c) 10^{-6}.]

6. Two busbars are mounted in air with their axes parallel to each other and 8 inches apart. If each busbar carries a direct current of 2500 amp., find the magnetic flux density at points lying between the busbars and in the plane of their axes, distant (a) 1 inch and (b) 2 inches from one of the axes. The currents in the busbars should be assumed to flow (1) in the same direction and (2) in opposite directions.

[(1a) 169; (1b) 65·6; (2a) 225; (2b) 131·2 lines per sq. cm.]

7. An outgoing and return cable carrying a steady current of 1000 amp. are parallel to each other, their axes being 20 cm. apart. Find the strength of the magnetic field at points in the plane

of the axes distant (a) 10 cm. and (b) 5 cm. from one of the axes and lying (1) between the cables and (2) outside either cable.

[(1a) 40; (1b) $53\frac{1}{3}$; (2a) $13\frac{1}{3}$; (2b) 32 c.g.s. units.]

8. Calculate the values of magnetising force and magnetic flux density, expressed in rationalised m.k.s. units, in the following cases. (a) At a point 20 cm. from a long straight conductor mounted in air and carrying a current of 500 amp. (b) At the centre of an air-cored solenoid of length 2 metres, uniformly-wound with 1500 turns and carrying a current of 12 amp. The diameter of the solenoid is small compared with its length.

[(a) 399 AT/m, 0·5 mWb/m; (b) 9000 AT/m, 11·3 mWb/m^2.]

9. (a) Convert to rationalised m.k.s. units (i) 25 oersteds, (ii) 7500 gauss, (iii) $2{\cdot}5 \times 10^6$ maxwells. (b) Convert to c.g.s. electromagnetic units (i) 1·5 milli-webers, (ii) 12,500 amp-turns per metre, (iii) 750 micro-webers per sq. metre.

[(a) (i) 1990 AT/m, (ii) 0·75 Wb/m^2, (iii) 25 mWb; (b) (i) $1{\cdot}5 \times 10^5$ maxwells, (ii) 157 oersteds, (iii) 7·5 gauss.]

10. A toroidal coil of 800 turns is uniformly-wound on a non-magnetic ring of mean diameter 50 cm. The circular cross-section of the ring has a diameter of 5 cm. Calculate the total magnetic flux in the ring, in webers, when the coil carries a steady current of 15 amp.

[18·8 μWb.]

*11. At what distance apart must two N. poles of 16 and 24 c.g.s. units be placed if they are to repel each other with a force of 4 dynes? Find the force that would be exerted on a unit N. pole placed between them and 3·8 cm. from the weaker pole.

[9·8 cm.; 0·44 dyne.]

*12. A bar magnet has a pole strength of 25 c.g.s. units, the separation of the poles being 20 cm. Find the magnetising force at points on the magnetic axis lying outside the magnet and distant (a) 5 cm., (b) 10 cm. and (c) 15 cm. from one of the poles.

[(a) 0·96; (b) 0·222; (c) 0·091 c.g.s. unit.]

*13. How much flux emanates from a magnetic pole which, when placed at the centre of a thin solenoid of 450 turns, 30 cm. in length and carrying a current of 3 amp., experiences a force of 198 dynes?

[44 lines.]

* *These exercises should be attempted only after reading the Appendix to Chapter VII.*

CHAPTER VIII

THE MOTOR AND DYNAMO EFFECTS

80. The Force on a Current-carrying Conductor in a Magnetic Field

It is a well-known mechanical principle that to every force there is an equal and opposite force of reaction. Since a conductor carrying current exerts a force on a magnetic pole placed in its vicinity, the conductor will itself experience a force of equal magnitude in the opposite direction. In fact, **all magnetic and electromagnetic**

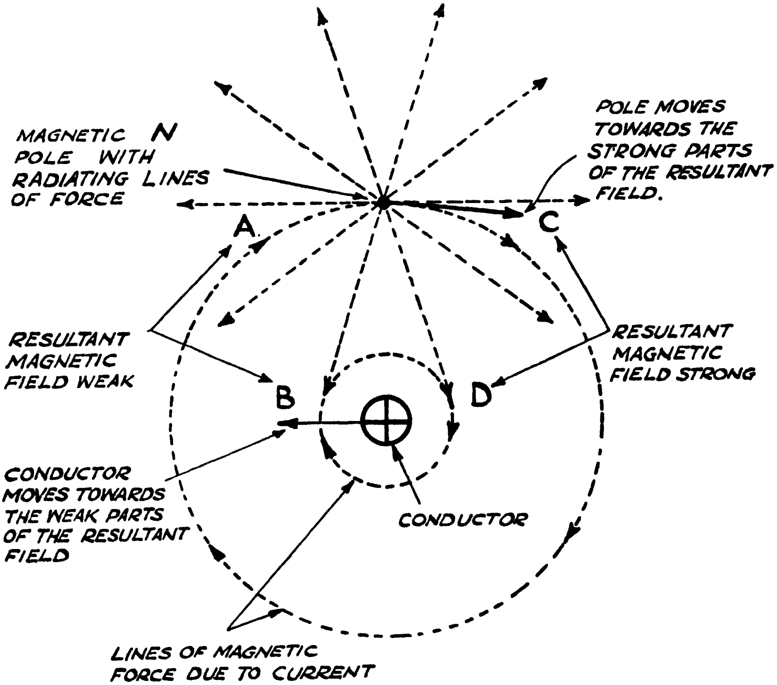

MAGNETIC N POLE WITH RADIATING LINES OF FORCE
POLE MOVES TOWARDS THE STRONG PARTS OF THE RESULTANT FIELD.
A
C
RESULTANT MAGNETIC FIELD WEAK
RESULTANT MAGNETIC FIELD STRONG
B
D
CONDUCTOR MOVES TOWARDS THE WEAK PARTS OF THE RESULTANT FIELD
CONDUCTOR
LINES OF MAGNETIC FORCE DUE TO CURRENT

Fig. 151.—Directions of Movement of Magnetic Pole and Adjacent Current-carrying Conductor.

forces are due to the interaction between two overlapping magnetic fields. *It should be noted that a current-carrying conductor does not experience a force unless it is placed in a magnetic field due to some external source.*

Fig. 151 shows the overlapping fields of a N. pole and a straight current-carrying conductor, the lines of force due to the current being concentric circles, and those due to the N. pole consisting of straight lines radiating from the pole. In regions A and B the lines due to the pole oppose in direction those due to the current, so that in these regions the resultant flux density will be relatively small. In regions C and D the flux density will be comparatively large, since the lines due to the pole have more or less the same direction as those due to the current. Thus at A and B we have weak magnetic fields and at C and D strong fields. *A magnetic pole tends to move in the direction of the resultant flux towards the strong parts of the field, whereas a current-carrying conductor tends to move towards the weak parts of the field.* Incidentally, the direction of motion of a current-carrying conductor placed in a magnetic field is perpendicular to the direction of this field in the region of the conductor.

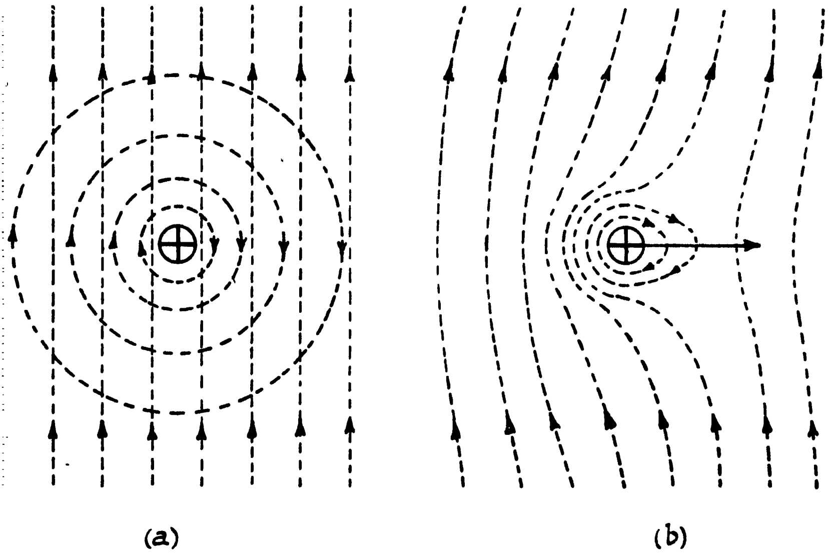


FIG. 152.—MOVEMENT OF A CURRENT-CARRYING CONDUCTOR PLACED AT RIGHT ANGLES TO A UNIFORM MAGNETIC FIELD.

Fig. 152 represents a straight conductor placed in and at right angles to a magnetic field of uniform strength. On passing a current through the conductor, the circular lines of force due to the current are superimposed upon the straight lines of the original field, producing a resultant magnetic field which is not symmetrical with respect to the conductor. The two constituent fields are as shown in Fig. 152a, and the resultant field as in Fig. 152b. From the principle stated above the conductor will move towards the right as shown in Fig. 152b.

If the conductor is placed parallel to the original lines of force, the resultant flux consists of lines spiralling round the conductor, but since the field is symmetrical with respect to the conductor, the latter experiences no tendency to move. Fig. 153 depicts some of the lines that spiral round the conductor at one particular radius, these lying on a cylindrical surface. It will, of course, be realised that there is an infinite number of such cylindrical surfaces of various radii, each containing a very large number of these spiralling lines of magnetic force. It should be remembered that *a current-carrying conductor experiences no tendency to move when placed in a magnetic field in a direction parallel to the lines of force.* If, however, a straight current-carrying conductor is inclined in the slightest to the lines of force of some external magnetic field, the conductor experiences a force in a direction perpendicular both to the conductor and the flux. It is proved in the Addendum that the force F dynes that would be experienced by the conductor illustrated in Fig. 154 is given by the formula:—

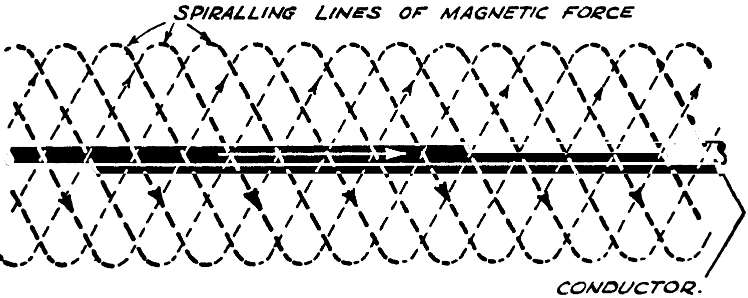


FIG. 153.—RESULTANT MAGNETIC FIELD ROUND A CURRENT-CARRYING CONDUCTOR PLACED IN A UNIFORM MAGNETIC FIELD PARALLEL TO ITS LENGTH.

$$F = \frac{1}{10} Bll \cdot \sin \theta$$

When the conductor and magnetic field are perpendicular to each other, $\theta = 90°$, so the formula becomes:—

$$F = \frac{1}{10} Bll$$

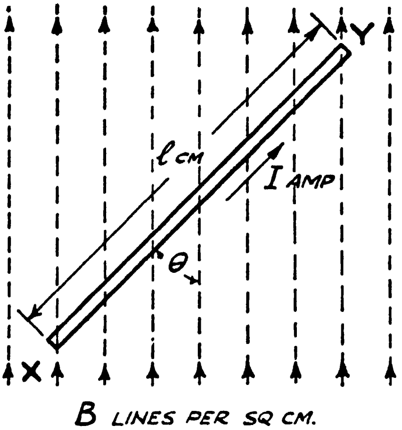


FIG. 154.—A CONDUCTOR PLACED OBLIQUELY IN A UNIFORM MAGNETIC FIELD.

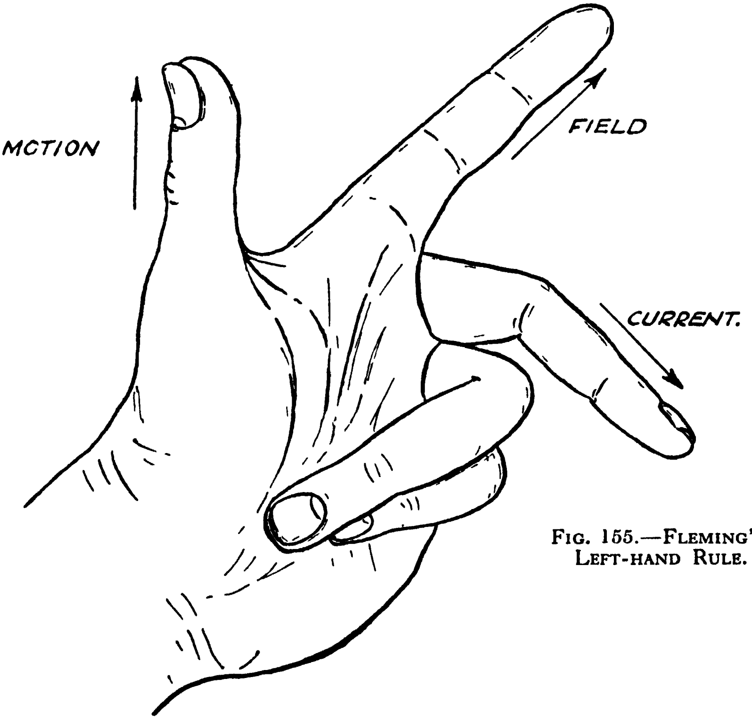


FIG. 155.—FLEMING'S LEFT-HAND RULE.

A straight current-carrying conductor placed in a uniform magnetic field always moves in a direction perpendicular both to its own length and to the direction of the original uniform magnetic field. If the conductor is perpendicular to the field—a condition which usually occurs in practical applications of this *motor effect*—the precise direction of movement may be determined by using **Fleming's left-hand rule :** place the thumb and first two fingers of the left hand mutually at right angles; then, if the first finger is pointed in the direction of the magnetic field and the second finger in the direction of the current, the thumb will indicate the direction in which the conductor tends to move (see Fig. 155).

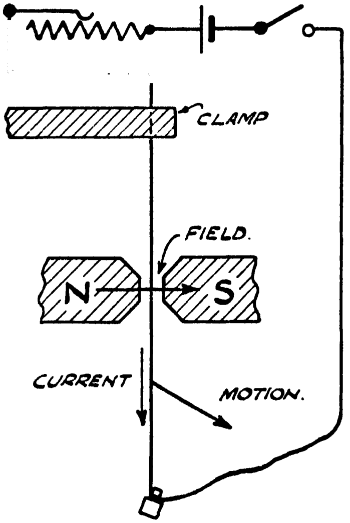


FIG. 156.—APPARATUS FOR DEMONSTRATING FLEMING'S LEFT-HAND RULE.

The behaviour of a current-carrying conductor in a magnetic field may easily be observed by suspending a long piece of fine-gauge copper wire between the poles of the laboratory electromagnet illustrated in Fig. 143 of the last chapter, using a 2-gm. weight to make the wire hang vertically (see Fig. 156). Connect the wire in series with a rheostat, accumulator and switch. On closing the switch the wire will move in the direction shown, in accordance with the left-hand rule. Reversing *either* the direction of the current *or* the direction of the field will cause the direction of the force to be reversed. Reversing both current and field will leave the force acting in the original direction.

A very interesting method of demonstrating that reversing the current in a conductor subject to a magnetic field reverses the direction of the force is to place the two poles of a horseshoe magnet on opposite sides of a carbon filament lamp. On passing an alternating current through the filament, the filament is observed to vibrate in a very pronounced manner, the frequency of vibration being, of course, equal to the frequency of the current supply. This experiment should not be performed with a metal filament lamp, since the method of suspension of the coiled filament does not lend itself so well to this demonstration; alternatively, the hairpin type of metal filament is likely to break owing to its brittle nature.

81. Electromagnetic Instruments and D.C. Motors

All electric motors and the majority of electrical instruments depend for their working upon the interaction between two overlapping magnetic fields. Examples are given below, but a much more comprehensive and detailed description is given in Vol. II.

(a) *The D.C. Motor*

The *armature*, which is the rotary part of a d.c. motor, is a steel cylinder with slots cut into the cylindrical surface parallel to the axis of the cylinder (occasionally these slots are slightly skewed as in the armature depicted in Fig. 158). In these slots are embedded copper conductors insulated from the steel core. Fig. 157 shows, in diagrammatic form, a sectional view of a small armature. At least 2 conductors are placed in each slot, and the conductors in the various slots so connected together that the complete armature winding consists of 2, 4, 6 or more paths for the current, each path containing the same number of conductors in series. These paths are then connected in parallel to a source of p.d. through a commutator and brush-gear—for further details see Vol. II, Chapter VI. Fig. 158 illustrates the appearance of the armature, commutator and

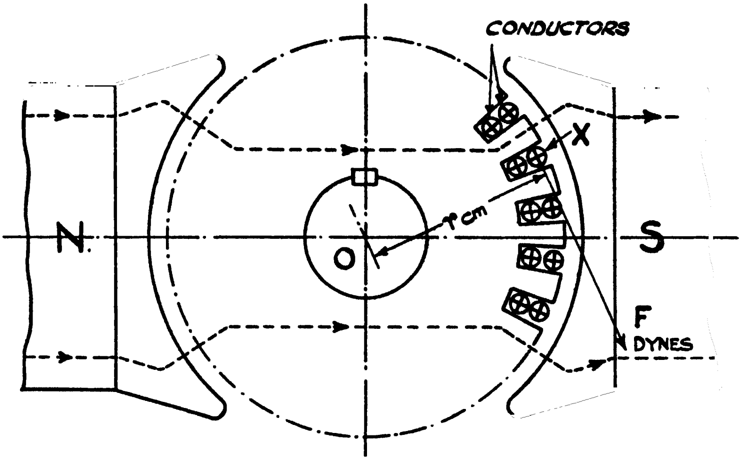


FIG. 157.—A SIMPLE TWO-POLE MOTOR.

cooling fan for a motor of medium size. In a complete motor an equal number of alternate N. and S. poles are arranged symmetrically round the armature, the poles being electromagnets.

It is, of course, important that the forces on each of the conductors should tend to rotate the armature in the same direction, not only at a given instant, but at all times. Obviously two conductors under opposite poles must have their currents flowing in opposite directions, and when a conductor moves from a position opposite a N. pole to one opposite a S. pole, the direction of the current must change. This is achieved by means of the commutator and brushes, using some particular armature winding scheme (see Vol. III). The construction of d.c. motors and generators is identical, so the above brief remarks apply equally to the d.c. generator, which is dealt with later in the chapter. Similarly, later remarks regarding the construction and types of d.c generators apply equally well to the d.c. motor.

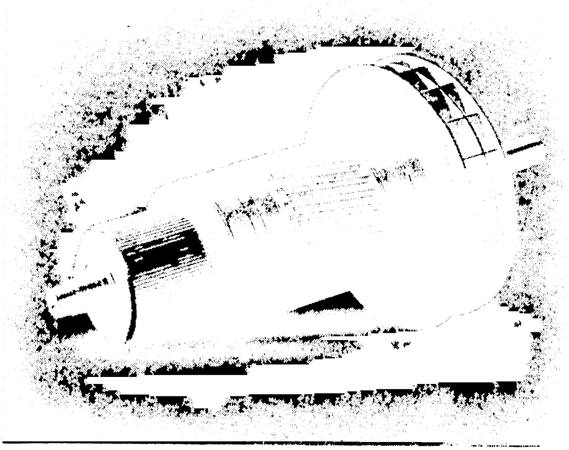


FIG 158—COMPLETE ARMATURE, COMMUTATOR AND COOLING FAN FOR A MEDIUM-SIZED D C MOTOR

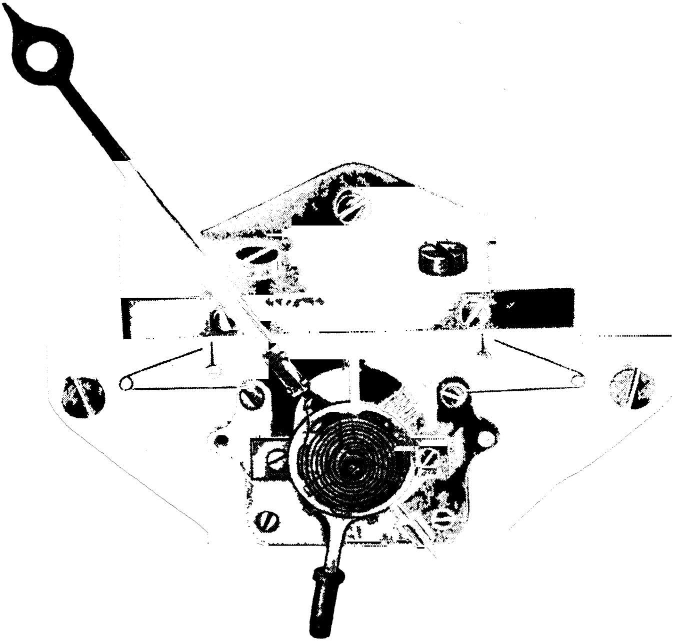
The function of a motor is to produce a *torque*, or turning effect (see Chapter I, Section 3). In the case of the d.c. motor the total torque is equal to the sum of the moments, about the armature axis, of the forces on the separate conductors. Thus if the force on the conductor X be F dynes, the moment of this force about the point O is F × r dyne-cm. (see Fig. 157), and the total torque produced is the sum of such moments for all the conductors lying under the poles. It is obvious from the left-hand rule that if the current in conductor X is flowing downwards, the force F will be in the direction shown. It should be noticed that the field produced by the magnet system in the region of the air-gap between pole and armature is a radial field, i.e., the magnetic lines of force, if produced, would be radii to the circular section. The forces on the various conductors are then always perpendicular to the radius joining the conductor to the axis of the armature.

(b) *Moving-coil Instruments*

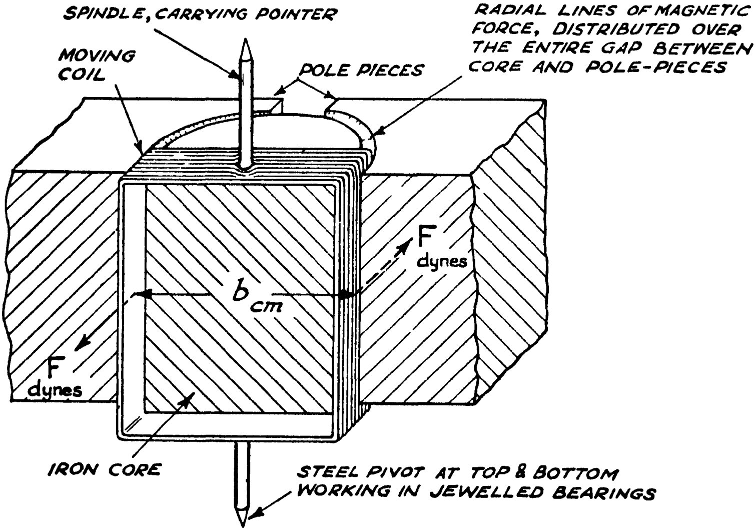
Most galvanometers, ammeters and voltmeters used for d.c. measurements are moving-coil instruments. In such instruments—see Fig. 159 (a) and (b)—a delicate coil, usually rectangular in shape and containing many turns of fine silk-covered copper wire, is usually pivoted so as to be free to turn through almost 90° between the cylindrical N. and S. pole faces of a permanent magnet.

The two vertical sides of the rectangular coil, being in a radial field and carrying current, each experience a force of F dynes, the two forces being parallel to each other, but in opposite directions, since if the current flows *up* one side of the coil, it flows *down* the other side. If the breadth of the coil is b cm., it experiences a torque of (F × b) dyne-cm. The current is led to and from the coil by two phosphor-bronze control springs. As the coil rotates, an opposing torque is set up by the springs, the coil eventually coming to rest when this controlling torque exactly balances the electromagnetic deflecting torque. A pointer attached to the spindle on which the coil is mounted moves over a scale, which may be calibrated in amperes.

In the case of an ammeter, only a portion of the current to be measured flows through the coil, most of the current flowing through a *shunt* of low resistance connected in parallel with the coil. This shunt often consists of several strips of manganin joined in parallel as illustrated in Fig. 160a. The moving-coil is usually so delicate that it would be burnt out by currents greater than a few milliamperes. However, in a sensitive galvanometer designed for very low currents the coil usually carries the whole current. In the case of M.C. voltmeters, a very high resistance is connected in series with the coil (see Fig. 160b). The p.d. to be measured is applied across the coil and series resistor or *multiplier*. According to Ohm's law, the current which flows through the coil will be proportional



(*a*) An assembled moving-coil movement


SPINDLE, CARRYING POINTER
RADIAL LINES OF MAGNETIC FORCE, DISTRIBUTED OVER THE ENTIRE GAP BETWEEN CORE AND POLE-PIECES
MOVING COIL
POLE PIECES
F dynes
b_{cm}
F dynes
IRON CORE
STEEL PIVOT AT TOP & BOTTOM WORKING IN JEWELLED BEARINGS

(*b*) A diagrammatic section

FIG 159 —THE MOVING-COIL TYPE OF INSTRUMENT

to the p.d. applied; the instrument scale can therefore be calibrated directly in volts.

A good M.C. ammeter usually possesses a resistance such that the voltage drop across it at full-scale reading is of the order of 75 millivolts, whereas a good M.C. voltmeter possesses a resistance of at least 60 ohms per volt of full-scale reading.

(c) *Moving-iron Instruments* (*Repulsion Type*)

There are two types of moving-iron instruments—namely, the attraction and repulsion types—the latter being the more common. The moving-iron instrument is the type primarily used for a.c. measurements; M.C. instruments cannot be used with an a.c. supply, for when the current is reversed the torque is reversed, and since the coil cannot follow the rapid alternations of current, it remains undeflected. In M.I. instruments, the current to be measured, or, in the case of the voltmeter, a current proportional to the p.d. to be measured, is passed through a short solenoid. This solenoid is wound with sufficient turns, so that when the maximum current for which the instrument is designed is passed through it, the number of ampere-turns is sufficient to produce a magnetising force strong enough to deflect the pointer to the full extent of the scale. In the repulsion type the movement consists of a piece of iron mounted on a spindle so as to lie in the undeflected position adjacent to a fixed piece of iron, as shown in Fig. 161 (a), (b) and (c). When the solenoid carries a current, both pieces of iron are similarly magnetised, i.e., with N. poles adjacent. If the current in the coil is reversed, the polarity of each piece of iron is reversed, but similar

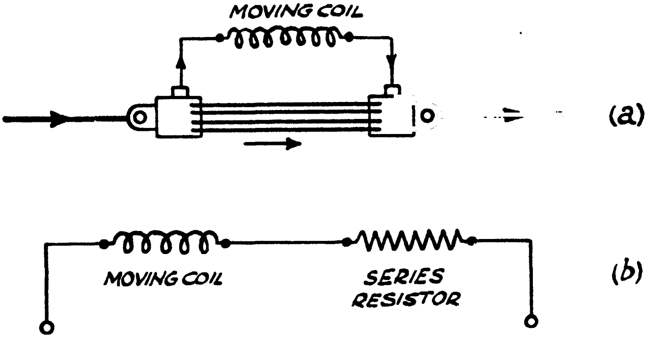
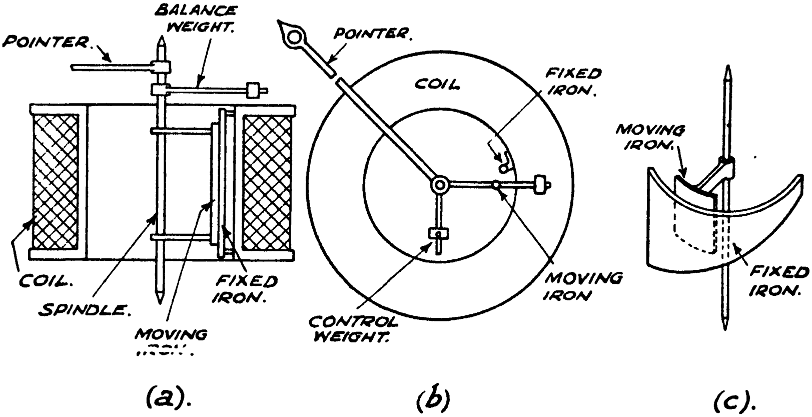


FIG. 160.—INTERNAL CIRCUITS OF MOVING-COIL AMMETER AND VOLTMETER.

poles remain adjacent. So, no matter what the direction of the current, repulsion ensues. The torque remains in the original direction when the current is reversed, so such an instrument may be used with an a.c. supply. The stronger the current, the greater the number of ampere-turns and the more intense the magnetisation induced in each piece of iron. Doubling the current causes the deflection to be more than doubled, since both pieces of iron have their magnetisation roughly doubled, and the deflecting torque depends upon the product of the pole strengths of the two pieces of iron. The deflecting torque depends also on the distance apart of the two pieces of iron. Although in some instruments the two pieces of iron are simple rods, in others they are given shapes as shown in Fig. 161c, in order to give rise to a more uniform scale. As the spindle and pointer rotate an opposing torque is established, usually by means of a phosphor-bronze spring, as in M.C. instruments, but sometimes by means of a control weight, as shown in Fig. 161b. In the case of ammeters used for large currents the solenoid consists of a few turns of comparatively thick copper wire, but in voltmeters a very large number of turns of fine-gauge wire is used.



(a). (b) (c).

Fig. 161.—The Moving-iron Type of Instrument.

82. The Force between Adjacent Current-Carrying Conductors

Since the magnetic fields due to two adjacent current-carrying conductors will overlap, there will usually be a mutual force of attraction or repulsion between the conductors. With two parallel conductors, attraction will occur when the two currents flow in the same direction, and repulsion when the currents are in opposite directions. This follows from the left-hand rule (see Fig. 162). This diagram represents portions of two very long parallel conductors, AB and CD, carrying currents of I_1 and I_2 amp. respectively. The magnetising force, H_2 c.g.s. units, at the point X due to the current I_2 amp. is $2I_2/10d$ c.g.s. units, and similarly the magnetising force, H_1 c.g.s. units, at the point Y due to the current I_1 amp. is $2I_1/10d$ c.g.s. units, i.e., the conductor AB throughout its length lies in a magnetic field where the flux density is $2I_2/10d$ lines per sq. cm. and the conductor CD in a field of flux density $2I_1/10d$ lines per sq. cm. Thus from the formula F = 1/10 $Bll \sin \theta$ the force on each centimetre of AB is equal to $(1/10 \times 2I_2/10d \times 1 \times I_1 \times \sin 90°) = 2I_1I_2/100d$ dynes per cm. By again using the formula we see that the force on each centimetre of CD is also equal to $2I_1I_2/100d$ dynes per cm., as would be expected from the general principle that it is impossible for a body AB to exert a force on a body CD without CD exerting an equal and opposite force on AB. Fig. 163 shows the patterns of the resultant magnetic fields round two parallel, straight, current-carrying conductors, and illustrates the general principle,

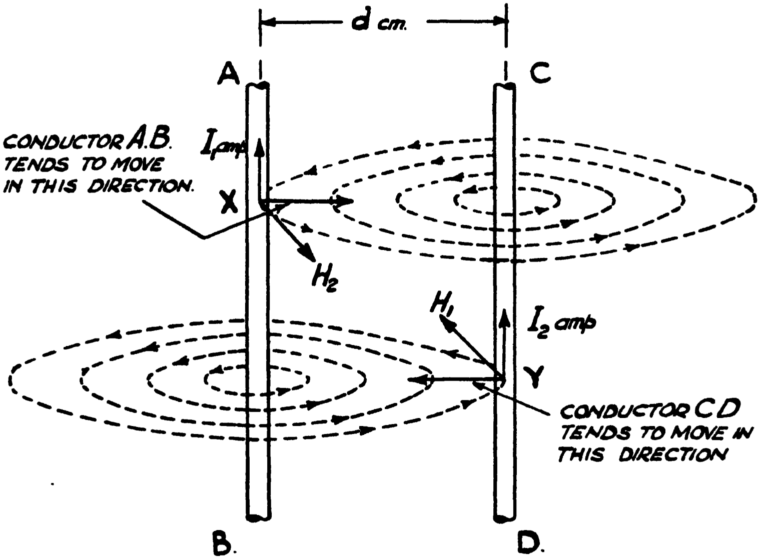

d cm.
A
C
CONDUCTOR A.B. TENDS TO MOVE IN THIS DIRECTION.
I_1 amp
X
H_2
H_1
I_2 amp
Y
CONDUCTOR CD TENDS TO MOVE IN THIS DIRECTION
B.
D.

FIG. 162.—THE FORCES BETWEEN ADJACENT PARALLEL CURRENT-CARRYING CONDUCTORS.

stated in Section 80, that current-carrying conductors tend to move towards the weak parts of the magnetic field.

From the formula derived above it is obvious that this mutual force of attraction or repulsion will, in general, be very weak. For example, even if $I_1 = I_2 = 100$ and $d = 5$; the mutual force $= \frac{20{,}000}{500} = 40$ dynes per cm., so two conductors, each 50 cm. long and carrying 100 amperes, would experience a force of only 2000 dynes, which is very little more than 2 gm. wt. But by arranging for one conductor to be suspended so that there is negligible opposition to its movement, it is possible for forces much smaller than this to produce a considerable deflection, as is done in the cases of the current balance and the dynamometer type of wattmeter (see Vol. II, Chapter IX).

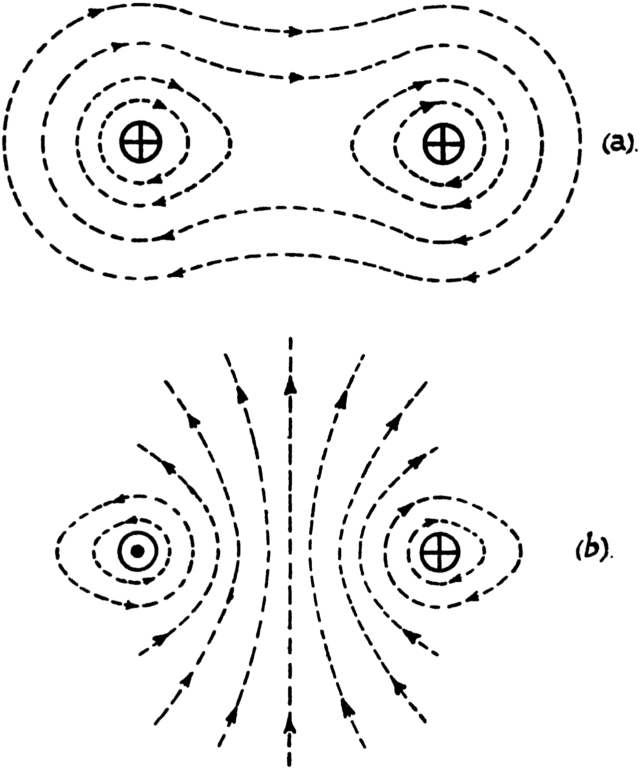


FIG. 163.—THE MAGNETIC FIELDS AROUND ADJACENT PARALLEL CURRENT-CARRYING CONDUCTORS.

83. Flux-linkages

If a conductor carries a current, it must be part of a closed circuit. A closed circuit gives rise to a magnetic field, and every line of force in this magnetic field is *linked* with the circuit. Thus in Fig. 128, where the conductor consists of a single loop, every line of magnetic force passes once through the loop. The number of lines threading the circular conductor is referred to as the number of *flux-linkages* with the loop. In the circuit of Fig. 130 the number of flux-linkages is much greater than the number of lines of magnetic force, since most of the lines link with *each turn* of the solenoid, although some of the lines do link with only one turn. In the case of the closely-wound solenoid of Fig. 129 practically all the lines link with all the turns. The total number of flux-linkages with a solenoid is the sum of the flux-linkages with each separate turn. With a very closely-wound solenoid, where it may be assumed that every line links with all the turns, the number of flux-linkages is simply the product of the flux, Φ lines, and the number of turns, n, i.e., the number of flux-linkages is Φn.

In calculating the number of flux-linkages made with any circuit it is not necessary that the flux should be due to the current flowing in that particular circuit; it is quite possible for a circuit not carrying a current to be linked with a large flux, due to some external source. Thus, referring to Fig. 164, if the switch S of circuit A be closed, flux will pass round the iron ring so long as the switch remains closed. This flux will link with circuit B. If a flux of 10,000 lines passes through a solenoid B consisting of 100 turns, then the number of flux-linkages with circuit B is 10^6. If, however, the switch S be opened, this flux will collapse, and the flux-linkages with the solenoid B will cease to exist. In 1831 Michael Faraday, working with apparatus similar to that illustrated in Fig. 164, made a discovery which may be said to have laid the foundations of electrical engineering, for he discovered how to generate an electro-motive force with the aid of a conducting circuit and a magnetic field.

84. Electromagnetic Induction

Using apparatus similar to that of Fig. 164, Faraday discovered that whenever the switch S was opened or closed, a momentary current flowed in coil B. His " galvanometer " consisted simply of the copper wire XY passing close over a pivoted compass needle.

On opening the switch the deflection of the needle was in the opposite direction to its deflection on closing the switch. It appears that **whenever the number of flux-linkages with a circuit is being changed an e.m.f. is induced in the circuit.** This phenomenon is known as **electromagnetic induction,** and should not be confused with *magnetic induction,* which, as already stated, simply means *induction density* or *flux density.* E.M.F's. produced by changes in a number of flux-linkages are always said to be *induced.* The manner in which the number of flux-linkages is changed is quite immaterial. If, for example, a magnet is thrust into a solenoid (Fig. 165) containing very many turns and with its ends connected to a galvanometer, the galvanometer registers a current while the magnet is being inserted. As soon as the magnet stops moving the current ceases. When the magnet is withdrawn a current flows in the reverse direction. When the magnet is removed to the position shown in Fig. 165a it may be assumed that none of the magnetic flux produced by the magnet links with any of the turns of the solenoid. When in the position of Fig. 165b there are quite an appreciable number of flux-linkages. So while the magnet is being moved from one position to the other the number of flux-linkages with the solenoid is changing, and during this change an e.m.f. is induced in the solenoid.

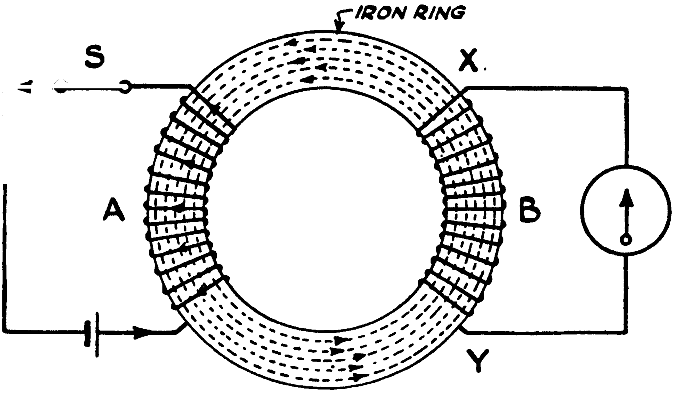


FIG. 161.—APPARATUS FOR DEMONSTRATING ELECTROMAGNETIC INDUCTION.

In Faraday's original experiment there was no relative movement of magnetic field and conductor. The flux was simply established by switching the current on, and removed by switching the current off. In the experiment of Fig. 165 the magnetic field is moved, and the conductor remains stationary; had the solenoid been moved towards the magnet and the magnet kept stationary, exactly the same result would have been obtained. By moving a single copper wire

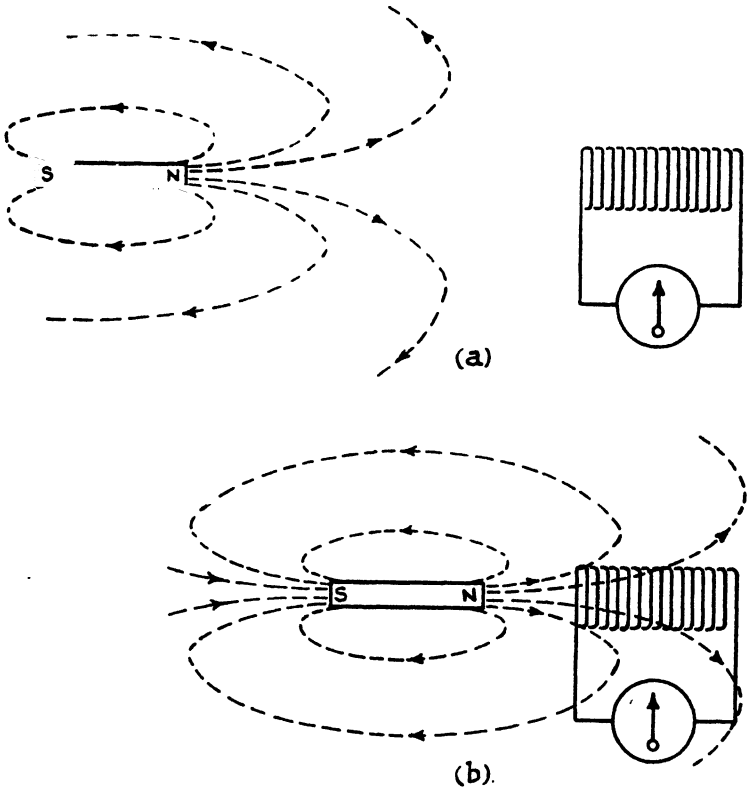


Fig. 165.—The Production of an Induced E.M.F. by means of a Moving Bar Magnet.

AB (Fig. 166) between the poles of a powerful electromagnet, an e.m.f. may be induced in the wire, and if the wire is part of a closed circuit a current will flow in the circuit. In all these cases the common feature is that the number of flux-linkages with a given circuit is changed, with the result that an e.m.f. is induced in the circuit. In the latter case, however, the e.m.f. induced in the circuit may be regarded as due to the *cutting of lines of magnetic force* by the conductor AB. This alternative method of viewing electromagnetic induction is frequently more convenient than the method whereby the induced e.m.f. is attributed to a *change of flux-linkages* with the circuit. This is the case, for example, when considering the d.c. generator, in which a large number of conductors continuously cut lines of magnetic force produced by field magnets. The two alternative viewpoints do not conflict, since if 10,000 lines are cut by the conductor AB of Fig. 166, as it moves across the poles, the change in flux linked with the circuit ABCD is also 10,000 lines.

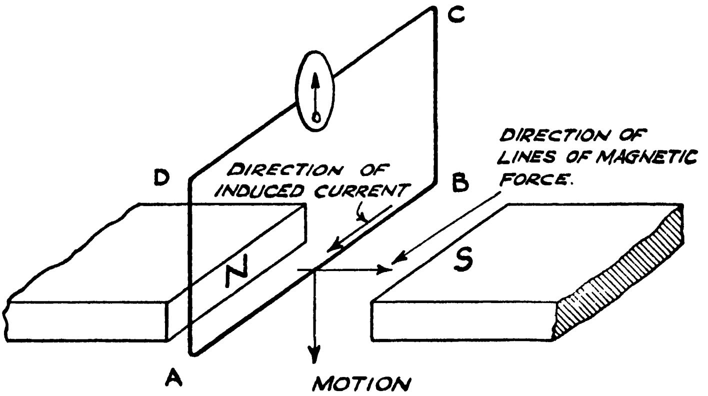

C
DIRECTION OF LINES OF MAGNETIC FORCE.
D
DIRECTION OF INDUCED CURRENT
B
N
S
A
MOTION

Fig. 166.—The Production of an Induced E.M.F. by Flux-cutting.

85. Lenz's Law and Fleming's Right-hand Rule

If the switch S of the circuit shown in Fig. 164 be closed and then opened, the galvanometer indicates a current first in one direction and then in the other direction. Similarly, using the apparatus of Fig. 165 the current flowing through the galvanometer while the magnet is being introduced into the solenoid is in the opposite direction to that flowing while the magnet is being withdrawn. **Lenz's law** is a useful rule for determining the direction of the induced e.m.f. in any case of electromagnetic induction. It states that: **The direction of an induced e.m.f. is such that it tends to oppose the change producing it.**

Consider its application to the case of electromagnetic induction illustrated by Fig. 165. While the N. pole of the magnet is being brought towards the solenoid an e.m.f. is induced, causing a current to flow round the solenoid in such a direction that the end of the solenoid near to the magnet becomes a N. pole, *thereby tending to repel the N. pole of the magnet.* This means that viewed from the magnet end the current passes round the solenoid in a counter-clockwise direction.

The reason Lenz's law is obeyed is that, if an induced current is to flow, energy must be supplied from some source, and in the case quoted it obviously comes from the movement of the bar magnet. Owing to the solenoid behaving as a magnet, and so exerting a force of repulsion on the bar magnet, work must be done to move the latter towards the solenoid. The energy necessary to do this work is changed to electrical energy. Whenever work is done against an opposing force, energy is changed from one form to another; in other words, work is the process whereby energy can be changed from one form to another. But mechanical work cannot be done unless there is an opposing force. Hence Lenz's law is not a mysterious rule—it is what one would expect from simple mechanical principles (see the Appendix, Section 95).

Whenever an e.m.f. is induced in a circuit the cause is a change in the amount of flux threading the circuit. Lenz's law therefore implies that the direction of the e.m.f. will be such that the induced current will produce flux of its own in a direction tending to neutralise the change in flux brought about by external means. Fig. 165a illustrates a magnet so far removed from a solenoid that none of the flux from the magnet links with any of the turns on the solenoid. When the magnet is in the position of Fig. 165b, flux passes from left to right through the inside of the solenoid. So, by Lenz's law, while the magnet is being moved into the position of Fig. 165b the current will flow in such a direction as to produce flux passing from right to left through the inside of the solenoid. It should be noted that the current only *tends* to neutralise the change in flux—it cannot permanently neutralise it. When the magnet is withdrawn from the position of Fig. 165b to that of Fig. 165a, the direction of the current is such that flux is produced, passing through the inside of the solenoid from left to right in an endeavour to maintain the flux which is linked with the solenoid as shown in Fig. 165b.

Every case of electromagnetic induction may be treated in this manner, but where the induced e.m.f. is due to a moving conductor cutting lines of force, **Fleming's right-hand rule,** which may be deduced from Lenz's law, is more convenient than Lenz's law itself for determining the direction of the induced e.m.f. It states that if the thumb and first two fingers of the *right* hand be placed mutually at right angles, then if the first finger is pointed in the direction of the magnetic field and the thumb in the direction of motion of the conductor, the second finger will give the direction of the induced e.m.f. Thus if the conductor AB of Fig. 166 be moved in the direction indicated, the right-hand rule informs us that the e.m.f. will be from B to A and a current will flow from B to A if AB is part of a closed circuit.

Experiment 16. *To verify Lenz's law*

Wind a solenoid of approximately 50 turns using a 1-inch test-tube as a former. Connect the ends of the solenoid to a galvanometer, after first determining the polarity of the galvanometer. To do this connect a Leclanché cell in series with the galvanometer, a switch and a high-resistance coil, the resistance being such that on closing the switch the current flowing will not be so high as to damage the galvanometer. Close the switch, observe the direction in which the pointer is deflected and note at which terminal the current enters the galvanometer. Thus whenever in further use the galvanometer pointer is deflected, the direction of flow of the current can be deduced. Insert the N. pole of a bar magnet into the solenoid as in Fig. 165; withdraw the magnet, insert the S. pole and again withdraw the magnet, in each case noticing the direction of flow of the current. Deduce in each case the polarity of the end of the solenoid near to the magnet and enter the results in a table as shown below:—

Operation.	Deflection of galvanometer.	Direction of current as viewed from magnet end.	Direction of flux inside solenoid produced by the temporary current.	Polarity of magnet end of solenoid.
N. pole inserted	To left (or right).	Counter-clockwise (or clockwise).	From right to left (or left to right).	N. (or S).
N. pole withdrawn	⋮	⋮	⋮	⋮
S. pole inserted	⋮	⋮	⋮	⋮
S. pole withdrawn	⋮	⋮	⋮	⋮

86. Magnitude of Induced E.M.F's.

Using the apparatus of Fig. 165, it can be shown that the deflection of the galvanometer pointer may be increased by increasing the rate of movement of the magnet into or out of the solenoid, by using a stronger magnet or by increasing the number of turns on the solenoid. The dependence of the galvanometer deflection on these factors is due to the fact that the **e.m.f. induced in a circuit is proportional to the rate of change of flux-linkages with the circuit.** In the case of a conductor cutting lines of magnetic force this statement may be expressed in an alternative way as follows: *The e.m.f. induced in a moving conductor cutting lines of magnetic force is proportional to the rate at which it cuts the lines.* Thus if a dynamo is run at various speeds, the flux density being kept constant,

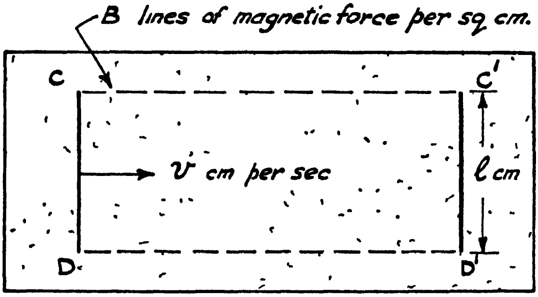


FIG. 167.—A STRAIGHT CONDUCTOR CUTTING LINES OF MAGNETIC FORCE AT RIGHT ANGLES

it is found that the e.m.f. generated is accurately proportional to the number of revolutions per minute at which the machine is driven.

Owing to the manner in which our units of p.d. were defined (see Section 77), **an e.m.f. of exactly 1 volt is induced in a conductor when it cuts magnetic flux at the rate of 10^8 lines per sec.** Similarly, if the number of flux-linkages with any circuit changes at the rate of 10^8 per sec., an e.m.f. of 1 volt is induced in the circuit.

Fig. 167 represents a uniform magnetic field of B lines per sq. cm., the lines of force flowing at right angles into the plane of the paper, with a conductor CD of length l cm. moving in the plane of the paper, in a direction at right angles to its own length, with a uniform velocity of v cm. per sec. In 1 sec. the conductor will have travelled to position C′D′, the distance CC′ being v cm. and the area of the rectangle swept out by the conductor being lv sq. cm. Since the flux density is B lines per sq. cm., the rate at which the conductor is cutting lines is Blv lines per sec. Since for every 10^8 lines cut per sec. an e.m.f. of 1 volt is induced in the conductor, the e.m.f., E volts, induced in the conductor CD is given by the equation:—

$$E = Blv \times 10^{-8}$$

This e.m.f. is maintained so long as the conductor CD continues to cut lines at this rate of Blv lines per sec.

87. The Quantity of Electricity Flowing during Electromagnetic Induction

We have seen that during electromagnetic induction:—

$$\text{The number of volts induced in the circuit} = \text{Rate of change of flux-linkages} \times 10^{-8}$$

So, the number of amperes flowing

$$= \frac{\text{Rate of change of flux-linkages} \times 10^{-8}}{\text{Resistance of the circuit, } R \text{ ohms}}$$

$$= \frac{\text{The change of flux-linkages per sec.} \times 10^{-8}}{R \text{ ohms}}$$

$$\text{But, the number of coulombs flowing} = \text{The number of amperes} \times \text{the number of seconds for which the current flows}$$

$\therefore$ The number of coulombs flowing

$$= \frac{\text{The change of flux-linkages per sec.} \times 10^{-8} \times \text{No. of secs.}}{R \text{ ohms}}$$

$$= \frac{\text{Total change of flux-linkages} \times 10^{-8}}{R \text{ ohms}}$$

$$\text{i.e., The quantity of electricity flowing in a circuit (in coulombs) in a given time, } t \text{ seconds} = \frac{\text{The net change of flux-linkages with the circuit in this time } t \text{ seconds} \times 10^{-8}}{\text{The circuit resistance, } R \text{ ohms}}$$

This relationship may be expressed in symbols, thus:—

$$Q = \frac{\Phi n}{R} \times 10^{-8}$$

88. Measurement of Flux with the Ballistic Galvanometer and Fluxmeter

The above formula is extremely important, since, with its aid, it is possible to measure flux, providing a *ballistic galvanometer* is available. A ballistic galvanometer is a galvanometer designed to measure an electric charge or quantity of electricity. Together with the fluxmeter, which is a modified form of ballistic galvanometer, it is described in some detail in Chapter IX of Vol. II. To use a ballistic galvanometer to measure the flux emerging from, say, the N. pole of a bar magnet, a search-coil must be used as described in Section 69, the ballistic galvanometer replacing the fluxmeter. If a flux Φ lines is produced by the magnet, the number of flux-linkages it makes with the search-coil will be Φn, providing there are n turns on the coil. So when the coil is slipped off the magnet there will be a decrease of Φn flux-linkages, and if the combined resistance of the galvanometer and search coil in series is R ohms, $\frac{\Phi n}{R} \times 10^{-8}$ coulombs will pass through the galvanometer. This charge will give an impulse to the galvanometer movement, causing a deflection which can be measured. If the galvanometer has been calibrated previously, the charge passing through the galvanometer can be deduced from this deflection. The measurement of charge with a ballistic galvanometer depends upon the amplitude of the first swing, and so in ballistic galvanometers the damping should be as small as possible, i.e., all frictional forces should be reduced to a minimum. Further, the moving system should have a long period, so that it will not have moved appreciably from its position of rest until the whole charge has passed through the galvanometer. To achieve this the moving system should be comparatively large and heavy.

The fact that the ballistic galvanometer must first be calibrated is rather inconvenient, so in the fluxmeter the ballistic galvanometer is modified so that the change of flux-linkages may be measured directly. The fluxmeter is a ballistic galvanometer designed to possess a very small controlling torque and with a scale calibrated in maxwell-turns (i.e., line-turns), so that by dividing the recorded measurement by the number of turns on the search-coil the number of maxwells (or lines) may be quickly ascertained. Readings are taken by observing the difference between the pointer readings before and after the change in flux-linkages has occurred (see previous chapter, Section 69).

89. The Single-loop Generator

A dynamo or generator is a machine in which magnetic flux is continuously cut by conductors, an e.m.f. being thus continuously generated. Consider a very simple form of dynamo consisting of a single rectangular loop, the sides AB and CD, each of length l cm., rotating with a linear velocity of v cm. per sec. in a uniform magnetic field of B lines per sq. cm. as illustrated in Fig. 168a. The rotating part of the dynamo, as in the case of the motor, is known as the

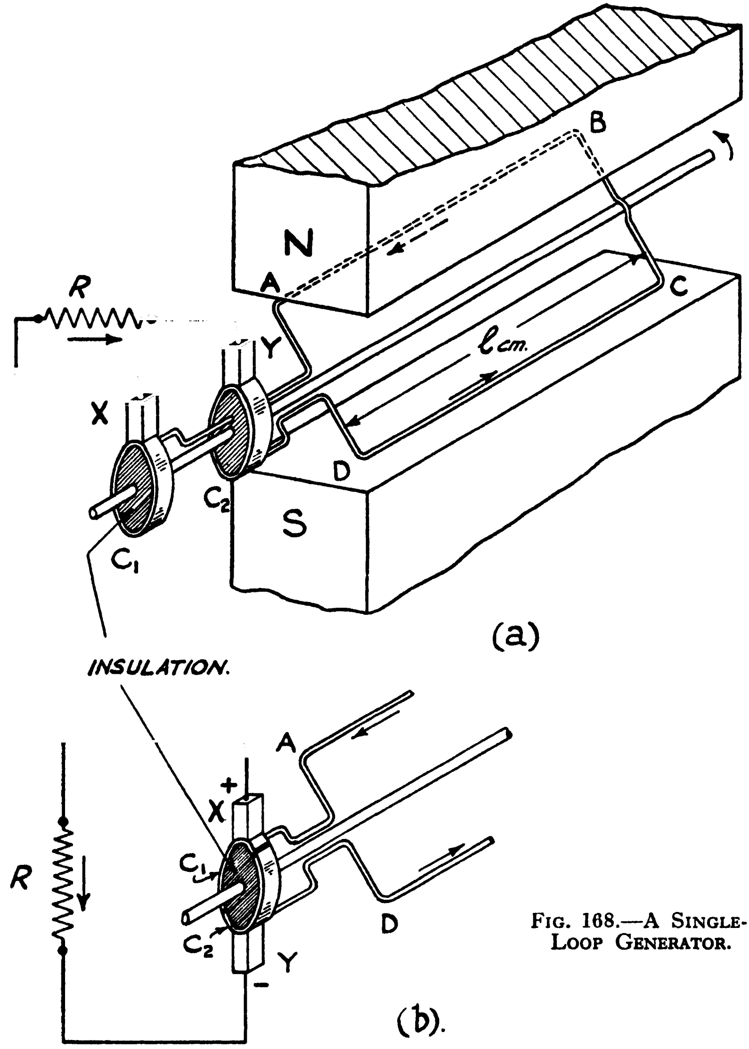
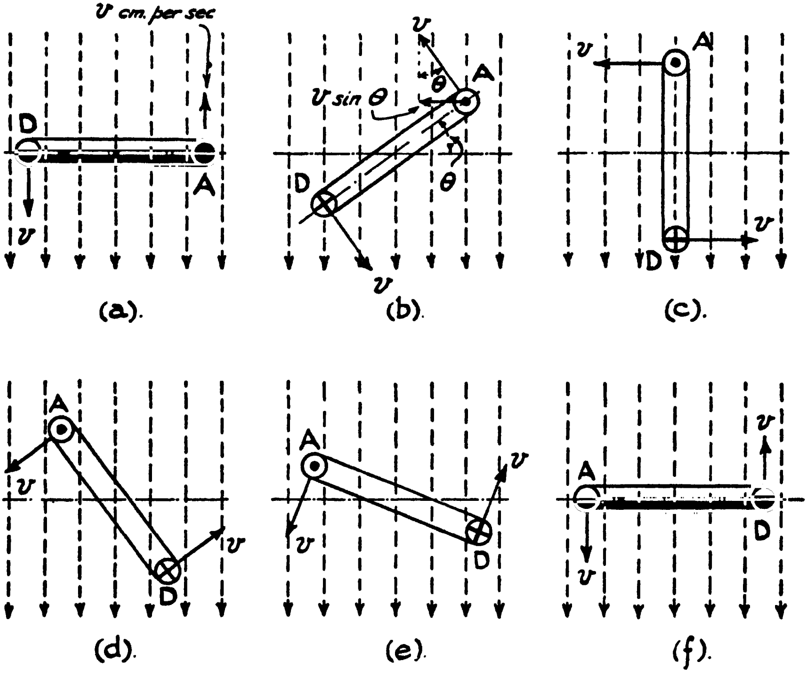


FIG. 168.—A SINGLE-LOOP GENERATOR.

armature. The ends of the loop are brought out to two metal rings, C_1 and C_2, known as *slip rings*, fixed to and insulated from the rotating shaft which carries the loop. Bearing on these rings are two carbon *brushes,* X and Y, which may be connected to some external circuit of resistance R ohms. Without some such device it would be impossible to connect a stationary external circuit to the ends of a rotating loop. As the shaft rotates, the two sides, AB and CD, of the loop cut across the vertical lines of magnetic force, but the two ends, AD and BC, do not.

Fig. 169 represents a number of diagrammatic end views of the loop in various positions during the first half-revolution, supposing the loop to start in the horizontal position of Fig. 169a. At the instant when the loop is in this position (a) the sides AB and CD are moving parallel to the lines of magnetic force and so, at this instant, no e.m.f. is induced. When the loop is in position (b) the lines are being cut fairly rapidly by the two coil-sides, AB and CD. Although, when the loop is in this position the side AB is moving obliquely across the lines, it is the component of the velocity in a direction at right angles to the lines which is responsible for the induced e.m.f. So, in using the right-hand rule to determine the directions of the



(a). (b). (c).

(d). (e). (f).

FIG. 169.—DIAGRAMMATIC END VIEWS OF A ROTATING SINGLE-LOOP GENERATOR.

e.m.f.'s induced in the two coil-sides during the first half-revolution, the side AB should be regarded as moving from right to left and CD as moving from left to right. The e.m.f's. induced in the two coil-sides, although in opposite directions, reinforce each other in so far as they tend to drive the current the same way round the coil, the direction of the current being as shown in Fig. 169b. In position (c) the coil-sides are cutting the lines at right angles. Thus the e.m.f. induced in either side of the loop at this instant will be a maximum. In position (d) the coil-sides are cutting the lines fairly rapidly, but not so rapidly as in position (c), so the e.m.f. will now have fallen slightly. When in position (e) the induced e.m.f. will

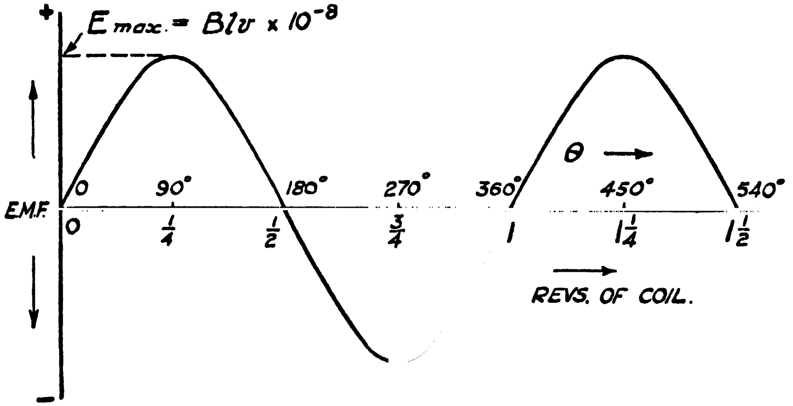


FIG. 170.—RELATIONSHIP BETWEEN E.M.F. AND POSITION OF COIL IN A SINGLE-LOOP GENERATOR.

have fallen almost to zero because the sides of the loop are moving almost parallel to the lines, and when the loop is again horizontal the e.m.f. will be again zero.

A similar process will occur in the next half-revolution, but the directions of the e.m.f's. induced in the two coil-sides will be reversed, since AB and CD are now interchanged. We thus have an *alternating e.m.f.* and also an *alternating current* if the brushes be connected through an external circuit.

When the loop has rotated from the horizontal position through an angle θ, the velocity with which the sides AB and CD are moving at right angles to the lines of force is $v \sin \theta$ (see Fig. 169b), and so the e.m.f., E volts, induced in one side of the loop is given by the equation

$$E = Blv.\sin \theta \times 10^{-8}.$$

Thus the magnitude of the e.m.f., E, is proportional to the sine of the angle, θ, through which the loop has rotated, and if E is plotted against θ, a *sine curve* will be obtained as in Fig. 170, the maximum value of the e.m.f. being $Blv \times 10^{-8}$ volts.

Obviously the total e.m.f. induced in the loop will, at any instant, be twice that induced in one of the sides. The direction of the e.m.f. will change every half-revolution, i.e., whenever the plane of the loop is horizontal. Whenever this plane is vertical the e.m.f. will be a maximum. The series of changes in the value of the e.m.f. obtained during 1 revolution of the loop is known as a *cycle*, and this cycle is repeated during each revolution of the loop. As was explained in Chapter II, the number of cycles occurring in 1 sec. is known as the *frequency* of the alternating e.m.f., the standard frequency for the mains supply being 50 cycles per sec. in this country. Although the a.c. generators (or *alternators* as they are usually termed) used in practice may appear to differ very much from the simple loop alternator described, their action is the same in principle, and the e.m.f's. they produce usually fluctuate in exactly the same manner.

Chiefly for economical reasons, alternating current is much more widely used nowadays than direct current, but for certain purposes (e.g., traction) d.c. is preferable, and in some cases (e.g., electrolysis) it is essential. *In order to produce direct current* in the external circuit of Fig. 168a it is necessary to *replace the slip rings by a commutator.* The type of commutator necessary for the single-loop armature of our simple generator is a two-part commutator, consisting of two semi-cylindrical metal segments, C_1 and C_2, as illustrated in Fig. 168b. The brushes are so placed that at the instant the e.m.f. is zero each brush makes contact with both segments, so short-circuiting the loop. At all other instants each brush makes contact with one segment only, as shown in Fig. 168b. Brush X is always connected to the top segment, which happens to be C_1 when the coil-side AB is higher than the coil-side CD, and C_2 when the coil-side CD is at the top. In other words, brush X is always connected through the commutator to the top conductor, in which the current always flows towards the commutator; similarly brush Y is always connected through the commutator to the bottom conductor, in which the current always flows away from the commutator. In the case where slip-rings are used (Fig. 168a), brush X is always connected to the coil-side AB, no matter whether it is above or below conductor CD;

so sometimes current flows out of brush X to the external circuit, but sometimes it flows towards X from the external circuit. But with the commutator of Fig. 168b the current *always* flows from the loop towards brush X, and so *always* flows from brush X to brush Y through the external circuit. In other words, X is the positive terminal of the d.c. generator of Fig. 168b.

With the single-loop commutator dynamo of Fig. 168b, the *brush e.m.f.* (i.e., the p.d. indicated by a high-resistance voltmeter connected across the brushes, with the external current path open-circuited), although uni-directional, still fluctuates from zero to a maximum value and back to zero during each half-revolution of the loop (see Fig. 171). The current in the external circuit pulsates in the same manner, and the graph of Fig. 171 could equally well be used to represent the fluctuations in the current flowing in the load resistance, *R* ohms, of Fig. 168b.

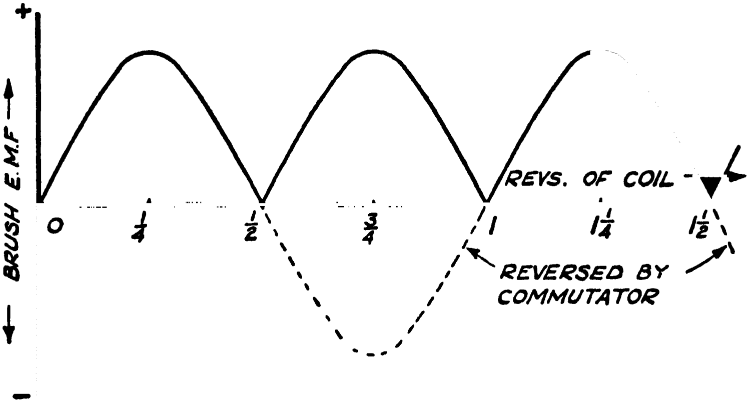


FIG. 171.—ILLUSTRATING THE EFFECT OF A TWO-PART COMMUTATOR.

Although the current in the external circuit of this simple dynamo is uni-directional, an alternating current still flows in the loop. This is also the case with practical dynamos—the direct current supplied is due solely to the ingenious manner in which the current is distributed from the armature coils to the external circuit.

90. The Practical Form of D.C. Generator

The pulsations in the brush e.m.f. obtained with the single-loop d.c. generator are almost completely smoothed out in the case of a commercial dynamo, since in such a case the armature consists not of two conductors only, but of a large number of conductors placed in slots round the periphery of a steel cylinder as described in Section 81(a) under the heading of the d.c. motor. As was stated in this section, the armature conductors are divided into an even number of groups joined in parallel across the brushes, each group consisting of a very large number of conductors in series. The conductors in any one of these groups are so arranged, with respect to the N. and S. poles, that the e.m.f's. induced in individual conductors are in directions such that they reinforce one another. The e.m.f's. in the various conductors, although acting the same way

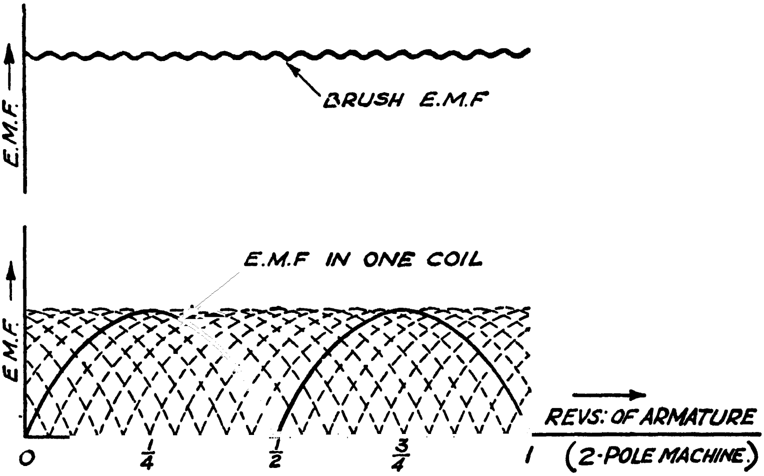


FIG. 172.—THE BRUSH E.M.F. OF A PRACTICAL D.C. GENERATOR.

round the circuit, reach their maximum values at different times, with the result that the total e.m.f. acquired by any one of the parallel groups remains roughly constant (see Fig. 172). This approximately constant e.m.f. available at the brushes is the sum of the e.m.f's. induced in all the conductors in any one group. The greater the number of armature coils in a group the steadier will be the brush e.m.f., and hence the current in the external circuit.

The magnetic flux is produced by electromagnets, two or more poles being used, the number varying from 2 poles for a small machine up to, say, 16 poles for a very large machine. The magnetic poles project inwardly from a circular yoke, the polarity being arranged alternately N. and S. by suitably arranging the directions of current flow in the *field coils* (i.e., the windings on the poles), these coils being connected in series with one another. A typical 4-pole frame is shown in Fig. 173, the dotted lines indicating the paths of the magnetic flux.

91. Types of D.C. Generators and Motors

Generators are classified according to the manner in which the *field current* is produced, the field current being the current which flows in the field coils. Excluding generators deriving their magnetic flux from permanent magnets, e.g., magnetos, there are 4 types:—

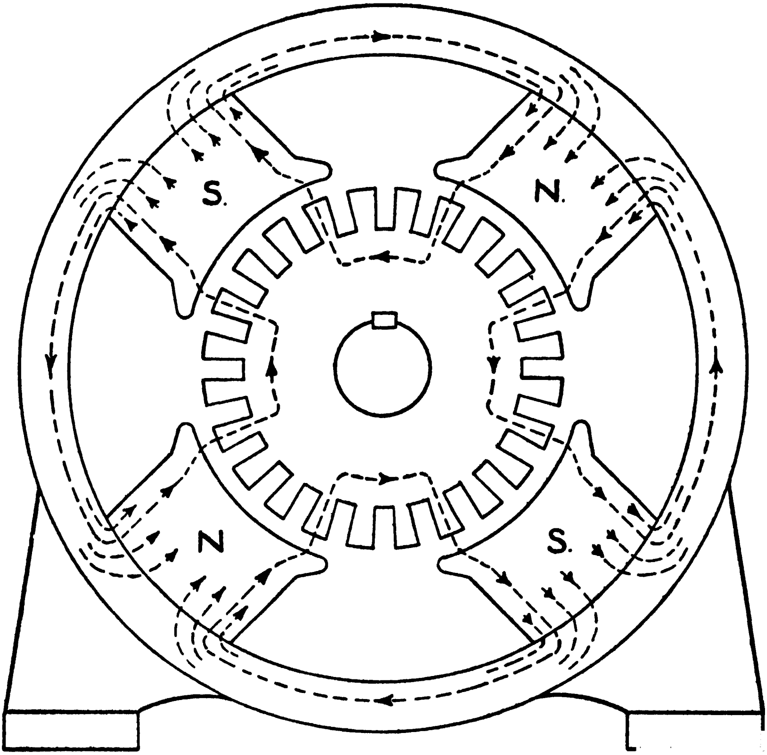


Fig. 173.—Illustrating the Paths of the Magnetic Flux in a Practical Four-pole Generator.

(a) *The Separately-excited Generator.* In this case the field current is taken from a separate supply, and is in no way dependent on the load current taken from the machine. Since a separate d.c. supply must be provided, this type of dynamo is used only in special cases, as, for instance, when a wide range of terminal voltage is required. Fig. 174 illustrates the manner in which the field coils are connected in the 4 types of d.c. generator, Fig. 174a depicting the separately-excited type.

(b) *The Series-wound Generator.* In this type the field coils are connected in series with the armature, and therefore take the same current. For this reason it is essential that the resistance of series field coils should be a small fraction of an ohm. This is easily arranged, since, owing to the fact that these coils take a heavy current, it is sufficient to have only a few turns on each coil in order to achieve the requisite number of ampere-turns. Series field coils therefore usually consist of several turns of heavy copper wire or strip. As will be seen in Volume III, series-wound generators are unstable in operation, and so, for this and other reasons, they are rarely used. Fig. 174b shows the connections.

(c) *The Shunt-wound Generator.* This type of machine is the one normally used for battery charging. The field coils are connected directly across the brushes of the machine (Fig. 174c), each coil consisting of a large number of turns of relatively thin copper wire. Thus the complete field circuit possesses a high resistance, usually of the order of several hundred ohms. Shunt field coils therefore carry a current which is only a small fraction of the armature current.

(d) *The Compound-wound Generator.* In this, the most generally useful form of d.c. generator, there are two sets of field coils, one of the series and the other of the shunt type. The shunt coils may be connected in either of the two ways shown in Fig. 174d. The " short-shunt " connection, shown by the continuous line, gives rise to a somewhat higher e.m.f. than the " long-shunt " connection, shown by the dotted line. The former is, therefore, the more usual arrangement.

There are similarly 4 types of d.c. motor, the only difference being that when the shunt and compound machines are used as motors, with the armature currents in the same directions as shown in Fig. 174, then the shunt field currents are reversed. Shunt motors are used when the speed has to be maintained approximately constant between no-load and full-load, series motors when a large starting torque is required, as in traction work, and compound motors when the load is of a fluctuating nature. The reasons that the various types of motor are used on such occasions are given in Volume III.

92. Comparison of Motor and Generator

The construction of a d.c. motor is the same as that of a d.c. generator, but in the motor a p.d. is applied to the brushes, with the result that the armature develops a driving torque due to the current flowing in it, whereas in the generator a torque is applied to the shaft carrying the armature and an e.m.f. is generated in the armature winding.

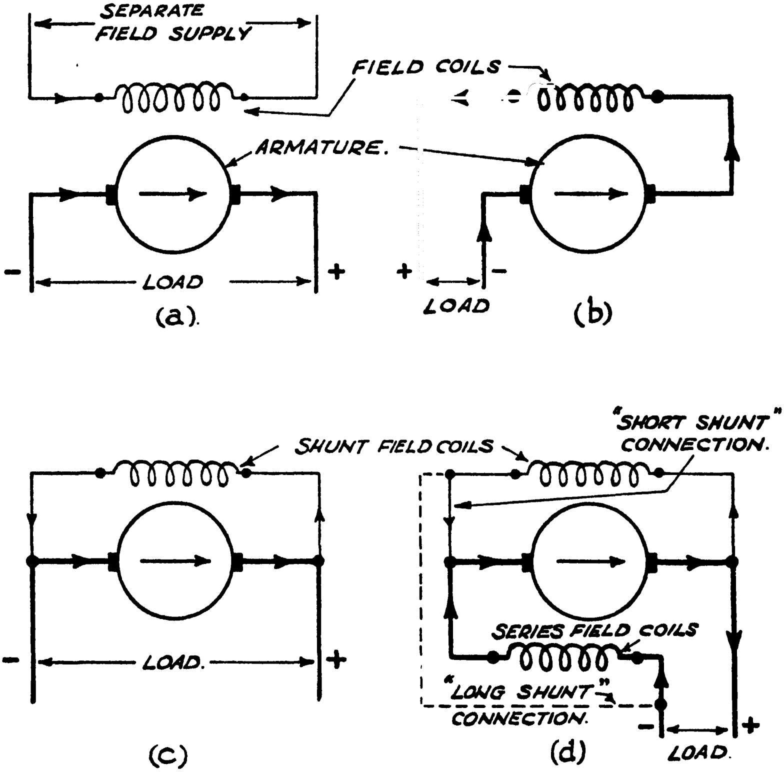


FIG. 174.—TYPES OF D.C. MACHINES.

(*a*) Separately excited.
(*b*) Series wound.
(*c*) Shunt wound.
(*d*) Compound wound.

In each case, however, we have rotating conductors carrying current and cutting lines of magnetic force. Because in each case the conductors carry current, the armature experiences a torque; because in each case the conductors cut lines of magnetic force, an e.m.f. is generated. Thus in both the motor and generator a torque and an e.m.f. are produced. The e.m.f. developed in the motor armature winding opposes the externally-applied p.d., and is often referred to as the *back e.m.f.* Similarly the torque developed by the dynamo armature opposes the externally-applied driving torque.

If the p.d. across the terminals of a d.c. machine be denoted by V volts, the e.m.f. generated in the armature by E volts and I_a amp. is the current flowing through the armature of resistance R_a ohms, then:—

For the generator: $V = E - I_aR_a$

For the motor: $V = E + I_aR_a$

The term I_aR_a represents the voltage drop in the armature due to its resistance. In the generator, the generated e.m.f., E volts, is equal to the internal voltage drop, I_aR_a, plus the voltage drop across the external circuit, i.e., $E = V + I_aR_a$. The student will recognise the similarity between this case and the case of a cell from which a current is being taken. In the motor the terminal p.d. V volts represents the p.d. applied to the brushes from some external source. When the motor is running, an e.m.f., E volts, is generated in a direction opposing this applied p.d., so that the net voltage is $(V - E)$. It is this net p.d. which determines the magnitude of the current taken by the motor since $I_a = \frac{V - E}{R_a}$. The higher the speed of the motor the greater the generated e.m.f., E volts, and thus the smaller the current taken by the armature. The equation given above for a motor will be recognised as similar to that for a cell on charge.

93. The Power Developed by Electrical Machines

When a conductor which is part of a closed circuit is moved so as to cut lines of magnetic force, an e.m.f. is induced in it and a current flows. But as soon as the current flows, since the conductor is in a magnetic field, it experiences a force which tends to oppose its motion. Thus, in Fig. 175, A represents a conductor which is being moved towards the left so as to cut lines of magnetic force at right angles. The right-hand rule indicates that the induced e.m.f. will cause a current to flow downwards into the plane of the paper, as shown in the diagram. But since this conductor now carries a current, it will experience a force which the left-hand rule indicates will be towards the right, thus opposing the force causing motion. This happens in the case of the conductor forming the armature winding of a dynamo.

In practice sufficient mechanical power must be supplied to the armature of a dynamo to keep it rotating against the total forces which tend to prevent motion. The most important of these forces is the one just mentioned, namely, that due to the current produced, but in addition there are opposing frictional and magnetic forces, and these incidental but unavoidable opposing forces are partly responsible for the inefficiency of the generator. If the only opposing forces were those due to the current produced, the electrical power developed would be exactly equal to the mechanical power supplied to rotate the armature. But in practice this ideal state of affairs cannot be achieved. There is rather less electrical power developed than mechanical power supplied. Similarly, in the case of a motor, rather more electrical power must be supplied than the mechanical power the motor develops.

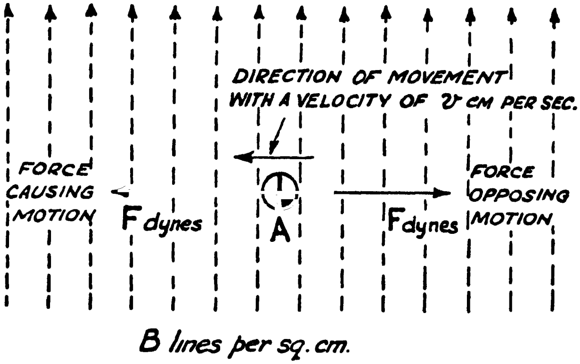


FIG. 175.—THE INTERDEPENDENCE OF THE DYNAMO AND MOTOR EFFECTS.

The efficiency of d.c. machines is discussed more fully in Volume III, but it is desirable that, even at this stage, the reader should be aware of the fundamental fact that the useful power developed is always less than that supplied. Students so often imagine that the power developed by a machine is power manufactured from nothing. It is always as well to bear in mind the **law of conservation of energy** which has already been stated in Section 36 of Chapter IV.

94. The Electromagnetic Force and E.M.F. Formulæ in Rationalised M.K.S. Units

We shall deal first with the formula giving the force exerted on a current-carrying conductor lying in a magnetic field (see Section 80). Referring to Fig. 154, if the length of the conductor XY is l' metres, and the flux density B' webers per sq. metre, the force, F' newtons, experienced by the conductor is given by the formula

$$F' = B'l'I \, . \, \sin\theta.$$

As before, when the conductor is perpendicular to the magnetic field, $\theta = 90°$, and the above formula becomes $F' = B'l'I$.

This formula may be used as the basis for defining the unit of *magnetic flux density* (see Section 78) in the rationalised m.k.s. system, as follows. *A magnetic field has unit flux density* (1 *weber per square metre*) *when a conductor carrying* 1 *ampere at right angles to that field has a force of* 1 *newton per metre acting upon it.*

The formula for the force between parallel current-carrying conductors, situated in air (see Section 82), may be derived in terms of rationalised m.k.s. units as follows. Referring to Fig. 162, let the distance between the conductors AB and CD be d' metres. The magnetising force, H_1' amp.-turn per metre, at Y due to the current I_1 amp. is $\frac{I_1}{2\pi d'}$ amp.-turn per metre and the corresponding magnetic flux density at Y, B_1' webers per sq. metre, is $\frac{\mu_0 I_1}{2\pi d'}$ webers per sq. metre. Thus the force, F' newtons, on each metre of conductor CD is $B_1'I_2$ or $\frac{\mu_0 I_1 I_2}{2\pi d'}$ newtons.

From this general result it follows that if the current in each of the two very long parallel conductors is 1 ampere, and their distance apart is 1 metre, then the force F' newtons, per metre of conductor length, is given by: $F' = \frac{\mu_0}{2\pi} = 2 \times 10^{-7}$. Hence the definition of the ampere in the rationalised m.k.s. system of units is as follows. *The ampere is that value of current which, when maintained in two parallel, straight conductors of infinite length and negligible cross-section, separated by a distance of* 1 *metre in vacuo, would produce between these conductors a force of* 2×10^{-7} *newtons per metre length.*

Turning now to the quantitative aspect of electromagnetically-induced e.m.f.s. We saw in Section 86 that, using c.g.s. units, if the flux linking with one turn changes at the rate of 10^8 c.g.s. lines per second, an e.m.f. of 1 volt is induced in that turn. As already stated in Section 78, the rationalised m.k.s. unit of magnetic flux is the *weber*, this being equivalent to 10^8 c.g.s. lines or maxwells. Hence, if the rate of change of flux linking with a single turn is 1 weber per second, an e.m.f. of exactly 1 volt is induced. The weber may be described, therefore, as a *volt-second.*

Alternatively, from the flux-cutting viewpoint, an e.m.f. of precisely 1 volt is induced in a conductor when it cuts magnetic flux at the rate of 1 weber per second. It follows that when a conductor of length l' metres moves in a direction perpendicular to its own length with a uniform velocity of v' metres per second, the direction of motion being also perpendicular to a uniform magnetic field of B' webers per sq. metre, the induced e.m.f., E volts, is given by:

$$E = B'l'v'$$

APPENDIX

95. The Relationship between the Force and E.M.F. Equations

Consider a conductor of length l cm. moving with a velocity of v cm. per sec. at right angles to a uniform magnetic field of flux density B lines per sq. cm. as depicted in Fig. 175. Suppose an e.m.f. of E volts is induced, giving rise to a current of I amperes. Then the power developed

$$\begin{aligned} &= EI \text{ watts} \\ &= EI \text{ joules per sec.} \\ &= EI \times 10^7 \text{ ergs per sec.} \quad . \quad . \quad . \quad . \quad \text{(a)} \end{aligned}$$

Supposing the force opposing motion to be F dynes; then, ignoring frictional forces, a force of F dynes must be applied to the conductor to move it to the left. But $F = \frac{1}{10}BlI$ and the work done per sec. by the force moving the conductor is the product of the force and the distance moved in 1 sec. by the conductor,

i.e., the power supplied $= Fv$ ergs per sec.

$$= \tfrac{1}{10}BlIv \text{ ergs per sec.} \quad . \quad . \quad . \quad \text{(b)}$$

Equating (a) and (b):—

$$EI \times 10^7 = \tfrac{1}{10}BlIv$$

i.e., $$\mathbf{E = Blv \times 10^{-8}}$$

In other words, the equation $E = Blv \times 10^{-8}$ may be derived from the equation $F = \frac{1}{10}BlI$, the proof of which is given in the Addendum.

Exercises on Chapter VIII

Any additional data required for the solution of these exercises may be obtained from the tables at the end of this book.

1. Calculate the torque exerted on the coil of a M.C. instrument if the coil possesses 50 turns, carries a current of 15 mA, is of rectangular shape 2 cm. × 2·5 cm. and lies in a radial magnetic field where the flux density is 1200 lines per sq. cm.

[450 dyne-cm.]

2. If a copper bar, lying in a magnetic field of 750 lines per sq. cm. perpendicular to the bar, carries a current of 1000 amp., express in lb. wt. the force experienced by each foot of copper bar.

[5·136.]

3. If the average flux density in the air-gap of a d.c. motor is 7500 lines per sq. cm. and there are 560 armature conductors, each of length 15 cm., lying under the poles, estimate (a) the force, in lb. wt. exerted on each armature conductor when carrying the full-load current of 50 amp., and (b) the total full-load torque, in lb.-ft. The overall diameter of the armature is 35 cm.

[(a) 1·264; (b) 406·2.]

4. A straight conductor of length 5 ft., carrying a current of 200 amp., is inclined at 40° to the horizontal and lies in a uniform vertical magnetic field of 25 lines per sq. cm. Calculate the magnitude of the force exerted on the conductor, in lb. wt., and specify the direction of this force.

[0·131 lb. wt., horizontally and perpendicular to the vertical plane containing the conductor.]

5. Two parallel busbars, each carrying 10,000 amp., are spaced 6 inches apart. Determine the approximate force exerted on each bar, in lb. wt. per foot of length.

[About 9.]

6. The axle of a train, moving horizontally at 60 m.p.h., is 5 ft. long and cuts a uniform magnetic field of 20 lines per sq. cm. Calculate the e.m.f. induced in the axle when the direction of the

magnetic field is (a) vertical and (b) inclined at 60° to the horizontal, whilst remaining perpendicular to the length of the axle.

[(a) 0·0818 V; (b) 0·0708 V.]

7. The 200-turn coil of a simple loop generator is 15 cm. long × 10 cm. wide and rotates at 3000 r.p.m. about a central axis parallel to the longer side of the coil in a uniform magnetic field of 1000 lines per sq. cm. Calculate the maximum value of the e.m.f. induced in the coil.

[94·26 V.]

8. If the flux from each pole of a 4-pole motor is 1·5 megalines and the armature rotates at 1500 r.p.m., calculate the average e.m.f. generated in each conductor.

[1·5 V.]

9. A d.c. machine with an armature resistance of 0·5 ohm generates an e.m.f. of 200 volts when run at 1500 r.p.m. If the armature current is 15 amp. and the speed is maintained at 1500 r.p.m., calculate the terminal voltage (a) when the machine is run as a generator, and (b) when run as a motor.

[(a) 192·5 V; (b) 207·5 V.]

10. Calculate the quantity of electricity induced in a 2500-turn, 20-ohm solenoid, which surrounds a bar-magnet at its mid-point, during the removal of the solenoid to a point some distance from the magnet, if the total flux in the magnet is 50,000 lines.

[0·0625 C.]

11. If a magnet is inserted into a 10-ohm solenoid at a speed of 100 cm. per sec., the change of flux-linkages being at the rate of 5×10^6 maxwell-turns per sec., what will be the average current flowing during the action—the terminals of the solenoid being short-circuited—and what will be the average force of repulsion experienced by the magnet?

[5 mA; 25 dynes.]

12. Two coils, A and B, possessing 50 turns and 4000 turns respectively, are wound on a single iron core, and a current of 2 amp. passed through coil A produces a flux of 10,000 lines in the core. Calculate the quantity of electricity which will flow through coil B on reversing the current in coil A, the resistance of coil B being 160 ohms and its terminals being short-circuited. If the current

reversal occupies one-twentieth of a second calculate the average current flowing in coil B, the average e.m.f. induced and average power supplied. How much work is involved in the reversal of the current in coil A?

[0·005 C; 0·1 A; 16 V; 1·6 W; 0·08 J.]

13. An armature conductor is 60 cm. in length and carries a current of 200 amp. If the average flux density in the air-gap of the machine is 0·725 Wb/m.^{2}, calculate the force on the armature conductor, (a) in newtons and (b) in kg. wt.

[(a) 87 newtons; (b) 8·87 kg. wt.]

14. Two parallel busbars are each 2·5 metres long and are spaced 10 cm. apart. Using the rationalised m.k.s. system of units, calculate the total force, in kg. wt., exerted between the busbars when the current in each bar attains a value of 20,000 amp. under fault conditions.

[204 kg. wt.]

15. A six-pole generator has a magnetic flux of 22·5 milli-webers in the air-gap under each pole. If the armature rotates at 1200 r.p.m., calculate the average value of the e.m.f. induced in each armature conductor.

[2·7 volts.]

16. A magnetic flux of 1·5 milli-webers links with all the turns of a 200-turn coil having a resistance of 10 ohms. If the terminals of the coil are short-circuited, calculate (a) the average e.m.f. induced in the coil when the flux is completely reversed in 0·25 sec. and (b) the quantity of electricity which flows in the coil.

[(a) 2·4 volts; (b) 0·06 coulomb.]

CHAPTER IX

ELECTROSTATICS

96. The Two Properties of a Potential Difference

IF between two conducting bodies there exists a potential difference, then (a) the two bodies will exert a force of attraction upon each other, and (b) if the bodies be connected by a conducting material a current will flow along this material, the strength of the current being directly proportional to the potential difference.

This second property of a potential difference is one which was thoroughly discussed in Chapter II, and forms the basis of the principle employed by the normal type of voltmeter. In using an electromagnetic type of voltmeter to measure the p.d. between two conductors or between two points on the same conductor, the voltmeter bridges the two points, so providing a conducting path along which flows a current proportional to the p.d. to be measured. The voltmeter is actually operated by the current which flows through it.

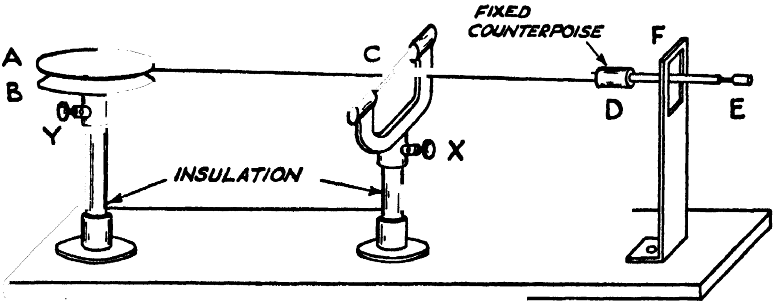


FIG. 176.—APPARATUS FOR DEMONSTRATING ELECTROSTATIC FORCES.

The first property, however, is one of which no mention has yet been made. Forces due to magnetic fields have been discussed, but we here become acquainted with a force caused simply by a potential difference, a force that occurs even in the absence of a current or a magnetic field. Such forces, known as *electrostatic* *forces*, are normally very minute, and rather delicate apparatus, such as that illustrated in Fig. 176, is necessary to demonstrate their existence.

In the apparatus shown, A and B are two thin aluminium discs, A being supported by a light aluminium tube pivoted at C. The small counterpoise E may be very slowly moved along a finely-threaded rod attached to the end of the tube. By adjustment of E the plate A may be arranged to lie a few millimetres above the plate B. A stop at F prevents the plate A moving too far either upwards or downwards. If the two terminals X and Y are connected to opposite poles of the supply mains (200–250 V.), the mains p.d. is applied across the two discs, A and B, and A is immediately attracted towards B. The stop F prevents the two plates touching and so short-circuiting the supply, but a thin sheet of mica attached to the top of the lower disc B is recommended as a further precaution.

The force of attraction is directly proportional to (a) the square of the potential difference and (b) the area of the plates. It is inversely proportional to the square of the distance between the plates. As an illustration of the smallness of electrostatic forces, two discs, each of 3 inches diameter, placed half an inch apart with a p.d. of 300 volts applied to them, would experience a force of attraction of only about 1 milligram weight. It is not surprising that these small electrostatic forces are of very little practical importance, apart from their use in electroscopes and electrostatic voltmeters.

97. Electroscopes, Electrometers and Electrostatic Voltmeters

By arranging one of two adjacent metallic discs, sheets or plates so that it is free to move towards the other against some controlling force, it is possible to employ the attraction property of a p.d. to measure the potential difference by observing the position to which the movable plate is deflected. Moreover, since attraction results no matter which plate is at the higher potential, such an instrument may be used to measure alternating p.d's.

Fig. 177 illustrates the construction of a *gold-leaf electroscope*. It consists of a metal case with two glass sides, through the top of which protrudes a brass rod insulated from the case. A horizontal brass disc is usually attached to the top of this rod, whereas the bottom end of the rod terminates in a metal plate and a strip of gold leaf, the two lying parallel to one another and side by side, when no p.d. exists between the rod and case. Sometimes two leaves are employed, the second one being in place of the metal plate. On establishing a p.d. between the rod and case, the gold leaf, being at the potential of the rod, is attracted towards the case and rises as shown in the diagram. When two leaves are used these diverge on application of the p.d. to the leaves and case. The greater the applied p.d. the greater the deflection, the deflecting torque being opposed by the gravitational effect on the deflected leaf or leaves.

Such an instrument is normally used for detecting a p.d. or for indicating an increase or decrease in p.d. But by marking a scale on one of the glass faces of the case and applying known p.d's. to the leaf and case it is possible to calibrate the instrument, and so use it as an *electrometer*. Unfortunately, unless p.d's. of the order of a thousand volts are employed the deflections are usually small. But by viewing the leaf through a microscope or projecting a shadow of the leaf on to a screen, the deflection may be so magnified as to make the instrument of considerable use for indicating variations in p.d's. of the order of 200 volts. Fig. 178 shows a projection electroscope useful for experimental work when employing the supply mains, a small projector lamp, a leaf electroscope and a lens system being contained in a single housing so as to be immediately available for projection purposes.

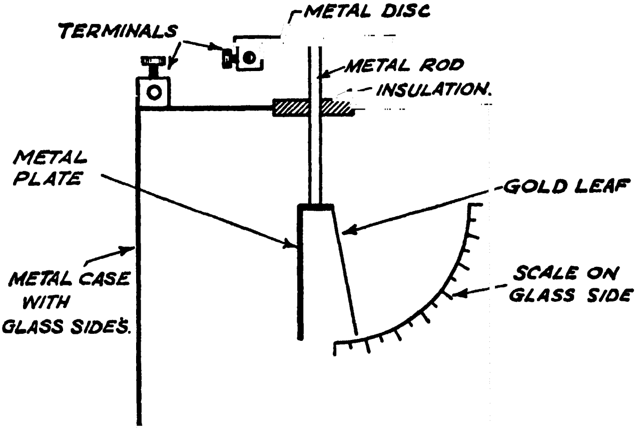


FIG. 177.—THE GOLD-LEAF ELECTROSCOPE.

Electrostatic voltmeters are really electrometers modified so as to be robust enough for general engineering practice. All types of electrometer and electrostatic voltmeter depend for their operation on an electrostatic force of attraction between two sets of plates, one fixed and one movable. In one type a vane, or set of vanes, rotates about an axis so as to lie opposite a fixed plate or set of plates; in another type, instead of a mere rotation, there is a bodily movement of one plate towards another.

Electrostatic voltmeters are particularly useful for high-voltage measurements, and for this purpose have certain advantages over other types of voltmeter, giving equally correct measurements on a.c. and d.c. circuits, being free from errors which arise due to the presence of magnetic fields, and above all possessing the great merit that they take no current at all from a d.c. circuit, and very little from an a.c. circuit. They are thus employed in cases where the current that would be taken by other types of voltmeter would seriously reduce the p.d. to be measured.

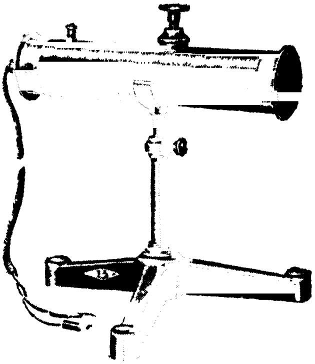


FIG. 178.—A PROJECTION ELECTROSCOPE.

98. The Charging and Discharging of a Simple Condenser

Any arrangement consisting of two conducting bodies insulated from one another is known as a **condenser or capacitor.** Thus a leaf electroscope could be regarded as a condenser, since it consists of the leaf system and the case separated by an insulator. In Fig. 179, X and Y represent two sheets of metal (e.g., tin foil), each about a yard square and placed face to face, but separated from each other by a sheet of waxed paper, so constituting a simple condenser. If such a condenser be connected in series with a very sensitive galvanometer, a resistance of 1 megohm and a double-pole switch connected to a 200-volt dry battery as in Fig. 179a, then on closing the switch it might be expected that the galvanometer would show no deflection owing to the absence of a closed circuit, due to the waxed paper intervening between the metal sheets. Actually the galvanometer would indicate that a momentary current flows when the switch is closed; at the same time an electroscope connected across the plates X and Y would indicate that the plates had acquired a p.d. of 200 volts. In performing this experiment it is important to insulate every item of the circuit from the bench so as to avoid a permanent deflection of the galvanometer due to minute currents leaking over the surface of the bench. Satisfactory insulation can easily be obtained by placing the apparatus on paraffin-wax blocks or sheets of waxed paper.

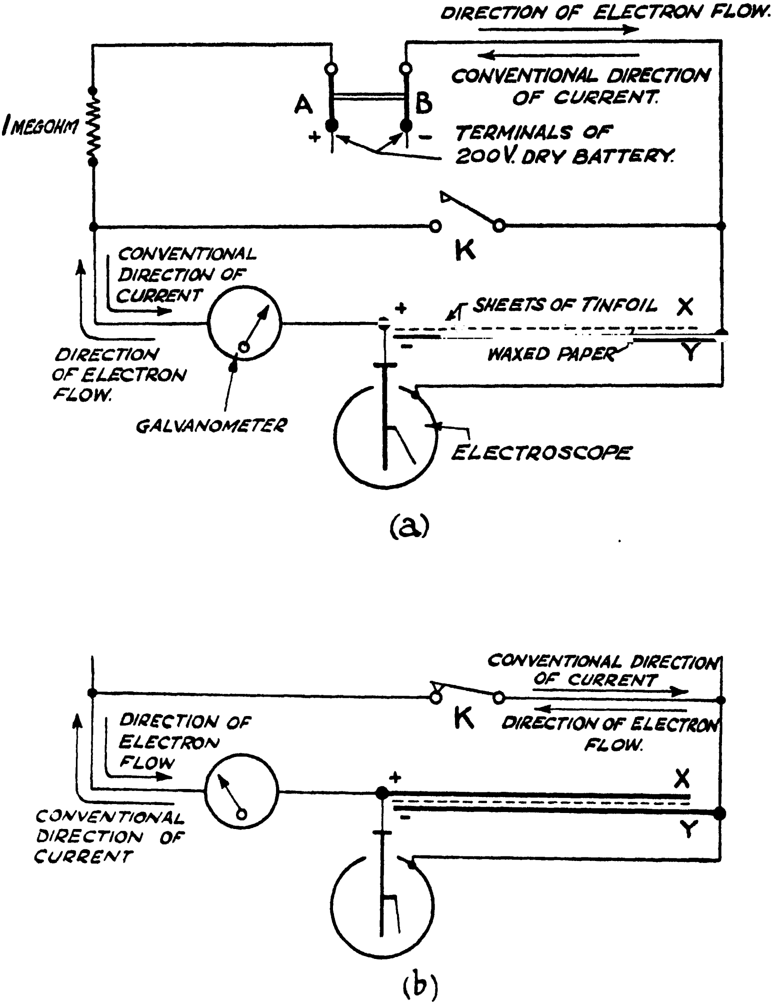

DIRECTION OF ELECTRON FLOW.
CONVENTIONAL DIRECTION OF CURRENT.
A
B
+
-
TERMINALS OF 200V. DRY BATTERY.
1 MEGOHM
K
CONVENTIONAL DIRECTION OF CURRENT
DIRECTION OF ELECTRON FLOW.
GALVANOMETER
+
SHEETS OF TINFOIL
X
-
WAXED PAPER
Y
ELECTROSCOPE
(a)
CONVENTIONAL DIRECTION OF CURRENT
K
DIRECTION OF ELECTRON FLOW.
DIRECTION OF ELECTRON FLOW
CONVENTIONAL DIRECTION OF CURRENT
+
X
-
Y
(b)

FIG. 179.—APPARATUS FOR DEMONSTRATING THE CHARGING AND DISCHARGING OF A SIMPLE CONDENSER.

The explanation of the momentary current is as follows. When the switch is open there is a surplus of electrons on the negative terminal B of the battery and a deficiency on the positive terminal A. On each of the plates X and Y there is neither a surplus nor a deficiency, the number of electrons being equal to the number of protons. When the switch is closed electrons flow from terminal B to plate Y and from plate X to terminal A until the potential, which depends upon the extent to which the electrons are crowded, is the same on plate Y as on terminal B and the same on plate X as on terminal A. The p.d. between plates X and Y is then equal to the p.d. between terminals A and B. It is, of course, conventional to say that the current flows from A to X and from Y to B, and also that plate X acquires a positive charge of, say, $+Q$ coulombs and plate Y a negative charge of $-Q$ coulombs. Although the positive and negative charges acquired by the two plates will be of the order of only a millionth of a coulomb, it should be realised that a millionth of a coulomb is equivalent to several billion electrons. Thus this condenser when *charged* will have a surplus of several billion electrons on one plate and a deficiency of an equal number of electrons on the other plate.

If the two plates X and Y are now short-circuited through the galvanometer by closing the key K, the condenser becomes discharged, as will be indicated by the electroscope connected across the plates. Discharge occurs by the passage of a momentary current through the galvanometer. What actually happens is that electrons flow from the negative plate Y through the galvanometer to wipe out the deficiency of electrons on the positive plate X (see Fig. 179b). This is confirmed by the momentary deflection of the galvanometer in the *opposite direction* to that occurring during the charging process. (When performing the latter part of this experiment it is advisable to close the key K with the battery still connected to the circuit; if the double-pole switch is first opened, some of the charge on the simple condenser will leak away before K can be closed, due to the imperfect insulation between the various parts of the circuit.)

99. " Electrostatics "

Electrostatics is the branch of electro-technology dealing with phenomena occurring because of electrons stored up in the above abnormal manner. Although a condenser usually consists of two sets of parallel *plates* or *sheets* separated by a thin sheet of insulation, it should be borne in mind that electrostatic conditions exist in other cases. It has already been pointed out that an electroscope is a form of condenser. So is an open switch. Whenever a switch is opened, thus stopping a current flow, we have electrostatic conditions, the two poles of the switch having a p.d. between them, the switch thus acting as a charged condenser. Two conductors in a multi-core cable form a condenser; they may be many miles long, and their behaviour as a condenser may have quite important effects.

Although it is possible to store positive electricity on one plate of a condenser, it is only because negative electricity is stored on the other plate. On a charged condenser, considered as a whole, there is no charge ! As a whole there is neither a surplus nor a deficiency of electrons. Nevertheless the fact that one plate possesses a surplus and the other a deficiency is very important, electrons stored in this manner possessing energy and producing a p.d. between the plates. A charged condenser thus behaves as a reservoir of electrical energy. The reason it is possible for a current to flow from one plate of a charged condenser to the other through a connecting wire is that the energy necessary to supply the current is stored in the condenser.

100. The Relation between the Charge on a Condenser and the P.D. between its Plates

When one plate of a condenser possesses a charge of $+Q$ coulombs and the other plate a charge of $-Q$ coulombs, it is said that a charge of Q coulombs is stored on the condenser, since this is the charge that flows from one plate round to the other when the two are joined by a connecting wire. By performing an experiment as described below, it can be shown that *the charge which can be stored on a condenser is directly proportional to the p.d. applied across its plates.* This relationship is of fundamental importance.

Experiment 17. *To investigate the relationship between the charge on a condenser and the p.d. applied to it.*

Connect up the circuit shown in Fig. 180, G being a ballistic galvanometer (see Section 88 of the previous chapter), C a simple condenser of the type described in Section 98, R a resistance of a few thousand ohms, A and B the terminals of a variable-voltage d.c. supply such as a dry battery and S a two-way switch. Put the switch S in position 1, when the condenser will be charged to the p.d. of the supply, the magnitude of which is indicated by the voltmeter V. Throw the switch over to position 2, when the condenser will be discharged through the ballistic galvanometer. Observe the first throw of the galvanometer; this is directly proportional to the charge passing through the galvanometer. Alter the p.d. of the supply, put the switch in position 1 again and repeat, recording the readings of the voltmeter and ballistic galvanometer in a table as shown below.

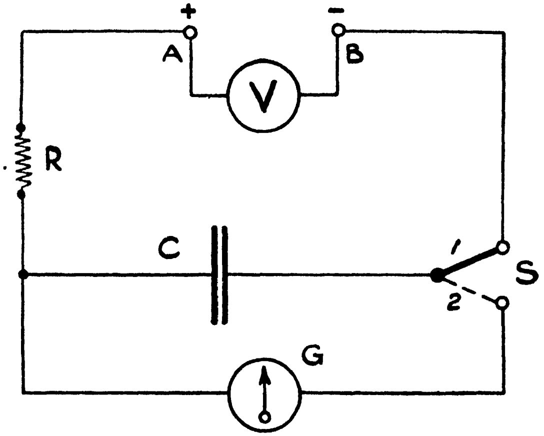


FIG. 180.—THE CIRCUIT EMPLOYED IN EXPT. 17.

Voltmeter reading.	Reading of ballistic galvanometer.	$\frac{\text{Reading of galvanometer}}{\text{voltmeter reading}}$.

Within the limits of experimental error it will be found that the values in the third column of the table are constant, showing that

$$\frac{\text{The charge on the condenser}}{\text{The p.d. across the condenser}} = \text{a constant}$$

or, in symbols $$\frac{Q}{V} = C$$

101. Capacitance

If the above experiment be repeated with another condenser having plates of different size, it will be found that although $\frac{Q}{V}$ remains constant as V is varied, yet the value of the constant will be different from that obtained when using the first condenser. Thus the amount of charge which can be stored on a condenser for a given p.d. depends upon the dimensions of the condenser, and *the ratio of the charge on a condenser to the p.d. between its plates is known as the capacitance of the condenser.* Thus the constant, C, in the above equation, represents the capacitance of the condenser used. The symbol C is actually the standard symbol used in connection with capacitance. Since the capacitance of a condenser is measured by the charge which must be given to it to raise its p.d. by unity, *a condenser will have unit capacitance if* 1 *coulomb raises the p.d. by* 1 *volt. The unit of capacitance is termed the* **farad** (*symbol:* F).

Thus the relationship between charge, p.d. and capacitance is:—

$$\frac{\textbf{Charge}}{\textbf{P.D.}} = \textbf{Capacitance}$$

or, in terms of the practical units of charge, p.d. and capacitance:—

$$\frac{\textbf{The no. of coulombs}}{\textbf{The no. of volts}} = \textbf{The no. of farads}$$

This very important relationship may be expressed symbolically in the following three forms:—

$$C = \frac{Q}{V}, \quad Q = CV \quad \text{and} \quad V = \frac{Q}{C}$$

A capacitance of 1 farad is found in practice to be abnormally large, a much more useful unit being the *micro-farad* (μF), which is, of course, a millionth of a farad. In radio circuits, condensers are often used with capacitances of the order of a few micro-micro-farads, a *micro-micro-farad*, sometimes termed a *pico-farad* (1 $\mu\mu$F), being equal to 10^{-12} of a farad.

The capacitance of a parallel-plate condenser is determined by three factors: it is directly proportional to the area of overlap of the two plates, inversely proportional to the distance between them and dependent upon the nature of the **dielectric,** *which is the name given to the insulating medium between the two plates.*

102. Practical Condensers

A condenser with plates each having the size of a postage stamp, placed 1 mm. apart and with air as the dielectric, has a capacitance of about 4 $\mu\mu$F, and a condenser with plates having an area as large as 20 square metres and placed 1 mm. apart with mica as the dielectric will have a capacitance of only 1 μF. It is therefore obvious that condensers having a capacitance of the order of a micro-farad cannot, for reasons of space, take the form of two flat sheets.

Several types of condenser are available, the most common being the rolled-paper or Mansbridge type. This consists of two very thin strips of waxed paper, each backed with an extremely thin layer of tinfoil, placed together as shown in Fig. 181a, and then rolled in spiral form as shown in Fig. 181b. In another type of rolled-paper condenser the paper is impregnated with an insulating oil and the complete condenser immersed in a similar type of oil. The wax and oil used in these paper condensers not only prevent the absorption of moisture by the paper, but result in an increased capacitance. Such condensers also possess fixed values of capacitance.

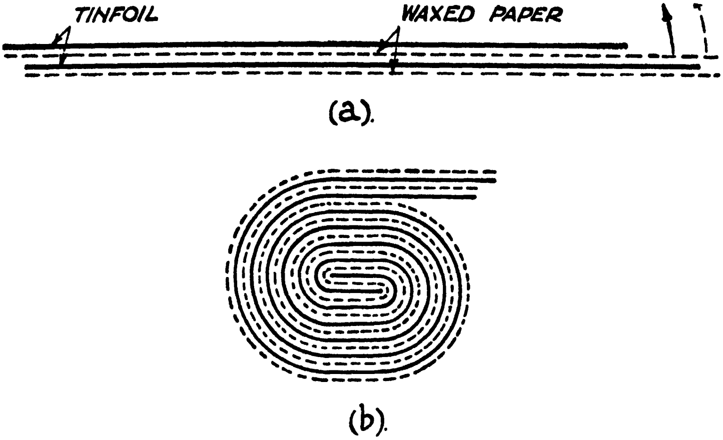


FIG. 181.—ILLUSTRATING THE CONSTRUCTION OF A ROLLED-PAPER CONDENSER.

Another type of *fixed condenser* consists of a large number of flat sheets of metal foil interleaved with mica or waxed paper, alternate sheets of foil being connected together and to one of two terminals as shown in Fig. 182. The sheets are normally clamped firmly together between end-plates. The effective area of such a condenser is equal to the area of overlap of two successive sheets of foil multiplied by the number of sheets of dielectric.

A further type of fixed condenser—the electrolytic type—is particularly useful where a large capacitance is desired in a condenser of comparatively small size, and hence *electrolytic condensers* are widely used in radio circuits. One plate consists of a corrugated aluminium rod which is immersed in a solution or paste containing ammonium or aluminium borate, or some similar salt, enclosed in an aluminium or copper can. The solution or paste acts as the second plate, and the dielectric is a thin film of aluminium oxide formed on the aluminium rod when a direct current is passed from the rod to the case. This current soon ceases, owing to the formation of the insulating film of oxide. The film is exceptionally thin, being of the order of 10^{-5} cm., thus accounting for the large capacitances possessed by these condensers in spite of their small plate area.

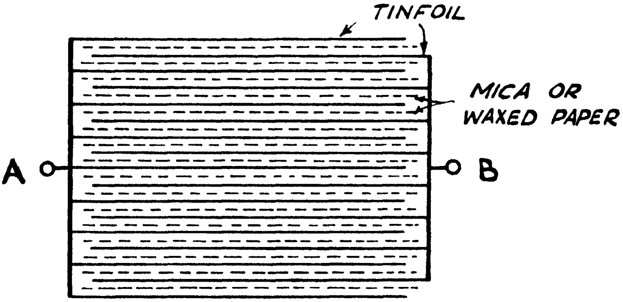


FIG. 182.—DIAGRAMMATIC ARRANGEMENT OF A MULTI-PLATE FIXED CONDENSER.

Normally electrolytic condensers can only be used in cases where the central aluminium rod can be maintained at a higher potential than the case. Moreover, such condensers are not very suitable for large p.d.'s, since the film is very easily broken down by large voltages; but where condensers with reasonably large capacitances are required for medium d.c. voltages they are preferred to paper condensers, on account of their relatively low cost.

Another very important type of condenser is the *variable capacitance air condenser*, widely used in radio circuits for tuning. The reader is probably familiar with the general appearance of small condensers of this type, as used in radio receivers. Such condensers normally consist of two sets of interleaving plates or vanes, one set of vanes being capable of rotation into the spaces between the plates of the other set. Thus the area of overlap with the fixed set can be varied. Variable condensers of a similar type are sometimes oil-immersed, one function of the oil being to increase the capacitance. Alternatively, the dielectric may consist of a gas under pressure, the interior of a large variable condenser of this type, employed in radio transmitting equipment, being illustrated in Fig. 183. The range of capacitance values for this condenser is from 130 to 420 $\mu\mu$F and the dielectric normally employed is capable of withstanding p.d's. up to about 90 kV before flash-over occurs.

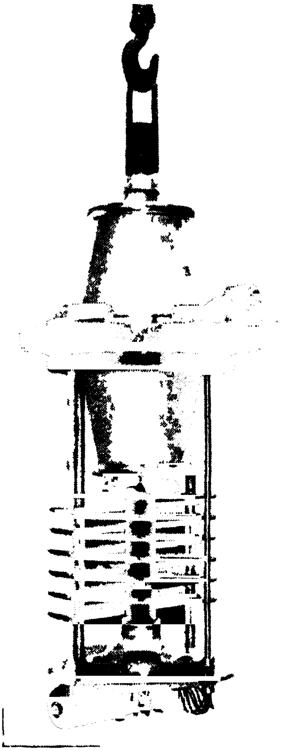


FIG. 183.—INTERIOR VIEW OF A COMPRESSED-GAS DIELECTRIC VARIABLE CONDENSER.

103. Condensers in Parallel

Condensers connected up as shown in Fig. 184 are said to be connected *in parallel*. The total charge, Q coulombs, flowing from the positive supply terminal in Fig. 184 on to the positive plates of the three condensers is obviously equal to the sum of the separate charges Q_1, Q_2 and Q_3 coulombs, i.e.:—

$$Q = Q_1 + Q_2 + Q_3$$

If C farads be the effective capacitance of the group as a whole, then $Q = CV$. Also, if C_1, C_2 and C_3 farads be the capacitances of the individual condensers, then, since each condenser is subjected to a p.d. of V volts:—

$$Q_1 = C_1V; \quad Q_2 = C_2V; \quad Q_3 = C_3V$$
$$\therefore \; CV = C_1V + C_2V + C_3V$$
$$\text{i.e., } \mathbf{C = C_1 + C_2 + C_3}$$

The similarity with the case of resistors connected in parallel is most marked, a common p.d. being applied across each resistor, and the effective conductance, G mhos, being equal to the sum of the conductances, G_1, G_2 and G_3 mhos, of the individual resistors.

A box of condensers connected in parallel, each condenser supplied with its individual switch or plug as shown in Fig. 184, is, on occasion, very useful. In such a box of four condensers, having capacitances of 1, 2, 2 and 5 μF respectively, any capacitance from 1 μF to 10 μF, in steps of 1 μF, is instantly available across the terminals A and B, by the simple process of closing and opening switches.

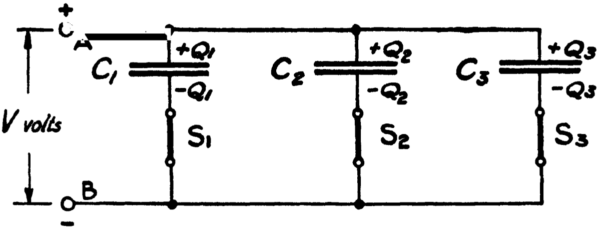


Fig. 184.—Condensers Joined in Parallel.

104. Condensers in Series

Condensers connected up as shown in Fig. 185 are said to be connected *in series*. When a p.d. of V volts is applied to such an arrangement, a charge of Q coulombs flows from the positive supply terminal on to plate A, off plate B on to plate C, off plate D on to plate E, and finally off plate F to the negative supply terminal. Thus each condenser is charged with Q coulombs. (This, of course, is merely the conventional way of regarding the process; actually a number of electrons moves from the negative supply terminal on

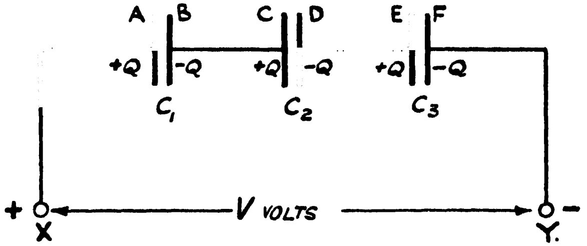


Fig. 185.—Condensers Joined in Series.

to plate F, from plate E on to plate D and so on, this same number of electrons moving from plate A on to the positive supply terminal.)

If the capacitances of the various condensers are C_1, C_2 and C_3 farads respectively and the p.d.'s across the various condensers V_1, V_2 and V_3 volts then $V_1 = \frac{Q}{C_1}$, $V_2 = \frac{Q}{C_2}$ and $V_3 = \frac{Q}{C_3}$. The total drop in potential, V volts, from X to Y will be the sum of V_1, V_2 and V_3 volts.

Thus, $$V = V_1 + V_2 + V_3$$

Regarding the effective capacitance of the group of condensers as C farads, then, since the group receives Q coulombs from the supply, $V = \frac{Q}{C}$.

$$\therefore \quad \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

i.e., $$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Once again there is a close analogy with the case of resistors connected in series. In such a case the same current flows through each resistor in turn, just as in the case of condensers connected in series the same charge flows on to each condenser. Moreover, the total voltage drop across the series of resistors is equal to the sum of the p.d.'s across the separate resistors. Further, the sum of the reciprocals of the conductances is equal to the reciprocal of the effective conductance of the group as a whole.

It should be noted that when condensers are connected in series, the effective capacitance of the whole group is less than the least of the capacitances of the individual condensers.

105. The Formula for the Capacitance of a Condenser

In Section 101 it was stated that the capacitance of a condenser was determined by three factors. That this is the case may be demonstrated by connecting up the circuit of Fig. 180 and repeating Experiment 17 firstly with condenser plates of different size, then, keeping the area of the plates constant, with different distances between the plates, and finally with different dielectrics of the same thickness. If in these three experiments a fixed p.d. is employed, the galvanometer throw will be directly proportional to the area of the plates, inversely proportional to the thickness of the dielectric and dependent upon the type of dielectric used. In performing these tests two rigid condenser plates capable of being held firmly in position should be used. Moreover, their separation should be capable of fine adjustment.

If, when the whole of the space between the plates is occupied by a certain dielectric of thickness, say, 2·5 mm., a galvanometer throw of 76 divisions is obtained, and if, on removing the dielectric without altering the separation of the plates, a throw of 20 divisions is obtained, the dielectric is said to have a **relative permittivity** of $\frac{76}{20}$, i.e., 3·8. In other words, the relative permittivity, which, for the time being, will be referred to merely as the *permittivity* (symbol ϵ, pronounced *epsilon*) of a dielectric, x, is defined as the ratio of the capacitance of a condenser with dielectric x to the capacitance of an identical air condenser. Permittivity is sometimes referred to as *dielectric constant*, or alternatively as *specific inductive capacity.*

So the formula for the capacitance of a condenser is of the form

$$C = \text{a constant} \times \frac{\epsilon A}{d}$$

where A represents the effective area of the plates, d their distance apart and ϵ the permittivity of the dielectric. The value of the constant depends upon the units employed. Where the area of the plates is A sq. cm., the distance apart d cm. and the capacitance of the condenser C farads, it can be shown from both theory and experiment that

$$C = \frac{\epsilon A}{4\pi d \times 9 \times 10^{11}}$$

The reason for the cumbersome constant is explained in Section 108, where another system of electrical units—the c.g.s. electrostatic system—is defined. It will be seen there that the awkward figures are partly due to the fact that 9×10^{11} c.g.s. electrostatic units of capacitance are equal to 1 farad.

The above formula applies only to the *parallel-plate* condenser, i.e., one in which the dielectric has a uniform thickness and in which the areas of the two plates are equal.

106. The Uses of Condensers

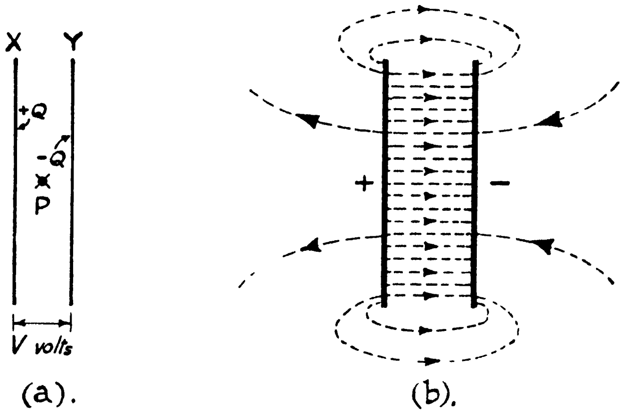
In this chapter we have been chiefly concerned with defining and describing a condenser, stating how it can be charged and what we mean by its capacitance, and on what factors the size of its capacitance depends. We have not attempted to discuss how condensers behave when connected in various kinds of circuit. For example, when a condenser is charged by the application of a d.c. potential difference the charging current is not a steady one, but varies in a very special manner, and use can be made of this, as, for example, in vehicle-operated traffic signals. The value of a high resistance can also be determined by measuring the time taken for part of the charge on a condenser to leak away through the resistor. A break in a cable may be located by regarding the cable as a condenser and measuring its capacitance. Sparking which occurs at a contact-breaker can be suppressed with the aid of a suitable condenser. Such practical applications cannot, however, be appreciated until the properties of condensers have been more fully discussed, and this is done in Chapter V of Vol. II.

Condensers are of particular importance in radio and a.c. engineering. Although a condenser cannot transmit power in a d.c. circuit, it can in an a.c. circuit, strange as this may appear. Condensers in radio circuits are frequently employed for the purpose of preventing the passage of direct currents, while at the same time permitting the flow of alternating currents. They are also used in radio circuits in the process of separating two alternating currents of widely different frequencies. Tuning a radio receiver to a particular wavelength is also done by adjusting the capacitance of a variable air condenser. A further very important use of condensers is for the reduction of the current taken from the supply by a.c. equipment possessing what is known as a low power factor (see Chapter VIII, Vol. II). Unfortunately, not until a.c. theory has been studied can the significance of these a.c. applications be perceived.

107. The Electric Field

Fig. 186a represents a simple condenser consisting of two plates X and Y charged with Q coulombs, the p.d. between the plates being V volts. The existence of this p.d. implies that the energy carried by a charge of 1 coulomb on plate X is greater than that carried by a charge of 1 coulomb on plate Y. The p.d. between two points has been defined in Chapter II, Section 10, as the energy given up by a coulomb as it moves from the point of high to the point of low potential. Conversely, if a coulomb is moved from plate Y to plate X, which is at a potential V volts higher than plate Y, V joules of energy must be given to the coulomb, i.e., V joules of work must be done if the coulomb is to be moved from plate Y to plate X. But if work has to be done in moving a coulomb, it follows that the charge must experience a force opposing its motion. *When space is in such a state that an electric charge placed in that space experiences a force, then we say that the charge is in an* **electric field,** just as when a magnetic pole experiences a force we say that the pole is in a magnetic field.

Suppose the plate Y to be at zero potential; then plate X is at a potential of V volts. Since V joules of energy must be expended in moving 1 coulomb completely across the space between the plates, it can be assumed that $\frac{V}{2}$ joules of energy would have to be expended in moving a coulomb to the point P half-way across the space. The potential at the point P is therefore $\frac{V}{2}$ volts. Thus it is possible to speak of the *potential at a point in space.* Previously the word *potential* has always been used in connection with a point on a conductor. But the potential at *any* point can be defined as the work necessary to move a unit charge to the point from a region of zero potential. So as we proceed from plate Y to plate X we move through points of gradually increasing potential. In other words, a *potential gradient* exists in the space between the plates, and wherever there is a potential gradient there exists an electric field. A potential gradient, and therefore an electric field, exists in the space between



(a). (b).

Fig. 186.—The Electric Field between Condenser Plates.

any two conductors which are at different potentials. Further, the *strength of the electric field* (or the *electric force*) is measured by the force experienced by a unit charge placed in the field.

The cause of the abnormal state of the space between the plates X and Y may be attributed to the tendency of the positive and negative charges to approach each other. There certainly is a strong tendency for the charges to approach one another, since (1) if the plates are joined by a connecting wire, charge flows along it from one plate to the other, and (2) if one plate is free to move, the charge on that plate moves towards the other charge, taking the plate with it ! But if the plates are immovable and if no conducting path is provided, the *stress* thus set up by the force of attraction causes the ether to be *strained.* This strained state we call an *electric field.* It is a different type of strain from that known as a *magnetic field,* since a stationary magnetic pole experiences no force when placed in an electric field, nor does a stationary electric charge experience any force when placed in a magnetic field.

Lines of electric force may be used to show the *strain pattern* of an electric field, just as lines of magnetic force can demonstrate a magnetic field pattern. A *line of electric force* is a curve drawn in an electric field such that its direction at any point is the direction of the electric field at the point, i.e., the direction in which a small positive charge placed at that point would tend to move. Unlike lines of magnetic force, lines of electric force are not endless; they do not pass through a conductor, but start from the surface of one conductor and end on the surface of another.

The type of electric field obtained with a charged condenser consisting of two parallel plates is shown in Fig. 186b. Between the plates there is a field of uniform strength, as indicated by the parallel lines, for, as in magnetism, the flux density *in air* gives an indication of the field strength. The lines converge to the strong parts of an electric field.

The electric field pattern between the plates of a condenser can be demonstrated in one of two ways:—

(1) If strips of tin-foil are glued to a plate of glass, as shown in Fig. 187a, and the terminals connected to a Wimshurst machine, which establishes a p.d. of a few thousand volts, then freshly-powdered crystals of gypsum scattered on the glass plate will arrange themselves along the lines of electric force.

(2) If two metal plates are placed, as in Fig. 187b, in a

beaker of transformer oil containing fine particles of sawdust of a heavy wood such as mahogany, then on establishing a high p.d. between the plates, the particles of sawdust set themselves in very marked lines along the lines of electric force. These can be seen very distinctly if a lamp is placed behind the beaker.

It is interesting to note that whenever we break a d.c. circuit a magnetic field collapses and an electric field, across the terminals of the switch, is established. In fact, an electric field exists even while the current is flowing, since lines of electric force pass from points in the circuit which are at high potentials to those which are at lower potentials. Thus electrostatics and current electricity go hand in hand. Electrostatics has very little practical importance except in so far as it is related to current electricity. A condenser, for example, cannot be charged or discharged without a current flowing, and in most practical applications the condenser is being continuously charged and discharged.

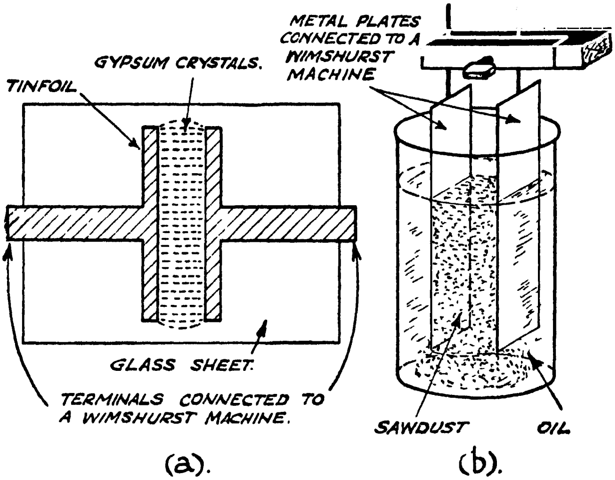

METAL PLATES CONNECTED TO A WIMSHURST MACHINE
GYPSUM CRYSTALS.
TINFOIL
GLASS SHEET.
TERMINALS CONNECTED TO A WIMSHURST MACHINE.
SAWDUST
OIL
(a).
(b).

FIG. 187.—APPARATUS FOR DEMONSTRATING THE PRESENCE OF AN ELECTRIC FIELD.

108. The C.G.S. Electrostatic System of Units

This system of units was established in the early days of electrical science as a convenient system for dealing with quantitative electrostatics and is based on the force exerted between electric charges. Here the abbreviation *e.s.u.* will be used for *c.g.s. electrostatic unit.*

The *e.s.u. of charge* is defined as that charge which, when situated in a vacuum, 1 cm. from an identical charge, repels it with a force of 1 dyne. It is found that, for practical purposes, **1 coulomb is equivalent to 3 × 10^9 e.s.u. of charge** (actually the constant is 2·998).

It is an experimental fact that the force, F dynes, exerted between two charges of Q_1 and Q_2 e.s.u., situated d cm. apart is given by:

$$\mathrm{F} = \frac{Q_1 Q_2}{\epsilon d^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(a)}$$

where ϵ is the *permittivity* (see below) of the medium separating the charges.

The **electric force** or **electric field strength** (symbol E) at a point is defined as the force in dynes exerted on a unit positive charge placed at that point. It follows from expression (a) above that, in a medium of permittivity ϵ, the electric force, E e.s.u., at a distance d cm. from a charge of Q e.s.u. is given by:

$$E = \frac{Q}{\epsilon d^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(b)}$$

i.e., *the electric force is inversely proportional to the permittivity of the surrounding dielectric.*

When dealing with electric fields, a magnitude is assigned to the **electric flux density** (symbol D), or number of lines of electric force per sq. cm., as with magnetic flux density. In the c.g.s. electrostatic system of units, it is assumed that *unit electric force produces unit electric flux density* (1 line of electric force per sq. cm.) in a vacuum (or, for practical purposes, in air).

For any dielectric, the ratio $\frac{\text{electric flux density}}{\text{electric force}}$ is termed the **permittivity** of the dielectric. Clearly, in the c.g.s. electrostatic system of units, the value of the permittivity for a vacuum (denoted by ϵ_o) is unity; for air the value is 1·0006 and may thus be taken as unity for most practical purposes. It is now possible to distinguish between *relative permittivity* (denoted by ϵ_r and defined in Section 105) and *permittivity* (denoted by ϵ and defined as above). Actually, $\epsilon = \epsilon_o \epsilon_r$, and since in the c.g.s. electrostatic system of units, ϵ_o has a value of precisely unity, the values of ϵ and ϵ_r are numerically equal to one another. The relative permittivity (ϵ_r) is a pure number and, for a given dielectric, has the same value in any system of units.

It may be shown, by reasoning similar to that adopted in the case of the c.g.s. unit magnetic pole (see Section 79), that *unit electric charge* (1 *e.s.u.*) *gives rise to* 4π *lines of electric force*, this value being independent of the medium surrounding the charge.

The **potential difference** between two points is a measure of the energy given up by a unit positive charge as it moves from the point of higher to the point of lower potential (see Chapter II, Section 10).

Thus, $$1 \text{ e.s.u. of p.d.} = \frac{1 \text{ erg}}{1 \text{ e.s.u. of charge}}.$$

But, $$1 \text{ volt} = \frac{1 \text{ joule}}{1 \text{ coulomb}} = \frac{10^7 \text{ ergs}}{3 \times 10^9 \text{ e.s.u. of charge}}$$
$$= \tfrac{1}{300} \text{ of an e.s.u. of potential difference.}$$

Alternatively, **1 e.s.u. of p.d. = 300 volts.**

The relationship between the *farad* and the electrostatic unit of **capacitance** may be derived in a similar manner.

$$1 \text{ e.s.u. of capacitance} = \frac{1 \text{ e.s.u of charge}}{1 \text{ e.s.u. of p.d.}} \quad \text{(see Section 101)}$$

But, $$1 \text{ farad} = \frac{1 \text{ coulomb}}{1 \text{ volt}} = \frac{3 \times 10^9 \text{ e.s.u. of charge}}{\frac{1}{300} \text{ e.s.u. of p.d.}}.$$

Hence, **1 farad = 9×10^{11} e.s.u. of capacitance.**

The reader will now appreciate the reason for the factor 9×10^{11} in the denominator of the expression for the capacitance, C farads, of a parallel-plate condenser (see Section 105). The corresponding formula for the capacitance, C' e.s.u., of this condenser is as follows:

$$C' = \frac{\epsilon A}{4\pi d}$$

The factor 4π appears here, since, as stated above, 1 e.s.u. of charge gives rise to 4π lines of electric force.

109. The Rationalised M.K.S. System in Electrostatics

As already stated, the rationalised m.k.s. system of units embodies all the practical electrical units, these including the joule, coulomb, ampere, volt and farad. In this system a *unit charge—the coulomb—* is assumed to give rise to 1 *line of electric force*, whereas in the c.g.s. electrostatic system of units, 4π lines of electric force were associated with the unit charge. This change has the effect of eliminating the factor 4π from the expression for the capacitance of a parallel-plate condenser. Since the rationalised m.k.s. unit of capacitance is the farad, the numerical constant 9×10^{11} is also removed from this expression.

The capacitance, C farads, of a parallel-plate condenser is therefore given by the formula: $C = \frac{\epsilon A'}{d'}$, where A' metres² represents the effective plate area, and d' metres is the thickness of the dielectric. The permittivity of the dielectric, ϵ, must now be expressed in the form given by the relationship—$\epsilon = \epsilon_0 \epsilon_r$, where ϵ_0 is the permittivity of a vacuum, in rationalised m.k.s. units, and ϵ_r is the relative permittivity of the particular dielectric (see the preceding Section). Hence, to avoid any possibility of error, it is preferable to state the formula for the capacitance, C farads, of a parallel-plate condenser in the form:

$$C = \frac{\epsilon_0 \epsilon_r A'}{d'}.$$

As stated in Section 78, the value of ϵ_0 in the rationalised m.k.s. system must be chosen so that it conforms with the relationship connecting μ_0, ϵ_0 and the velocity of propagation of electromagnetic waves (c metres per second). Using this relationship and taking c as 3×10^8, we get:

$$\epsilon_0 = \frac{1}{\mu_0 c^2} = \frac{10^7}{4\pi \times 9 \times 10^{16}} = \frac{1000}{36\pi} \times 10^{-12}.$$

Or, $$\epsilon_0 = \mathbf{8{\cdot}85 \times 10^{-12}}.$$

The rationalised m.k.s. units for *electric force* and *electric flux density* are dealt with in Chapter V of Vol. II, in which chapter the electric field aspect of electrostatics is discussed more fully than is desirable at this stage.

Exercises on Chapter IX

Any additional data required for the solution of these exercises may be obtained from the tables at the end of this book.

1. Calculate the p.d. which must be applied to the plates of a condenser of capacitance 0·2 μF in order to give the condenser a charge of 5·5 μC. What would be the capacitance of a condenser the plates of which received such a charge from an applied p.d. of 11 volts? What charge would be given to the plates of a condenser of capacitance 50 μμF if subjected to an applied p.d. of 2·5 volts?
[27·5 V; 0·5 μF; 125 μμC.]

2. Using rationalised m.k.s. units, calculate the capacitance, in $\mu\mu$F, of a condenser consisting of two metal plates, each 20 cm. × 20 cm., separated by a dielectric of thickness 2 mm. and of relative permittivity 2·5.
[442·5 $\mu\mu$F.]

3. A condenser is constructed of alternate layers of tinfoil and waxed paper, odd sheets of tinfoil being connected to one terminal A and even sheets to a second terminal B. If 99 sheets of tinfoil are used and if the area of overlap of one sheet with the next is 450 sq. cm., calculate the capacitance of the condenser, taking the permittivity of the waxed paper to be 3·0 and its thickness 0·12 mm.
[0·975 μF.]

4. Determine the voltage between the plates of a parallel-plate condenser with plates 0·25 mm. apart, the dielectric having a permittivity of 3·5 and the surface charge on the plates being 0·0025 μC per sq. cm.
[201·9 V.]

5. A 0·5 μF condenser of the rolled-paper type is constructed of two strips of waxed paper each backed with tinfoil, as shown in Fig. 181. If the waxed paper is 0·001 in. thick and has a permittivity of 3·2, the overlap of the tinfoil being 2 in. in width, determine, in feet, the total length of paper strip required.
[29 ft.]

6. (a) Convert to c.g.s. electrostatic units (i) 25μC, (ii) 15 volts, (iii) 50 $\mu\mu$F. (b) Convert to rationalised m.k.s. units (i) 20 c.g.s. electrostatic units of p.d., (ii) 18,000 c.g.s. electrostatic units of capacitance, (iii) 15,000 c.g.s. electrostatic units of charge.
[(a) (i) 75,000, (ii) 0·05, (iii) 45; (b) (i) 6000 V, (ii) 0·02 μF, (iii) 5 μC.]

7. A variable air condenser consisting of two plates is given a charge of 0·04 micro-coulomb by applying a p.d. of 160 volts to the plates, their separation being 1·5 mm. Express the capacitance of the condenser in micro-micro-farads. If the distance between the plates of the condenser is increased by 5 mm., what will now be (a) the capacitance of the condenser, (b) the p.d. between the plates? Calculate the area of each plate.
[250 $\mu\mu$F; (a) 57·7 $\mu\mu$F; (b) 693·3 V; 424 sq. cm.]

8. Two parallel-connected condensers have a combined capacitance of 80 μF. If the charges on the two condensers be 450 μC and 100 μC respectively, calculate the voltage applied and the capacitance of each condenser.

[6·875 V; 65·45 μF; 14·55 μF.]

9. If the voltage drop across two series-connected condensers of capacitances 0·3 and 2·5 μF respectively is 400 volts, what is the voltage drop across each, and what is the capacitance of the arrangement?

[357·1 V; 42·9 V; 0·268 μF.]

10. Three series-connected condensers of equal dimensions have dielectrics of permittivities 1, 2 and 3 respectively. Determine the voltage drop across each condenser given that a p.d. of 1·5 kV is applied across the whole arrangement.

[0·82 kV; 0·41 kV; 0·27 kV.]

11. Two condensers A and B of capacitance 1 μF and 0·5 μF respectively are joined in series. In parallel with this series arrangement is connected a third condenser C of capacitance 0·1 μF. The complete arrangement is connected to a d.c. supply of 200 volts. Calculate the charge on the plates of each condenser and the combined capacitance of the whole arrangement.

[66·67 μC on A and B; 20 μC on C; 0·433 μF.]

12. A condenser consists of two rectangular metal plates, each 50 cm. × 30 cm., separated by a dielectric of thickness 2 mm. and relative permittivity 5. A p.d. of 3000 volts is maintained between the plates. Calculate (a) the capacitance of the condenser in e.s.u. and in $\mu\mu$F, (b) the charge on the plates in e.s.u. and in μC, (c) the electric flux density and the electric force in the dielectric, in e.s.u.

[(a) 2980 e.s.u.; 3320 $\mu\mu$F. (b) 29,800 e.s.u.; 9·96 μC. (c) 250 electric lines/cm.^{2}; 50 e.s.u.]

ADDENDUM

(For reference after a knowledge of the calculus has been acquired. The treatment is in terms of the c.g.s. electromagnetic system of units.)

110. Laplace's Law

Referring to Fig. 188, AB represents a portion of a closed electric circuit through which a current of I' c.g.s. units is flowing, CD being an element of the conductor of infinitesimal lengths ds cm. At the point P is a N. pole of strength m c.g.s. units. Let the distance CP be r cm. in length and θ the angle that CP makes with the direction of the current at the point C.

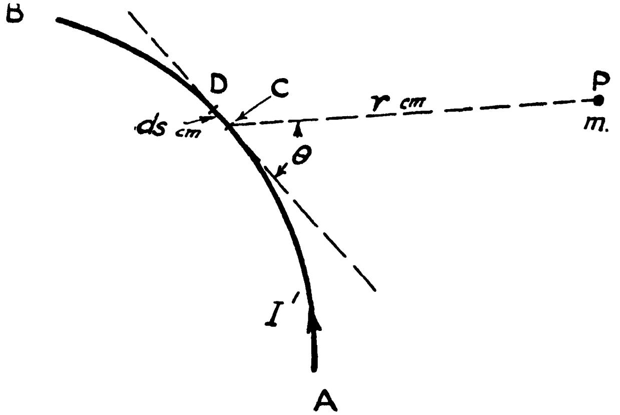


FIG. 188.

According to Laplace's law, the force, F dynes, exerted on the pole by the current flowing in the infinitesimal element of length ds is given by the equation:—

$$F = \frac{mI'ds \,.\, \sin\theta}{r^2}$$

This force acts in a direction at right angles to the plane containing the element ds and the line CP. In other words, the force acts in a direction at right angles to the plane of the paper, the *sense*, in accordance with the corkscrew rule, being *downwards* into the plane of the paper.

The magnetic pole will, of course, exert an equal and opposite force on the current element, i.e., the element of length ds will experience a force of $\frac{mI'ds \,.\, \sin\theta}{r^2}$ dynes *upwards* at right angles to the plane of the paper.

From this fundamental law the electromagnetic formulæ of Chapters VII and VIII may be derived, the justification for the law being that the formulæ so derived are found to be in accordance with experimental results.

If the pole at the point P be a unit pole, the force in dynes exerted on the unit pole gives a measure of the magnetising force, expressed in c.g.s. units, at the point P due to the current element. Thus the magnetising force at the point P due to the current element is $\frac{I'ds \,.\, \sin\theta}{r^2}$ c.g.s. units.

To determine the magnetising force at a point due to an entire electric circuit involves the vectorial summation of the magnetising forces due to each separate element, and this usually demands the application of the integral calculus. Only in special cases is this summation, or integration, a simple matter, but these happen to be cases of some importance. In the sections which follow, Laplace's formula is used to determine the magnetising force in these cases.

111. The C.G.S. Electromagnetic Unit of Current

Fig. 189 represents a conductor AB bent in the form of a circular arc of length s cm., the centre of the arc being at the point P and the radius of length r cm. Consider the element CD of infinitesimal length ds cm. The line CP is perpendicular to this element. If a current of I' c.g.s. units flows through the conductor, the magnetising force due to the current-carrying element will be $\frac{I'ds}{r^2}$ c.g.s. units, since $\sin 90° = 1$. For all other elements the angle 'θ' and the distance 'r' have the same values. So the magnetising force, H c.g.s. units, at the point P due to the length s of current-carrying conductor is given by the equation

$$H = \int \frac{I' \,.\, ds}{r^2} \qquad \text{i.e.,} \quad H = \frac{I'}{r^2}\int ds = \frac{I's}{r^2}$$

If now I', s and r are each made equal to unity, H is also unity. In other words, when unit current flows in an arc of a circle of length

1 cm., the radius of the arc being also 1 cm., the magnetising force produced at the centre is 1 c.g.s. unit; or, conversely, 1 c.g.s. electromagnetic unit of current is that current which, flowing in a conductor in the form of a circular arc of length 1 cm. and radius 1 cm., would exert a force of 1 dyne on a unit magnetic pole placed at the centre of the arc. This is the definition of unit current given in Section 77.

Had the c.g.s. unit of current been given any other value, it would have been necessary to write Laplace's formula not in the manner given in the preceding section, but in the form

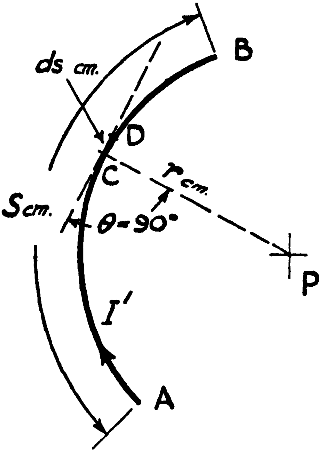


FIG. 189.

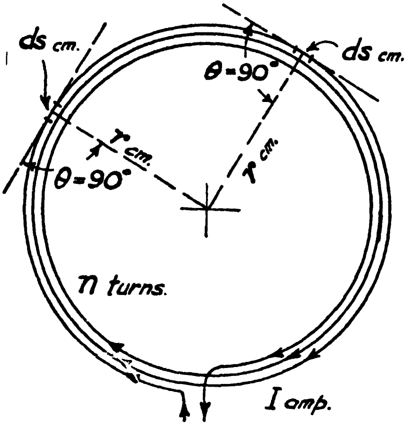


FIG. 190.

$$F = k \times \frac{mI'ds \, . \sin \theta}{r^2}$$

where k is a constant the value of which would depend on the size of the unit current. Had the value of the constant k been anything else but unity, it would have had to appear in all the electromagnetic formulæ (employing c.g.s. units) of Chapters VII and VIII.

112. The Magnetising Force at the Centre of a Circular Current-carrying Conductor

Fig. 190 represents a circular conductor of n turns bunched together so that they may all be regarded as lying in the same plane and as possessing the same radius r cm. Then for each infinitesimal element of length ds the distance to the centre of the circle is r cm.

and the line joining any element to the centre of the circle is perpendicular to the element. So if the current carried be I amp., i.e., $\frac{I}{10}$ c.g.s. units, the magnetising force at the centre of the circle due to each current element is

$$\frac{\left(\frac{I}{10}\right) ds \,.\, sin\ 90^\circ}{r^2} \text{ c.g.s. units,}$$

and since the total length of conductor is $2\pi r.n$ cm., the magnetising force, H c.g.s. units, due to the whole current-carrying conductor is:—

$$\frac{\left(\frac{I}{10}\right) \,.\, 2\pi r \,.\, n \,.\, sin\ 90^\circ}{r^2} \text{ c.g.s. units}$$

i.e.,
$$H = \frac{2\pi In}{10r}$$

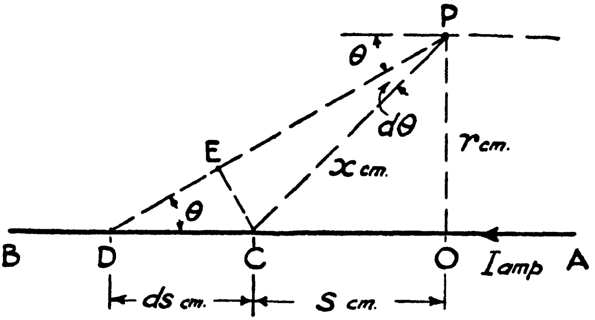


FIG. 191.

113. The Magnetising Force at a Point Distant r cm. from an Infinitely Long Straight Current-carrying Conductor

In Fig. 191 AB represents a portion of an infinitely long, straight conductor and P a point a distance r cm. from the conductor. Let OP be a perpendicular drawn from the point P to the conductor, OC a length s cm. measured along the conductor from the point O, and CD an infinitesimal increment, ds, in this length. Let θ be the angle shown and $d\theta$ the angle subtended at the point P by the element CD of length ds. Drop a perpendicular CE from the point C on to the line DP. Suppose CP to be of length x cm.

Then:— $$CE = ds \,.\, sin\ \theta$$

But, $$CE = x \,.\, d\theta$$

So, $$x..\, d\theta = ds \,.\, sin\ \theta$$

$$\therefore \quad sin\ \theta = x \,.\, \frac{d\theta}{ds}$$

But since $$O\hat{C}P \simeq O\hat{D}P = \theta,\ sin\ \theta = \frac{r}{x}$$

$$\therefore \quad \frac{r}{x} = x \,.\, \frac{d\theta}{ds}$$

i.e., $$\frac{ds}{x^2} = \frac{d\theta}{r}$$

If a current of I amp., i.e., $\frac{I}{10}$ c.g.s. units, flows through the conductor, the magnetising force at P due to the current element CD $= \frac{(I/10)ds \,.\, sin\ \theta}{x^2}$ c.g.s. units and the magnetising force due to the whole conductor

$$= \frac{I}{10} \int_{\theta = 0}^{\theta = \pi} \frac{sin\ \theta \,.\, ds}{x^2} \text{ c.g.s. units}$$

Substituting $\frac{d\theta}{r}$ for $\frac{ds}{x^2}$, the magnetising force, H c.g.s. units, due to the whole conductor is thus given by the equation:—

$$H = \frac{I}{10} \int_0^{\pi} \frac{sin\ \theta \,.\, d\theta}{r}$$

i.e., $$H = \frac{I}{10r} \int_0^{\pi} sin\ \theta \,.\, d\theta = \frac{I}{10r} \Big[-cos\ \theta \Big]_0^{\pi}$$

So, $$H = \frac{2I}{10r}$$

114. The Magnetising Force at a Point on the Axis of a Circular Current-carrying Conductor

Fig. 192 represents a circular conductor of radius R cm. carrying a current of I amp., i.e., $\frac{I}{10}$ c.g.s. units. Consider the magnetising force, F c.g.s. units, due to the infinitesimal element CD of length *ds*, at a point P, situated along the axis of the conductor at a distance

x cm. from the centre O of the conductor. The line CP is at right angles to the current element CD. If the length of CP be r cm., then,

$$F = \frac{\left(\frac{I}{10}\right) . ds . \sin 90^\circ}{r^2}$$

i.e.,

$$F = \frac{I . ds}{10r^2}$$

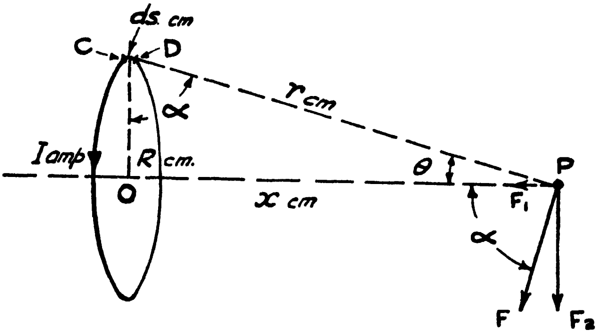


FIG. 192.

The magnetising force F c.g.s. units acts in a direction at right angles to the plane containing CD and CP. Although the force due to each element of length ds is of magnitude $\frac{I . ds}{10r^2}$ c.g.s. units, yet the

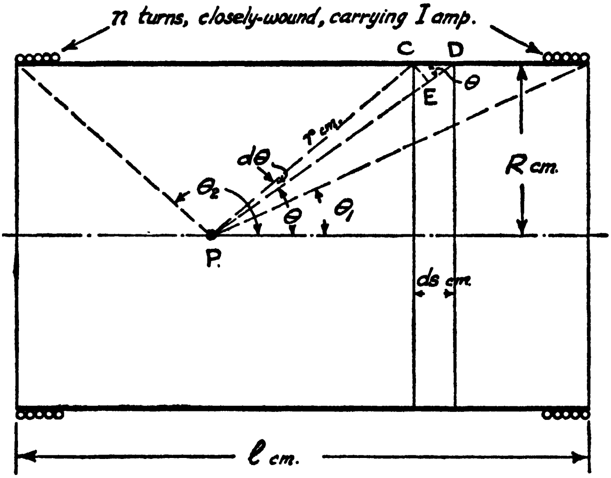


FIG. 193.

forces due to the different elements lie in various directions, although in each case making an angle a with the axis OP. By resolving each of these forces into two components, one along the axis and one at right angles to it, it will be seen that, by symmetry, the components at right angles to the axis cancel, so that the resultant magnetising force due to the whole conductor, H c.g.s. units, is the sum of the components along the axis.

If F_1 c.g.s. units be the component along the axis of the force F c.g.s. units, then:—

$$F_1 = F \cos a = \frac{I \, . \, ds \, . \cos a}{10r^2}$$

So $$H = \frac{I \cos a}{10r^2} \int ds$$

$$= \frac{I \cos a}{10r^2} \, . \, 2\pi R$$

and since $$\cos a = \frac{R}{r},$$

$$H = \frac{2\pi R^2 \, . \, I}{10r^3}$$

This formula is used in the next section to determine a formula for the magnetising force within a current-carrying solenoid.

Since $r = \sqrt{R^2 + x^2}$, $$H = \frac{2\pi R^2 \, . \, I}{10(R^2 + x^2)^{3/2}}$$

When $$x \longrightarrow \infty, H \longrightarrow 0,$$

and when $$x = 0, H = \frac{2\pi I}{10R}$$

as was proved in Section 112.

115. The Magnetising Force at a Point within and on the Axis of a Closely-wound Solenoid

Fig. 193 represents a very closely-wound solenoid of radius R cm. with n turns distributed over a length l cm. and carrying a current, I amp. CD represents an infinitesimally short length, ds, of the solenoid, the number of turns in this length being $n \times ds/l$. Thus the effective current in this "circular conductor" is $In \, . \, ds/l$ amperes and, if the distance CP be r cm., the magnetising force it produces at the point P on its axis is $\frac{2\pi R^2(In\,.\,ds/l)}{10r^3}$ c.g.s. units $=$ $\frac{2\pi In}{10l} \times \frac{R^2 ds}{r^3}$ c.g.s. units. If CE be a perpendicular dropped from the point C on to the line PD, then

$$\text{CE}/ds = \sin\theta$$

but
$$\text{CE} = r\,.\,d\theta$$

so
$$\text{CE} = ds\,.\,\sin\theta = r\,.\,d\theta$$

$$\therefore \quad ds = \frac{r\,.\,d\theta}{\sin\theta}$$

Also
$$\sin\theta = R/r$$

$$\therefore \quad \frac{R^2}{r^3}\,.\,ds = \frac{R^2}{r^2}\,.\,\frac{ds}{r}$$

$$= \sin^2\theta \times \frac{d\theta}{\sin\theta}$$

$$= \sin\theta\,.\,d\theta$$

So, substituting $\sin\theta\,.\,d\theta$ for $\frac{R^2}{r^3}\,.\,ds$ in the above expression for the magnetising force due to the infinitesimal length of solenoid, we get for this magnetising force, $\frac{2\pi In}{10l}\,.\,\sin\theta\,.\,d\theta$. c.g.s. units. Hence the magnetising force, H c.g.s. units, due to the whole solenoid is given by the equation:—

$$H = \frac{2\pi In}{10l}\int_{\theta_1}^{\theta_2}\sin\theta\,.\,d\theta$$

$$= \frac{2\pi In}{10l}\Big[-\cos\theta\Big]_{\theta_1}^{\theta_2}$$

$$= \frac{2\pi In}{10l}\,[\cos\theta_1 - \cos\theta_2]$$

Where the solenoid is infinitely long, $\theta_1 = 0$ and $\theta_2 = \pi$, for points well removed from the end faces of the solenoid, in which case,

$$H = \frac{4\pi}{10}\,.\,\frac{In}{l}$$

116. The Force Experienced by a Straight Current-carrying Conductor in a Magnetic Field

It was stressed in Section 110 that the force $\frac{mI'ds\,.\,\sin\theta}{r^2}$ dynes was a force experienced not only by the pole of strength m c.g.s. units, but also by the infinitesimal current element CD (see Fig. 188). This current element lies in the field of the magnetic pole, the magnetising force due to the pole in the vicinity of the element being m/r^2 c.g.s. units. In other words, the element CD is subject to a magnetic flux density of m/r^2 lines per sq. cm., the lines being inclined to the current element at an angle θ.

Thus, owing to a flux density of m/r^2 lines per sq. cm., inclined at angle θ to the current element, the element experiences a force of $(m/r^2 \times I'ds \times \sin\theta)$ dynes. So if a straight conductor, XY, carrying a current of I' c.g.s. units, lies in a uniform field of flux density B lines per sq. cm. as shown in Fig. 154, we should expect each element of length ds to experience a force of $(B \times I'ds \times \sin\theta)$ dynes upwards at right angles to the plane of the paper. Further, we should expect the total upward force on a length l cm. to be $(B \times I'l \times \sin\theta)$ dynes.

So, if a straight conductor of length l cm. carrying a current of I amperes, i.e., $\frac{I}{10}$ c.g.s. units, lies in a uniform magnetic field of flux density B lines per sq. cm., the angle of inclination between the lines and the conductor being θ, the force F dynes exerted on the conductor is given by the equation:—

$$\mathrm{F} = \frac{I}{10} \cdot BlI \cdot \sin\theta$$

TABLE I

Miscellaneous Data

1 inch = 2·54 cm. $\pi = 3{\cdot}1416$.
1 pound = 453·6 gm. $\log_{10}\pi = 0{\cdot}4972$.
1 gm. wt. = 981 dynes.
1 kg. wt. = 9·81 newtons.
1 newton = 10^5 dynes.
1 gallon of water weighs 10 lb.
1 cu. ft. of water weighs 62·4 lb.
1 joule = 10^7 ergs.
1 ft.-lb. = 1·356 joules.
1 H.P. = 33,000 ft.-lb. per min. = 746 watts.
1 B.Th.U. = 778 ft.-lb. = 252 calories = $\frac{5}{9}$ C.H.U.
1 ampere-hour = 3600 coulombs.
1 watt-hour = 3600 joules.
1 kilowatt-hour = $3{\cdot}6 \times 10^6$ joules.
1 calorie = 4·185 joules.
1 kilowatt-hour = 3414 B.Th.U. = 1896 C.H.U.
1 ampere = $\frac{1}{10}$ of a c.g.s. electromagnetic unit of current.
1 coulomb = $\frac{1}{10}$ of a c.g.s. electromagnetic unit of quantity or charge.
= 3×10^9 c.g.s. electrostatic units of quantity or charge.
1 volt = 10^8 c.g.s. electromagnetic units of e.m.f. or p.d.
= $\frac{1}{300}$ of a c.g.s. electrostatic unit of e.m.f. or p.d.
1 ohm = 10^9 c.g.s. electromagnetic units of resistance.
1 farad = 9×10^{11} c.g.s. electrostatic units of capacitance.

TABLE II

Specific Gravities and Specific Heats of Various Substances

Substance.	Specific gravity.†	Specific heat.‡
Aluminium	2·63	0·21
Carbon (graphite)	2·3	0·16
Chromium	7·1	0·11
Copper	8·93	0·092
Iron (pure)	7·87	0·106
Steel	7·7 to 7·9	about 0·1
Lead	11·37	0·031
Mercury	13·56	0·033
Nickel	8·9	0·11
Platinum	21·5	0·32
Silver	10·5	0·056
Tin	7·29	0·055
Tungsten	19·3	0·034
Zinc	7·1	0·093
Brass *	8·5	0·090
Cadmium–copper *	8·95	about 0·09
Constantan * (Eureka)	8·88	0·098
Nickel Silver * (German Silver)	8·5	0·095
Manganin *	8·5	about 0·09
Platinoid *	about 9	about 0·09
Nichrome * (80/20)	8·35	0·106
Oil (transformer)	0·865	about 0·45

* See Table IV.

† The above values of specific gravity are approximate only, the precise value for any given substance being dependent upon the condition of the material (i.e., cast, rolled or drawn) and upon the exact composition.

‡ The values of specific heat are also approximate, for the reasons stated above. The values quoted are mean values for the temperature range 0–100° C.

TABLE III

Standard Wire Gauge Equivalents

S.W.G. No.	Equivalent diameter or thickness.	
	Mm.	Inch.
0	8·23	0·324
1	7·62	0·300
2	7·01	0·276
—	—	—
12	2·64	0·104
14	2·03	0·080
16	1·63	0·064
18	1·22	0·048
20	0·914	0·036
22	0·711	0·028
24	0·559	0·022
26	0·457	0·018
28	0·376	0·0148
30	0·315	0·0124
—	—	—
40	0·122	0·0048
—	—	—
50	0·0254	0·0010

TABLE IV

The Composition of some Alloys used as Conducting Materials

Alloy.	Typical Composition.
Brass	Copper 66% ; Zinc 34%.
Cadmium–Copper	Copper with 0·8 to 1·0% Cadmium.
Constantan (Eureka)	Copper 60% ; Nickel 40%.
Manganin	Copper 86% ; Manganese 12% ; Nickel 2%.
Monel	Nickel 67% ; Copper 28% ; remainder Iron, Manganese and Silicon.
Nichrome	Nickel 80% ; Chromium 20% ; often some Iron.
Nickel Silver (German Silver)	Copper 62% ; Nickel 18% ; Zinc 20%.
Phosphor Bronze	Copper 95% ; Tin 4% ; Phosphorus 0·3%.
Platinoid	Nickel Silver with a little Tungsten.

Note.—The proportions of the elements in the above alloys often vary considerably from those stated.

TABLE V

Resistivities and Conductivities at 20° C.

Material.	Resistivity.		Conductivity.	
	Microhms per inch cube.	Microhms per cm. cube.	Mhos per inch cube $\times 10^{-6}$.	Mhos per cm. cube $\times 10^{-6}$.
Aluminium (hard-drawn)	1·12	2·86	8·9	3·5
Copper (hard-drawn) .	0·70	1·78	14·3	5·6
Copper (annealed) .	0·68	1·72	14·7	5·8
Iron (pure) . . .	4·77	12·1	2·10	0·83
Steel (0·1% carbon) .	about 7·9	about 20	about 1·3	about 0·5
Lead	8·27	21·0	1·21	0·48
Mercury . . .	37·7	95·8	0·265	0·104
Nickel (99–100% pure) .	3·80	9·66	2·63	1·03
Platinum . . .	4·37	11·1	2·29	0·90
Silver	0·65	1·64	15·5	6·1
Tin	4·45	11·3	2·25	0·88
Tungsten . . .	2·18	5·55	4·59	1·8
Zinc	2·40	6·10	4·16	1·64
Cadmium–copper * .	0·80	2·03	12·5	4·93
Constantan (Eureka) * .	19–20	48–49	0·50–0·53	0·20–0·21
Manganin * . . .	16·9	43	0·59	0·23
Monel * . . .	17·7	45	0·57	0·22
Nichrome (80/20) * .	40·6	103	0·25	0·097
Nickel silver (German silver) * . . .	10·9	27·6	0·92	0·36
Phosphor bronze * .	5·4	13·7	1·85	0·73
Platinoid * . . .	about 13·4	about 34	about 0·75	about 0·29

* See Table IV.

The precise value of the resistivity is dependent upon the exact composition and condition of the material.

TABLE VI

Temperature Coefficients of Resistance (from 0° C.)

Material.	α_0, ohm/ohm/degree C.
Aluminium (hard-drawn) . . .	$43\cdot5 \times 10^{-4}$
Copper (annealed and hard-drawn) . .	$42\cdot6 \times 10^{-4}$
Iron (pure)	62×10^{-4}
Lead	43×10^{-4}
Mercury	9×10^{-4}
Nickel (99–100% pure)	41×10^{-4}
Platinum	38×10^{-4}
Silver	40×10^{-4}
Tin	45×10^{-4}
Tungsten	51×10^{-4}
Zinc	37×10^{-4}
Cadmium–copper *	31×10^{-4}
Constantan (Eureka) *	$-0\cdot8$ to $+0\cdot6 \times 10^{-4}$
Manganin *	$-0\cdot2 \times 10^{-4}$
Monel *	19×10^{-4}
Nichrome (80/20) *	$0\cdot98 \times 10^{-4}$
Nickel silver (German silver) * . .	-1 to $+5 \times 10^{-4}$
Phosphor bronze *	$5\cdot9 \times 10^{-4}$
Platinoid *	$2\cdot5 \times 10^{-4}$

* See Table IV.

The above values of α_0 are average values for the temperature range 0–100° C., i.e., assuming a linear relationship of the form :—

$$R_\theta = R_0(1 + \alpha_0\theta).$$

The reader is referred to Sections 22 and 32 (Chapter III).

TABLE VII

Electro-chemical Equivalents

Element.	Valency.	E.C.E. Milligrams/ coulomb.	E.C.E. Lb. per 1000 Ah.
Aluminium	3	0·0945	0·750
Chromium	6 (chromic acid)	0·091	0·722
Copper	2 (copper sulphate)	0·329	2·61
Copper	1 (copper cyanide)	0·658	5·22
Hydrogen	1	0·0104	0·0826
Iron	2	0·289	2·29
Lead	2	1·072	8·51
Magnesium	2	0·124	0·985
Nickel	2	0·305	2·42
Oxygen	2	0·083	0·66
Silver	1	1·118	8·88
Sodium	1	0·238	1·89
Tin	2	0·615	4·88
Zinc	2	0·337	2·67

Note.—Owing to the fact that ions in different solutions frequently have different valencies the valency corresponding to the given E.C.E. is stated. It is essential to ascertain that the valency of the element in a given case is the one stated in the table, where the most common valency is given.

TABLE VIII

An Abbreviated Electro-chemical Series of Elements

Potassium
Sodium
Calcium
Magnesium
Aluminium
Chromium
Manganese
Zinc
Cadmium
Iron
Cobalt
Nickel
Tin
Lead

Hydrogen

Antimony
Bismuth
Copper
Mercury
Silver
Platinum
Gold
Silicon
Carbon

TABLE IX

Standard Symbols for Quantities

Capacitance	C	Magnetising force	H
Charge	Q	Permeability	μ
Conductance	G	Permittivity	ϵ
Electric current	I	Potential difference	V
Electric flux density	D	Power	P
Electric force	E	Quantity of electricity	Q
Electromotive force	E or V	Resistance	R
Energy	W	Resistivity	ρ
Frequency	f	Temperature	θ
Magnetic flux	Φ	Time	t
Magnetic flux density	B		

Note.—The above symbols are in accordance with the recommendations of the British Standards Institution (B.S. 1991 : Part I : 1954).

TABLE X

British Standards

These Standards are drawn up and issued by the British Standards Institution, 28 Victoria Street, London, S.W. 1, a body founded in 1901 and incorporated by Royal Charter in 1929. The principal objects of the Institution are:—

(1) To co-ordinate the efforts of producers and users for the improvement, standardisation and simplification of engineering and industrial materials.
(2) Simplification of production and distribution.
(3) Elimination of the waste of time and material caused by an unnecessary variety of patterns and sizes of articles for one and the same purpose.
(4) To set up standards of quality and dimensions and promote the general adoption of British Standards.

The list of British Standards given below is not complete, even so far as those relating to electrical engineering are concerned. It includes, however, those standards dealing with materials and equipment to which direct reference has been made in Vol. I. The complete range of British Standards is being continually modified and extended, the list below being up-to-date at the time of publication.

CABLES, CONDUCTORS AND WIRES

B.S.	
7	**Rubber-Insulated Cables** and flexible cords.
23	**Trolley and Contact Wire.**
115	**Metallic Resistance Materials** for electrical purposes.
125	**Hard-Drawn Copper Conductors** (circular, solid and stranded) for overhead power transmission purposes.
128	**Bare Annealed Copper Wire** for electrical machinery and apparatus.
156	**Enamelled** round **Copper Wires** (oleo-resinous enamel).
174–181	**Overhead Line Wire Material** (non-ferrous) for telegraph and telephone purposes.
182–184	**Galvanised Iron and Steel Wire** for telegraph and telephone purposes.
215	**Hard-Drawn Aluminium and Steel-Cored Aluminium Conductors** for overhead power transmission purposes.
446	**Braided Cables** with copper conductors, for overhead transmission lines.
480	**Impregnated Paper-Insulated Cables,** for electricty supply.
608	**Varnished Cambric Insulated Cables** for electricity supply.
672	**Hard-drawn Cadmium Copper Conductors** (circular, solid and stranded) for overhead power transmission purposes.
708	**Trailing Cables** for **Mining Purposes.**
760	**Paper-insulated Cables** for use in **Mines.**
837	**Steel-cored Copper Conductors** for overhead transmission purposes.
883	**Cables** and **Flexible Cords** for electrical equipment of **Ships.**
937	**Flexible Cords** for **Miners' Cap Lamps.**
977	**Flexible Cables** for Power and Control Circuits of **Electric Lifts.**
1116	**Flexible Trailing Cables** for **Quarries and Metalliferous Mines.**
1117	**Fine Resistance Wire** for **Telecommunications** and similar purposes.
1231	**P.V.C. Cables and Cords** for switchboard panel wiring.
1258	**Textile-covered** bunched enamelled copper wire **Conductors.**
1327	Insulated asbestos roved **Flexible Cords.**
1497	Impregnated asbestos-covered solid **Copper Conductors.**
1557	**Polythene-Insulated Cables** sheathed with P.V.C. for electric power and lighting up to 250 V.
1791	Cotton-covered round **Copper Wires.**
1815	Enamel (oleo-resinous) and cotton-covered round **Copper Wires.**

B.S.	
1844	Enamelled round **Copper Wires** (enamel with vinyl acetal base).
1862	**Cables for Vehicles.**
1933	Varnish-bonded, glass-covered round **Copper Wires.**
2004	**P.V.C. Insulated Cables** and **Flexible Cords** for electric power and lighting.
2008	**Wire Armoured Cables** (vulcanised rubber insulated) for collieries.
2479	Enamel (oleo-resinous) and Silk or Rayon-covered round **Copper Wires.**
2480	Silk or Rayon-covered round **Copper Wires.**

ELECTRIC LAMPS

161	**Tungsten Filament** General Service **Electric Lamps.**
469	**Electric Lamps** for **Railway Signalling.**
535	**Miners' Lamp Bulbs.**
555	**Tungsten Filament Electric Lamps** (other than General Service Lamps).
867	**Traction Lamps** (Series Burning).
941	**Electric Lamp Bulbs for Automobiles** (6 V. and 12 V. Bulbs for Head, Side and Rear Lamps).
1015	**Exciter Lamps for 35 mm.** projectors.
1050	**Visual Indicator Lamps.**
1075	**Studio Spotlight Lamps.**
1522	Schedule of **Projector Lamps.**
1546	**Lighthouse Lamps** (electric).

ELECTRIC HEATING APPLIANCES

744	Testing of **Electric Boiling-Plates** for domestic purposes.
843	Thermal-storage **Electric Water Heaters.**
1315 (Part 1)	Domestic **Electric Ovens.**
1326	**Electric Wash-boilers.**
1556	**Electric Immersion Heaters** for domestic hot water supply.
1670	Safety requirements for **Electric Fires.**
1671	Domestic **Electric Kettles.**
1732	Domestic **Electric Irons.**
1789	**Electric Heating Pads.**
2607	**Electric Coffee Percolators** and Brewers for domestic use.
2608	**Electric Toasters** for domestic use.
2612	**Electrically Heated Blankets.**

FUSES

88	**Electric Fuses** for circuits of voltage-ratings up to 660 V.
646	Ordinary-duty 250 V. **Cartridge Fuses** (rated up to 5 A.) for A.C. and D.C. service.
1361	**Cartridge Fuses** for domestic consumers' units.
1362	**Cartridge Fuse-links** for use in plugs.
2692	**Fuses for A.C. circuits** above 660 V.

BATTERIES AND CELLS

397	**Leclanché-type Primary Cells** and **Batteries.**
440	**Stationary Accumulators** (lead-acid Planté positive type) for general electrical purposes.
454	**Lead-Acid Train Lighting Accumulators** (Planté and Faure type).
683	**Alkaline Train Lighting Accumulators.**

B.S.	
966	**Batteries** for valve-type hearing aids.
1335	Air-depolarised **Primary Cells.**
1766	**Dry Batteries** for domestic radio receivers.
1975	**Primary Cells** and **Batteries** for intrinsically safe bell signalling circuits in coal mines.
2550	**Lead-acid Traction Batteries** for battery electric road vehicles.

CAPACITORS

1082	**Fixed Capacitors.**
1650	**Capacitors** for connection to power-frequency systems.

NOMENCLATURE AND SYMBOLS

108	**Graphical Symbols** for general electrical purposes.
205	**Glossary of Terms** used in electrical engineering.
1991	**Letter Symbols, Signs and Abbreviations.** Part I. General.

INDEX

A

Absolute units. See C.g.s. electromagnetic system of units.
A.C., 38, 139–42, 272–3
A.C. generator, 269–73
Accumulators. *See* Secondary cells.
Acid, 147, 167
Acid cells. *See* Secondary cells.
Alkali, 167
Alkaline cells. *See* Secondary cells.
Alloys, composition of, 320
"Alni", 237
Alternating current, 38, 139–42, 272–3
Alternator, 273
Aluminium, extraction of, 161
Ammeter, 25, 64, 65
—— and voltmeter method of measuring resistance, 66–8
——, hot-wire, 134
——, moving-coil, 65, 255–7
——, moving-iron, 65, 239, 257–8
Ampere, 25, 30–1, 242, 281
Ampere-hour, 37–8, 179
—— efficiency, 192–3, 201
Ampere-second, 37
Ampere-turn per metre, 243
Ampere-turns, 228, 257
Annealed conductors, 52–3
Anode, 156
Applied p.d., 82
Arc, carbon, 117–18, 123–6, 137–9
—— furnace, 118–20
—— lamps, 124–6
—— welding, 122–4
Armature of d.c. machine, 253–5, 270, 274–5
—— of electric bell, 235–6
Atom, 11, 163–8
Atomic weight, 175–6
Attraction type of M.I. instrument, 239, 257
Automatic cut-in and cut-out, 237

B

Back e.m.f. in electrolysis, 157–8
—— —— in motors, 279
Ballistic galvanometer, 268–9
Bare conductors, 52–3
Base, 147, 167
Batteries, 24–5, 83–5
——, stationary, 184–7
Bayonet type of lamp cap, 129
Bell, electric, 235–6
Board of Trade Unit, 105
Boiling plate, 114–16
Box, Post Office, 74–6
——, standard resistance, 62–4
Bridge, metre, 71–4
——, Wheatstone, 67, 69–71
British standards, 324–6
—— Thermal Unit, 17, 108
Brush e.m.f., 274
Brushes, 270–1, 273, 275

C

Cables, 52–3, 324–5
Cadmium cell, Weston standard, 153–4
—— copper, 54
Calorie, 17, 108, 110–11
Capacitance, 294, 297–300, 306
——, electrostatic unit of, 306
—— in a.c. circuits, 301
—— in d.c. circuits, 300–1
—— of parallel plate condenser, 299–300, 306–7
——, practical unit of, 294
Capacitor. *See* Condenser.
Capacity of an accumulator. *See* Secondary cells.
Carbon arc, 117–18, 123–6, 137–9
—— composition type of resistor, 58
—— filament lamp, 126–7
—— plate type of rheostat, 61–2
Cathode, 156
Cell, 23–4
——, capacity of. *See* Secondary cells.
——, dry, 151–3
——, efficiency of. *See* Secondary cells.
——, e.m.f. of, 156, 189–90, 198
——, Leclanche, 151–3, 156, 172
——, polarisation of, 150
——, primary, 150–4
——, internal resistance of, 78–85, 192, 200–1
——, reversible, 180
——, secondary. *See* Secondary cells.
——, simple voltaic, 148–9, 156, 168–71
——, Weston standard cadmium, 153–4
Cells connected in parallel, 83–5
—— —— in series, 24–5, 83–5
——, mixed grouping of, 84–5
Centigrade Heat Unit, 17, 108
C.g.s. electromagnetic system of units, 13, 241–2, 311–12
—— electrostatic system of units, 13, 241, 304–6

Charges, electric, 291–4, 302, 305–6
Charging of cells. *See* Secondary cells.
Chemical equivalent, 159
—— reactions, 147–8
—— —— in alkaline cell, 206–8
—— —— in lead-acid cell, 203–6
Chromium plating, 162
Cinema projection arc lamp, 125–6
Circuit breaker, 237
Circular coil, field due to a, 217–19, 227, 244, 312–16
Clockwise rule for polarity of a solenoid, 219–20
Coil and plunger principle, 238–9
Coiled-coil lamps, 127–8
Commutator, 253–4, 273–4
Comparison of e.m.f.'s, 156
—— of motor and generator, 278–9
Composition of alloys, 320
Compound-wound d.c. machine, 277–8
Compounds, 13
Condenser, 289–301
Condensers, capacitance of, 294, 297–300
——, electrolytic type, 296
——, fixed, 295–6
——, in parallel, 297–8
——, in series, 298–9
——, Mansbridge type, 295
——, practical, 295–8
——, use of, 300–1
——, variable capacitance, 296–7
Conductance, 42–4
Conductivity, 51, 321
Conductor, carrying current in a magnetic field, 248–60, 317–18
——, circular, magnetic field, due to 217–19, 227, 312–16
——, moving in a magnetic field, 263–83
——, straight, magnetic field due to, 216–17, 226–7, 312–14
Conductors, 324–5
—— and insulators, 22
——, force between parallel, 258–60
——, gaseous, 91
——, liquid, 191
——, stranded, 52–3
Conservation of energy, law of, 107, 280
Constantan, 56, 154, 319–22
Construction of resistors, 56–64
Contact p.d.'s, 169
—— resistance, 91
Controlling torque, 255, 258
Copper-plating, 157, 160, 172–4
Corkscrew rule for solenoids, 219
—— —— for straight conductors, 217–18
Cost of electrical energy, 105–7
Coulomb, 25, 30, 179, 242, 291–4, 305–6
Crest value, of an alternating current, 141–2
Crystal structure, 164
Current efficiency, 162
——, measurement of, 25
——, strength, 25, 30–31, 33–8
——, units of, 25, 241, 281, 311–12
Currents, alternating, 38, 139–42, 272–3
——, induced, 261–83
Cut-in and cut-out, automatic, 237
Cycle, 38
Cyclic state, 193

D

D.C., 38
—— generator, 273–80
—— motor, 248–55, 277–9
Demagnetisation of a ferro-magnetic material, 228–31
Density, 18
Depolariser, 150
Dial type of standard resistance box, 63–4
Dielectric, 294
—— constant. *See* Permittivity.
Direct-current, 38
—— —— generator, 273–80
—— —— motor, 248–55, 277–9
Domains, 240
Drumm cell, 196
Dry cell, 151–3
Dynamo effect, 263–83
Dynamometer wattmeter, 260
Dyne, 14

E

Edison cell, 196–202, 206–8
—— screw cap, 129
Effective value, of an alternating current, 141–2
Efficiency, current, 162
—— of cells. *See* Secondary cells.
—— of d.c. machines, 280
—— of lamps, 124, 128
Electric arc, 117–18, 122–6, 137–9
—— —— heating, 112, 117–20
—— bell, 235–6
—— —— indicator, 236
—— charge, 291–4, 302, 305–6
—— circuit, 22–39
—— —— breaker, 237
—— current, 22, 25, 30–1, 33–8
—— field, 301–4
—— —— strength, 302, 305
—— flux density, 305
—— force, 302, 305
—— ——, line of, 303–4, 306
—— furnaces, 116, 118–20
—— fuse, 130–33
—— generators, 269–80
—— heating appliances, 111–20, 325
—— lamps, 124–30, 325

Electric lines of force, 303–4
—— motor, 248–55, 277–9
—— welding, 120–24
Electrical energy, 27–30, 105–42
—— power, 31, 105
Electro-chemical equivalent, 159–60, 322
—— series of elements, 169, 323
Electro-chemistry, 147–76
Electrode type of salt bath, 112–13
Electrodes, 156
Electrolysis, 156–63, 172–6
——, back e.m.f., during, 157–8
——, laws of, 159, 175–6
—— of copper sulphate, 157, 160, 172–4
—— of water, 158, 174–5
Electrolytes, 91, 147, 163–8
Electrolytic cell, 156
—— condenser, 296
—— extraction of metals, 160–61
—— manufacture of chemicals, 161–2
—— refining of metals, 160–61
Electromagnet, 232–9
Electromagnetic induction, 261–83
—— medium, 225
—— relay, 236
—— system of units, 13, 241–2, 311–12
Electrometer, 287–9
Electromotive force, 23–4, 30–1, 154–6, 189–90, 198
—— —— induced, 261–283
—— ——, measurement of, 154–6
Electron, 11
—— drift, 25
——, free, 22
——, sharing, 166–7
——, transfer, 166–7
Electronic theory of magnetism, 239–40
Electroplating, 162
Electroscope, 287–9
Electrostatic field. *See* Electric field.
—— force, 286–7
—— voltmeter, 287–9
Electrostatics, 286–307
Electro-type, 162–3
Elements, 11
E.m.f. *See* Electromotive force.
——, back, 157–8, 279
—— of cells, measurement of, 154–6
Energy, 16, 27–30, 105–42
—— conservation of, 107, 280
—— costs, 105–7
—— in charged condenser, 292
Engraving by electrolysis, 163
Equivalent of heat, electrical, 107–11
—— weight, 159
Erg, 15
" Ether," 225–6
Eureka, 51. *See also* Constantan.
Excitant, 151
Excitation, separate, of d.c. generator, 276–7
Extraction of metals by electrolysis, 160–61

F

Farad, 294
Faraday, 159, 261
Faraday's laws of electrolysis, 159, 175–6
Faure accumulator plate, 180–3
Ferro-magnetic materials, 211
Field coils, 275
—— current, 276
——, electric, 301–4
—— magnet, 233
——, magnetic, 214
——, strength, 229
—— windings, 275
Fields, magnetic, 211–45
Filament lamps, 126–30
Firebar, electric, 114
Fixed condensers, 295–6
—— resistors, 56–9
Flat rate system, 107
Fleming's left-hand rule, 251–2, 279–80
—— right-hand rule, 264–6, 279–80
Flux density, magnetic, 222–3
——, magnetic, 222, 268–9
Flux-linkages, 261, 269
Fluxmeter, 222
Foot-pound, 15
Force, 14
—— between parallel conductors, 258–60, 281
——, moment of, 14
—— on conductor, 248–60, 317–18
——, units of, 14
Formed accumulator plate, 180–82
Free electron, 22
Frequency, 39, 273
Furnace, arc, 118–20
——, resistance, 116
Fuse, electric, 130–33, 325
—— element, 131
Fusing currents, table of, 133
—— factor, 131

G

Galvani, 149
Galvanic cell, 148–9
Galvanometer, 65–6
——, ballistic, 268–9
Gas-filled lamp, 127–30
Gaseous conductors, 91
Gauss, 223, 244
Generator, direct-current, 273–80
German silver. *See* Nickel silver.
Gram-calorie. *See* Calorie.
Gravity control, 258

H

Hard-drawn conductors, 52
Heat energy, 16–18, 107–42
——, electrical equivalent of, 107–11
—— generated in a circuit, 107–11
——, quantity of, 17, 107–11
——, specific, 17–18, 319
——, units of, 17
Heating appliances, 111–20, 325
—— effect of a current, 108
—— ——, applications of the, 111–39
—— —— with alternating current, 139–42
—— element, 112
—— resistor, 111
Horse-power, 16
Hot-wire ammeter, 134
Hydrometer, 189

I

Impedance, 68
Incandescent filament lamps, 124–30
Induced e m f's, 261–83
—— magnetism, 228–31
Inductance, self, 68
Induction density, 229
——, electro-magnetic, 261–83
—— heating, 112, 116–17
——, magnetic, 229
——, self, 76
Instantaneous value of an alternating current, 140
Instruments, measuring, 24–7, 64–8, 239, 255–8
Insulating materials, 22
Insulation resistance, 91
Insulators, 22, 52, 185
Internal resistance of cells, 78–85, 192, 197–8, 200–1
—— —— of cells, measurement of, 82–3
Inverse square law for magnetic poles, 244–5
Ionic theory, 163–76, 203–8
Ionisation, 22
Ions, 22, 163–8
" Ironclad " accumulator plates, 181–3

J

Joule, 15–16, 29–30, 105, 108, 110–11, 242
Jungner cell, 196–202, 206–8

K

Kilo-calorie, 17
Kilocycle, 39
Kilohm, 38
Kilovolt, 37–8
Kilovolt-ampere, 107
Kilowatt, 16, 105–7
Kilowatt-hour, 38, 105–6, 108
Kirchhoff's laws, 85–90

L

Lamps, 124–30, 325
——, electric arc, 124–6
——, —— filament, 126–30, 325
Laplace's law, 310–11
Law, Lenz's, 264–7
—— of conservation of energy, 107, 280
—— of inverse squares for magnetic poles, 244–5
——, Ohm's, 34
Laws, Kirchhoff's, 85–90
—— of electrolysis, Faraday's, 159, 175–6
Lead–acid cells *See* Secondary cells
Leclanche cell, types of, 151–3, 172
Left-hand rule, Fleming's, 251–2, 279–80
Lenz's law, 264–7
Lifting magnet, 234
Linear resistor, 136
Lines of force, electric, 303–4
—— ——, magnetic, 213, 216
Linkages, flux-, 261–9
Liquid conductors, 91
Load characteristics of accumulators, 189–90, 198–9
—— curves, 105–7
Local action, of cells, 149–50, 195–6
Lumen, 124
Luminous efficiency of electric lamps, 124, 128

M

Magnet, electro-, 231–9
——, lifting, 234
——, permanent, 211–13, 230
——, poles of a, 211–12
——, properties of a, 211–13
Magnetic axis, 211
—— brakes, 234
—— chuck, 234
—— clutch, 234
—— effect of a current, 216
—— field, 214
—— —— strength, 229
—— fields, 211–45
—— —— in the vicinity of various current-carrying conductors, 216–21, 249–50, 260
—— flux, 222, 268–9
—— —— density, 222–5, 229, 281
—— force, 216, 223–8, 248–60, 310–18
—— ——, lines of, 213, 216
—— induction, 229
—— meridian, 216
—— pole, 211–13

Magnetic pole strength, 213
—— ——, unit, 213, 245
—— retentivity, 230, 234
—— separator, 235
—— units, 213, 216, 222–3
Magnetisation, 228–31
Magnetising force, 216, 223–9, 310–17
—— ——, calculation of, 312–17
Magnetism, induced, 228–31
——, theory of, 239–40
Manganin, 56, 58, 63, 255, 319–22
Mansbridge condenser, 295
Mass and weight, 13
Maximum demand system, 105–7
—— value of an alternating current, 141
Maxwell, 222, 244
Maxwell's corkscrew rule, 217–18
Maxwell-turns, 222
Measurement of resistance, 64–78
Megacycle, 39
Megawatt, 38
Megohm, 38
Metal filament lamps, 127–30
Metre bridge, 71–4
Mho, 42
Microampere, 37–8
Microfarad, 294
Microhm, 38
Micro-micro-farad, 294
Microvolt, 38
Milliampere, 37–8
Millivolt, 38
M.k.s. system of units, 13
Molecular theory of magnetism. *See* Electronic theory of magnetism.
Molecule, 12, 164–8
Moment of a force, 14
Moments, principle of, 15
Monel metal, 54
Motor, d.c. electric, 248–55, 277–9
—— effect, 248–60, 317–18
Moving-coil instrument, 65–6, 255–7
Moving-iron instrument, 65–7, 239, 257–8
Multiple and sub-multiple units, 37–8
Multiplier, for voltmeter, 255

N

Networks, 69, 85–90
Neutron, 11
Newton, 14, 242
Nichrome alloy, 51, 54, 56, 95, 96, 113–14, 116, 135–7, 319–22
Nickel–cadmium cell. *See* Secondary cells.
Nickel–iron cell. *See* Secondary cells.
Nickel plating, 162
—— silver, 71, 154, 319–22
" Nife " cell. *See* Jungner cell
Nomenclature, 326
Non-conductors, 22
Non-electrolytes, 166–8
Non-linear resistor, 136–7, 139

O

Oersted, 216, 243
Ohm, 35, 242
Ohm's Law, 34
—— —— formula, 34–7
Open circuit terminal p.d., 80

P

Paper, as dielectric, 289, 295
Paraffin wax, 290
Parallel conductors, force between, 258–60, 281
—— connection, of cells, 83–5
—— ——, of condensers, 297–8
—— ——, of resistors, 43–6
—— connections, 26
—— plate condenser, 299–300, 306–7
Paramagnetic substances, 240
Pasted plates, 181–3
P.d. *See* Potential difference.
Permanent magnets, 230–31
Permeability, 223, 229, 233
" Permendur," 238
Permittivity, 168, 300, 305
——, relative, 300, 305
Phosphor bronze, 255, 258, 320–22
Pico-farad, 294
Plante plate, 180–81, 184
Platinoid, 71, 154, 319–22
Plug type of standard resistance box, 63
Polarity, electrical, 24
Polarisation of cells, 149–50
Pole, magnetic, 211–13, 244–5
—— ——, unit, 213, 245
—— strength, magnetic, 213
Portable accumulators, 187–8, 203
Post Office box, 74–6
Potential, 86–7, 302
—— difference, 23, 26, 29–38, 286–9, 292–4, 306
—— ——, applied, 82
—— ——, electromagnetic unit of, 241
—— ——, practical unit of, 26, 242
—— divider or slide, 61
—— gradient, 302
Potentiometer, 154–6
—— rheostat, 61
Power, 16, 31, 105
—— developed by machines, 279–80
Practical units, 13, 241–2
Primary cells, 150–54, 168–72
—— ——, internal resistance of, 78–85
Principle of moments, 15
Proton, 11

Q

Quantity of electricity, 25, 30, 159–60, 175–6, 268–9, 294
—— —— ——, measurement of, 268–9
—— —— ——, unit of, 25, 30, 37–8, 240–1
—— —— heat, 17
—— —— ——, units of, 17, 108, 110–11

R

Radial pattern of rheostat, 60
Rateable value tariff, 107
Rationalisation, of systems of units, 243
Rationalised m.k.s. system of units, 242–5, 281–2, 306–7
Reactor, 123
Relay, electromagnetic, 236
Repulsion type of moving-iron instrument, 257–8
Resistance, 34–8, 42–99
—— at contact surfaces, 91
—— box, dial type, 63–4
—— ——, plug type, 62–3
—— furnace, 116
—— heating, 111–16
——, insulation, 91
——, internal, of cells, 78–85, 192, 197–8, 200–1
——, measurement of, 64–78
—— ——, using ammeter and voltmeter, 66-8
—— ——, —— metre bridge, 71–4
—— ——, —— P.O. box, 74–6
—— ——, —— substitution method, 69
—— ——, —— Wheatstone bridge, 67, 69–71
—— of stranded conductors, 53
—— —— electrolytes, 91
—— —— gases, 91
——, specific. *See* Resistivity.
——, temperature coefficient of, 53–6, 77-8, 92–4, 322
—— thermometers, 96–9
——, units of, 35, 38, 241–2
——, variation with temperature, 53–6, 77–8, 92–6
—— welding, 120–22
Resistivity, 49–52, 76–7, 321
——, dimensions of, 92
Resistor, 42
Resistors, carbon composition, 58
——, construction of, 56–64
——, fixed, 56–9
—— in parallel, 43–6, 78
—— —— series, 42–3, 78
——, linear, 136
——, mixed grouping of, 46–9
——, non-linear, 136–7, 139
——, standard, 58–9
Resistors, variable, 59–64
——, vitreous enamelled, 57–8
——, wire-wound, 56–7
Retentivity, 230, 234
Reversible cells, 180
Rheostat, carbon plate type, 61–2
——, potentiometer, 61
——, radial pattern, 60
——, stud type, 60
——, tubular type, 59–60
Rheostats, 59–62
Right-hand rule, Fleming's, 264–6, 279-80
Root-mean-square value of an alternating current, 141–2
Rotational effect of a force, 14–15

S

Salt, 147
—— bath, electrode type, 112–13
Screw type of lamp cap, 129
Search coil, 222
Secondary cells, 150, 178–208, 325–6
—— ——, acid, 178–96, 200–6
—— ——, —— and alkaline compared, 200–3
—— ——, alkaline, 196–203, 206–8
—— ——, applications of, 202–3
—— ——, capacity of, 179, 190–92, 200–2
-, care of, 194–6, 199-200, 202
—— ——, charge and discharge curves, 189–90, 198–9
—— ——, charging of, 194–6, 199–200
—— ——, chemical reactions in, 203–8
—— ——, construction of, 180–87, 196–7
—— ——, efficiency of, 192–3, 200–1
—— ——, e.m.f. of, 189–90, 198
—— ——, internal resistance of, 78–85, 192, 197, 200–1
—— ——, specific gravity of electrolyte of, 188–9, 196, 198
—— ——, terminal p.d. during charge and discharge of, 189–90, 198–9
Self-inductance, 68
Separately-excited generator, 276–7
Series circuit, 25–6
—— connection, of cells, 24–5, 83–5
—— ——, of condensers, 298–9
—— ——, of resistors, 42–3
—— resistance for voltmeters, 255
—— wound d.c. machine, 277
Series-parallel connections, 46–9
Short-circuited cells, 81
Shunt for ammeters, 255
—— wound d.c. machine, 277
Silver plating, 162
Simple voltaic cell, 148–9, 156, 168–71
Sine wave, 272–3
Single-loop generator, 269–74

Slip rings, 270, 273
Solenoid, magnetic field due to, 218–21, 227–8, 244
Specific gravity, 18, 319
—— ——, of electrolyte, 188–9, 196, 198
—— heat, 17–18, 319
—— inductive capacity. *See* Permittivity.
—— resistance. *See* Resistivity.
Spring control, 255, 258
"Stalloy," 238
Standard cell, 153–4
—— resistance boxes, 62–4
—— resistors, 58–9
—— symbols, 36–7, 323, 326
—— wire gauge equivalents, 320
Standards, British, 324–6
Stationary batteries, 184–7, 202
Storage cells. *See* Secondary cells.
Straight conductor, field due to a, 216–17, 226–7, 244, 248–50, 313–14
—— ——, force experienced by a, 248–50, 281, 317–18
Stranded conductors, 52–3
Stud type of rheostat, 60
Sub-multiple and multiple units, 37–8
Substitution method of resistance measurement, 69
Sulphation of acid cells, 195
Symbols, 36–7, 323, 326

T

Tariffs, supply, 107
Telephone receiver, 237–8
Temperature coefficient of resistance, 53–6, 77–8, 92–4, 322
—— rise, 17
Terminal p.d. of cell, 79–83
Therm, 17
Thermal capacity, 18
—— radiator, 124
Thermocouple, 99
Thermometer, resistance, 96–9
Toroidal coil, 220, 227–8, 243
Torque, 14–15, 254–8, 277–9
Tractive force of electromagnet, 234
Tubular type of rheostat, 59–60
Tungsten filament lamp, 127–30
Turning moment. *See* Torque.

U

Unit, Board of Trade, 105
——, capacitance, 294
——, magnetic flux, 222, 244, 282
Unit, magnetic flux density, 223, 244, 281
——, —— pole, 213, 245
——, magnetising force (or magnetic force), 216, 243
Units, c.g.s. electromagnetic, 241–2
——, —— electrostatic, 13, 241, 304–6
——, multiple and sub-multiple, 37–8
——, practical, 13, 241–2, 294
——, rationalised m.k.s. 242–5, 281–2, 306–7

V

Vacuum lamp, 126–7
——, permeability of, 223, 242–3
——, permittivity of, 242–3, 305, 307
Valency, 164–6, 175–6
Variable condensers, 296–7
—— resistors, 59–64
Variation of resistance with temperature, 53–6, 77–8, 92–6
Virtual value, of an alternating current, 141–2
Vitreous enamelled resistors, 57–8
Volt, 26, 29–31, 242, 267
Volta, 149
Voltage drop, 26
Voltaic cell, 148–9, 156, 168–71
Voltameter, 160
Voltmeter, 26, 64–5
——, electrostatic, 287–9
——, moving-coil, 65–6, 255–7
——, moving-iron, 65–7, 239, 257–8
Volt-second, 282

W

Water equivalent. *See* Thermal capacity.
Watt, 16, 31, 242
Watt-hour, 38
—— efficiency, 192–3, 201
Wattmeter, dynamometer type, 258
Watt-second, 105
Weber, 244, 282
—— per sq. metre, 244, 281
Weight, 13
Welding, electric, 120–24
Weston standard cadmium cell, 153–4
Wheatstone bridge, 67, 69–71
Wimshurst machine, 303
Wire gauges, 320
—— tables, 52
Wire-wound resistors, 56–7
Wires, 324–5
Work, 15–16

Editor General: **W. E. FISHER, O.B.E., D.Sc.**

THE
TECHNICAL COLLEGE SERIES

The following titles have already been published and others are in preparation.

NATIONAL CERTIFICATE MATHEMATICS

Published in three volumes covering the First, Second and Third Years' Syllabus.

By **P. ABBOTT,** B.A., **C. E. KERRIDGE,** B.Sc., **H. MARSHALL,** B.Sc., and **G. E. MAHON,** B.Sc.

Vol. I. 8*s*. 6*d*. net. Vol. II. 8*s*. 6*d*. net.
Vol. III. 9*s*. 6*d*. net.

Designed to provide a systematic and progressive textbook in Mathematics for students taking Engineering Courses, Mechanical or Electrical, in a Technical Institute. The three volumes are planned to correspond with the work which is usually done in the first three years of the Senior Course, and endeavours have been made to provide a fundamental and theoretical basis such as is necessary for the study of Mathematics in more advanced work.

" Well-planned, systematic text books containing full and detailed explanations of each section of the syllabus."—*Schoolmaster.*

PRACTICAL ENGINEERING DRAWING

By **C. J. ATKINS**, Int.A.M.I.P.E., A.M.I.I.A.
Senior Lecturer, Mechanical Engineering Department,
Croydon Polytechnic,

and **G. B. WARD**
Croydon Polytechnic.
Engineering Drawing Master, Stanley Junior Technical School.

Vol. I. 12*s*. 6*d*. net. Vol. II. 12*s*. 6*d*. net.

"The authors have achieved the dual aim of combining instruction in elementary draughtsmanship with examples of recent practice."
Engineering.

" Can be recommended to the Machine Shop Engineering Student as well as The National Certificate Course Student."
Technical Education.

" It is an excellent production and can be highly recommended."
Higher Education.

" An excellent text-book for the student who wishes to gain a knowledge of the elementary methods of draughtmanship."
Society of Engineers.

GEOMETRICAL DRAWING

By **H. BINNS**, A.M.I.M.E., A.M.I.P.E.
Lecturer in Engineering Drawing, Halifax Municipal Technical College.

9*s*. 6*d*. net.

This volume includes the fundamental work in geometrical construction which is necessary for all forms of practical drawing.

" In this book the author gives us the fundamentals of plane geometry and the various types of projection. The drawings are clear and the text concise, while the short chapter on freehand sketching should prove most valuable." *Times Educational Supplement.*

EXAMPLES IN ENGINEERING DRAWING

By **H. BINNS**, A.M.I.M.E., A.M.I.P.E.

Lecturer in Engineering Drawing, Halifax Municipal Technical College.

Vol. I. 8*s*. 6*d*. net. Vol. II. 9*s*. 6*d*. net.
Vol. III. 8*s*. 6*d*. net.

" A highly commendable work." *Mechanical World.*

" The object of this book is to provide a set of drawing examples suitable for the homework of students taking the engineering course at technical colleges of the first year senior standard and also for students in junior technical schools. It is profusely illustrated by 52 examples. The author is an expert at his subject."

Society of Engineers.

WORKSHOP ENGINEERING: CALCULATIONS AND TECHNICAL SCIENCE

By **J. T. STONEY**, B.Sc.(Eng.)

Lecturer at the Wolverhampton and Staffordshire Technical College.

Vol. I. 11*s*. 6*d*. net. Vol. II. 12*s*. 6*d*. net.

A text-book in two volumes which will be suitable for the first and second years of the Normal Workshop Courses.

" This is a really excellent text-book which should meet the requirements of young students, but may even be recommended to the teachers in whose care they will be placed."

Electrical Review.